CONDUCTING AN INDEPENDENT-SAMPLES T-TEST

Example: Comparing Attractiveness Ratings

To test our hypothesis, we randomly assign 4 participants to view a filtered photo of someone and 4 participants to view an unfiltered photo of someone.

We then have them rate how attractive they think that person is on a scale from 1 to 10, with 10 being very attractive.







Filtered Photo Group

Unfiltered Photo Group

Subject	Attractiveness	Subject	Attractiveness
1	9	5	5
2	8	6	6
3	6	7	5
4	9	8	4

Step 1: Check Assumptions

• The distributions for each sample are normal (no skew/outliers)



For this: We don't have a lot of data for this example, but in normal circumstances, you would create a histogram of the data to check for normality

- o The samples are independent of one another
 - The participants in one condition are not the same participants in the other condition (there is no effect of one sample on the other)



For this: **We have to know how our** sample was collected and know that we controlled for confounding variables to ensure the two samples are collected from different participants

- Homogeneity of variance
 - The variance of the populations (represented by each sample) are equal—this means that the spread or variability in scores around the population mean is the same in both samples

For this: We need to run a **Levene's Test** (if our data is less or not normally distributed) or a **Barlett's Test** (if our data is close to or normally distributed) to test whether the variances are the same

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Both **Levene's** and **Bartlett's Tests** <u>are</u> hypothesis tests. They test the null and alternative hypotheses that the variances are equal or not, respectively. (P.s., always two-tailed!)

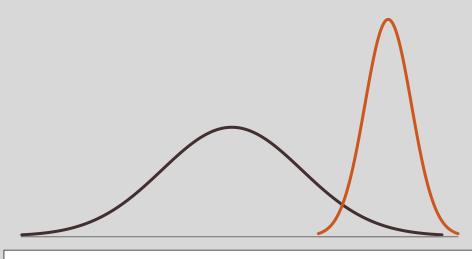
$$H_0: \sigma_1^2 = \sigma_2^2 = \dots \sigma_x^2$$

$$H_a: \sigma_1^2 \neq \sigma_2^2 \neq \dots \sigma_x^2$$

You will test whether you fail to reject the null hypothesis (homogenous/equal variances)

or

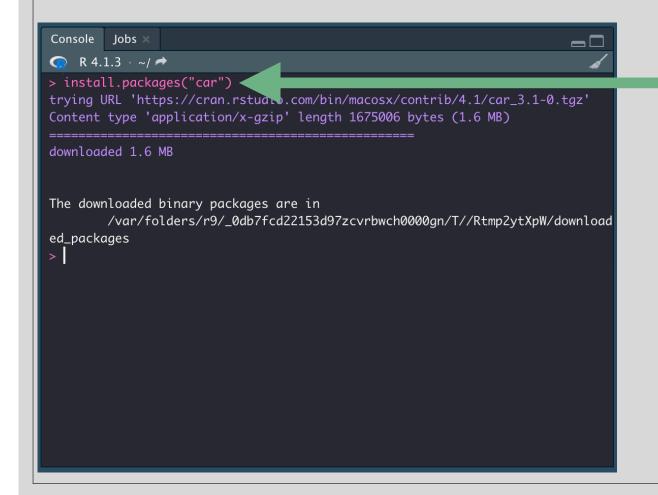
reject the null hypothesis (heterogeneous/unequal variances)



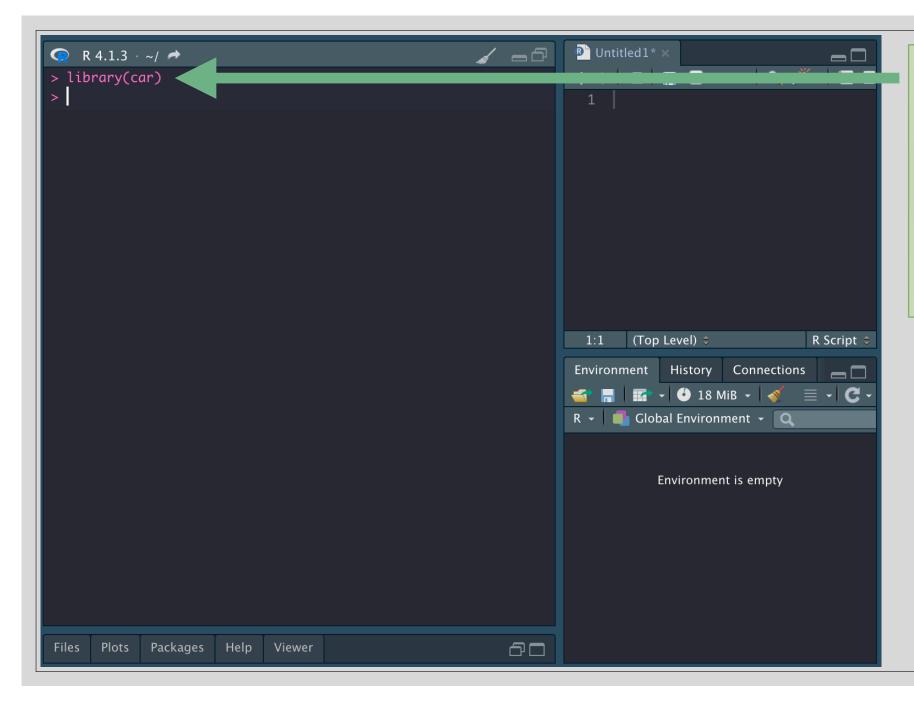
The variances of **Group 1** and **Group 2** are not equal if this were actually our distributions.

Step 1: Check Assumptions

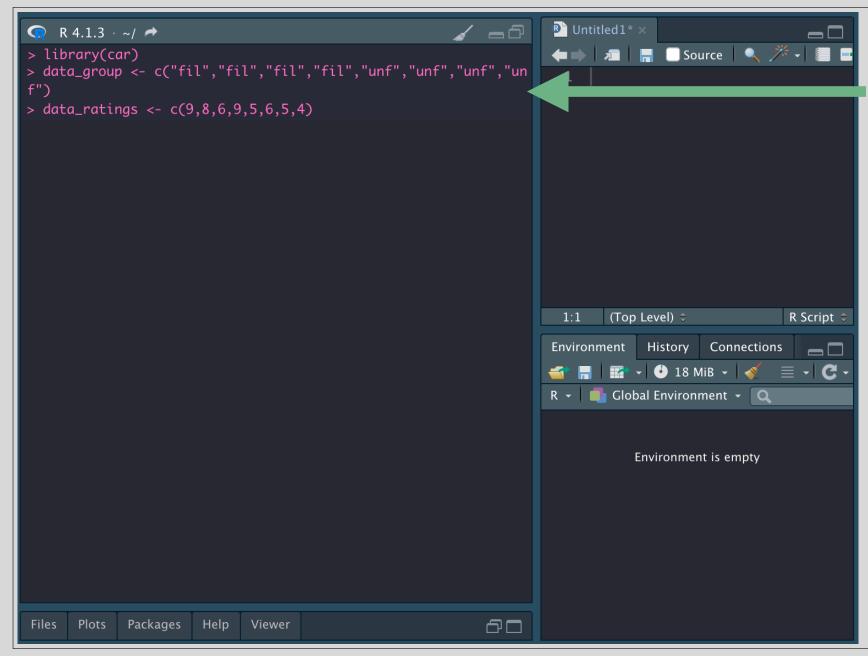
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Install the **car** package in R. Remember: You only ever need to install a package *once*.



Load the **car** package from your **library** in R. Remember: You need to do this *every* time you open RStudio and start coding a new script.



- You will do this just the same way as before using a name, assignment operator, and c().
- NOTE: R analyzes data in *long* format.
 This means that instead of having a column for each dataset (or condition), you need to have one column for your dependent variable (across datasets) and one column for your independent variable.

Let's say we wanted to conduct a study on how happiness is affected by the weather/season. So, on four selected days over the course of one year, we asked people to rate their happiness on a scale from 1 (really unhappy) to 10 (really happy).

Wide Format: A column for each level of the independent variable

Long Format: Multiple rows for each level of the independent variable

Wide Format

Rainy & Fresh Spring Day	Warm & Sunny Summer Day	Chilly & Bask Fall Day	Cold & Snowy Winter Day
5	7	6	2
6	8	2	1
4	9	5	3
3	10	7	4

Long Format

Happiness Rating
5
6
4
3
./
8
9
10
6
2
5
7
2
1
3
4

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3	10	7	4

Long Format

Season	Happiness Rating
Spring	5
Spring	6
Spring	4
Spring	3
Summer	7
Summer	8
Summer	9
Summer	10
rall	O
Fall	2
Fall	5
Fall	7
Winter	2
Winter	1
Winter	3
Winter	4

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Long Format

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Spring	4
Spring	3
Summer	7
Summer	8
Summer	9
Summer	10
Fall	6
Fall	2
Fall	5
Fall	7
Winter	2
Winter	1
Winter	3
Winter	4

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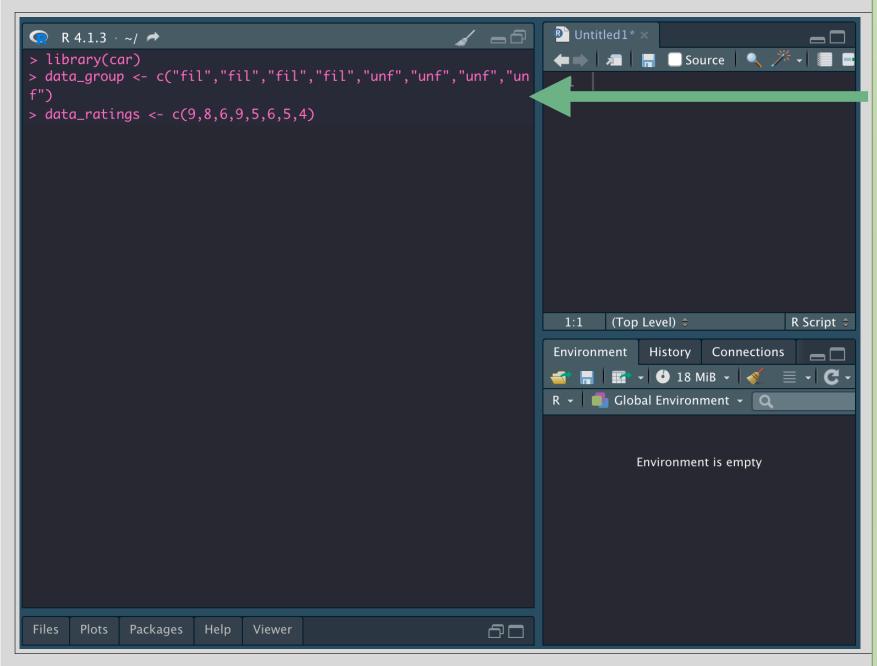
Wide Format

Rainy & Fresh Spring Day	Warm & Sunny Summer Day	Chilly & Brisk Fall Day	Cold & Snowy Winter Day
5	7	6	2
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4	9	5	3
3	10	7	4

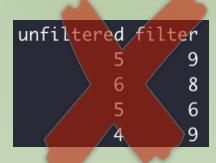
Long Format

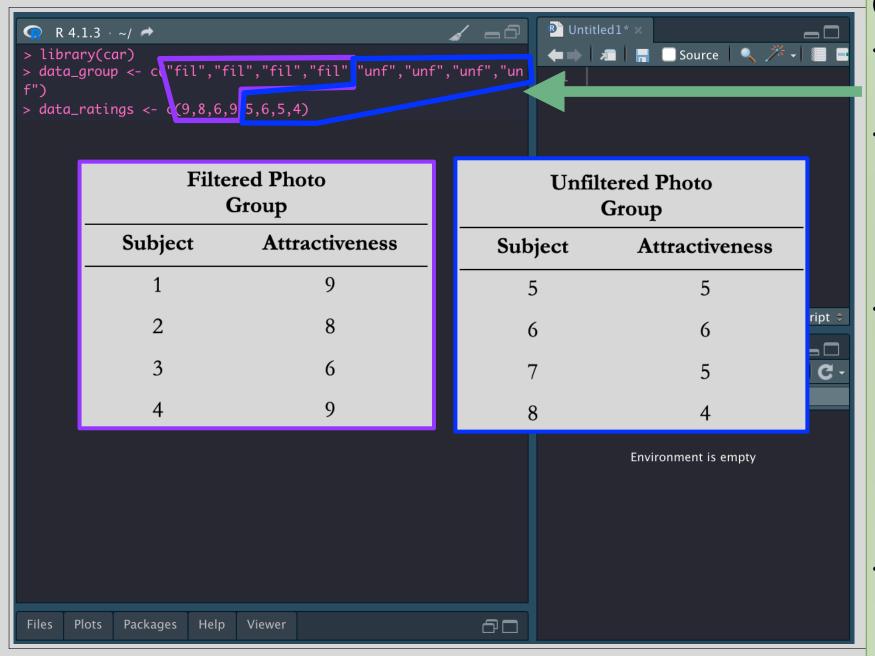
Season	Happiness Rating
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Spring	4
Spring	3
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Fall	6
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Fall	7
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Winter	1
Winter	3
Winter	4

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- Be careful about writing your code because you need to line up your values for your IV and DV.

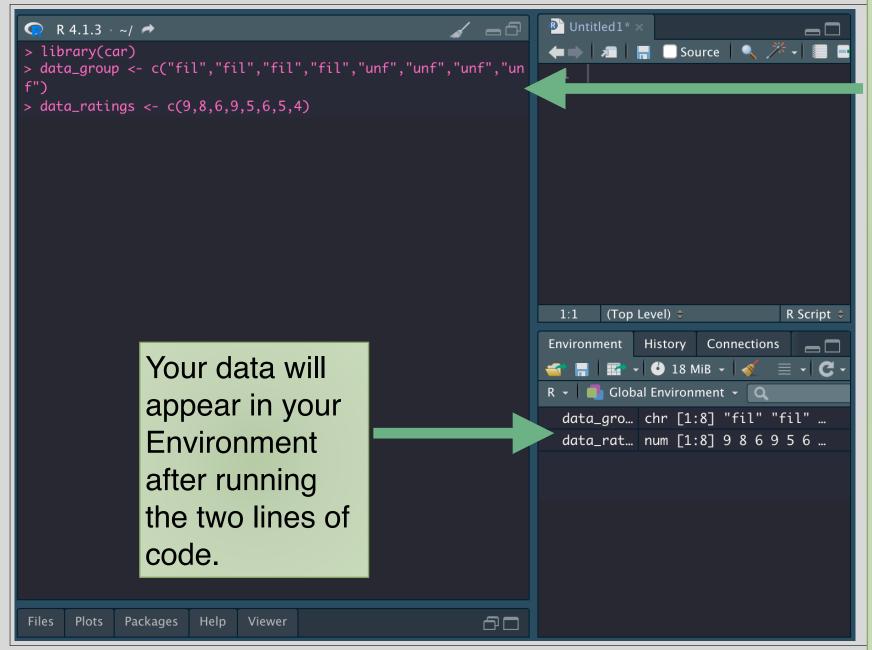




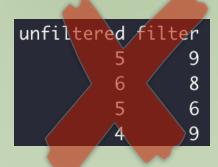
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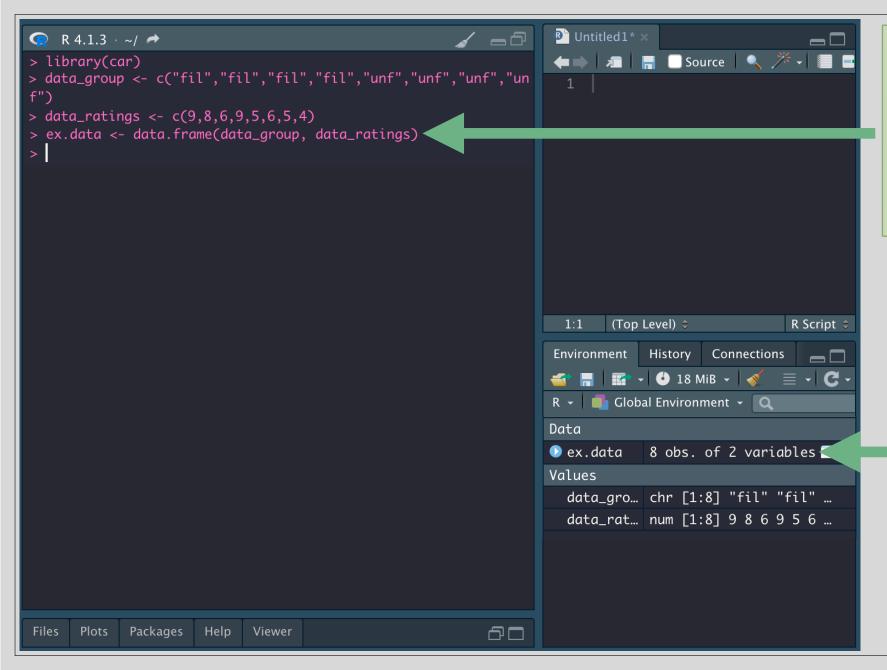
I decided to assign values for the
 Filtered group = "fil" and values for the
 Unfiltered group = "unf". The first four
 values listed are attractiveness ratings
 for Filtered and then Unfiltered.



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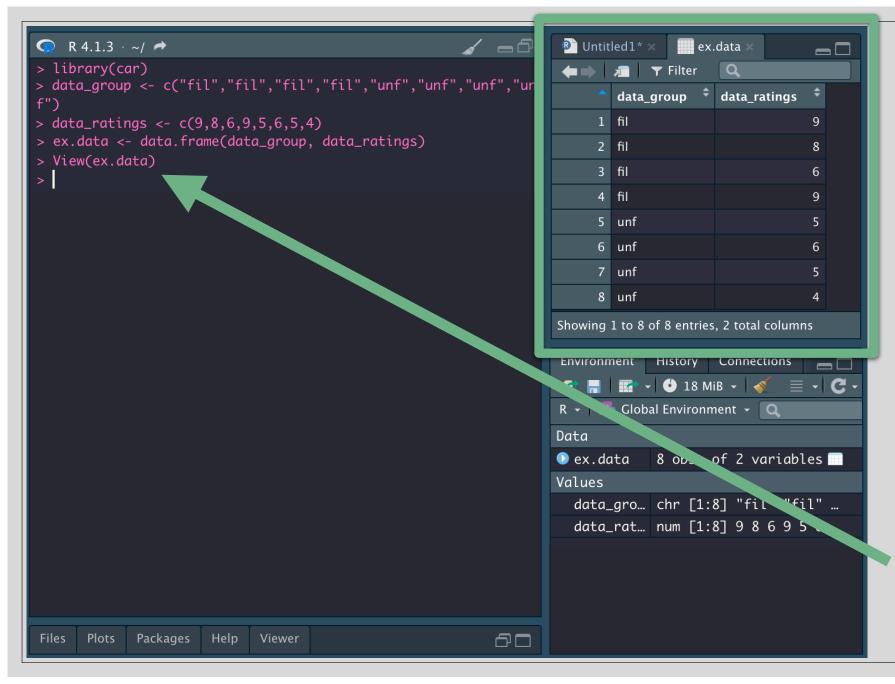
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Use data.frame() to create your combined data matrix.

 Type the names you assigned for your data in the parentheses in no particular order.

> Your data matrix will appear in your Environment after running the line of code.

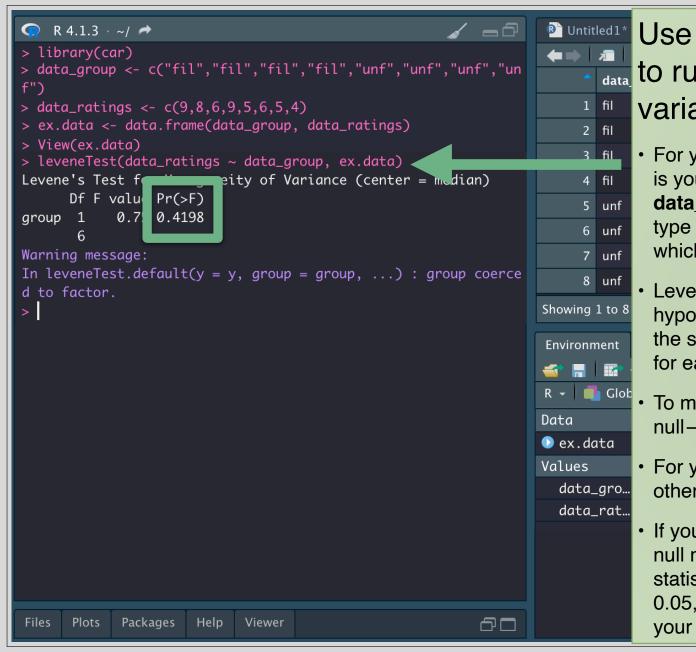


You can see now how the data is encoded into R (i.e., *long* format).

The data_group column corresponds to whether the score in data_ratings is from the filtered or unfiltered group.

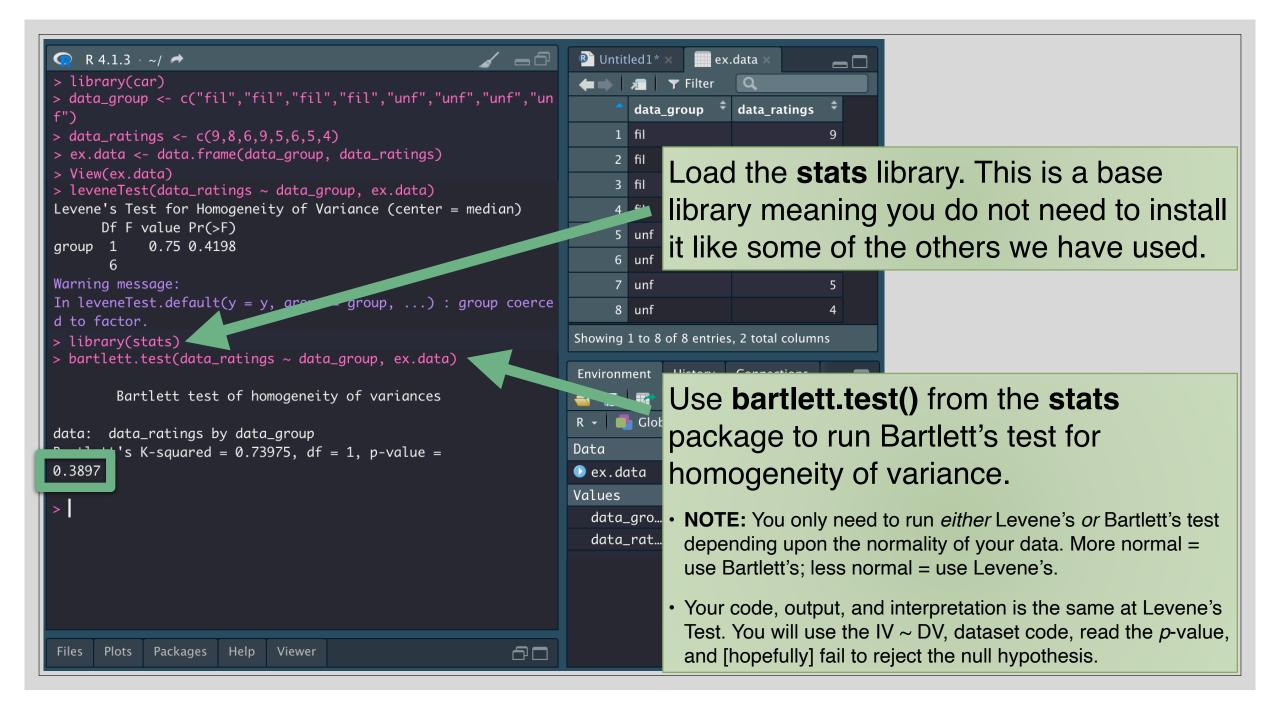
Use **View()** to see your data matrix.

 Type the name you assigned your data matrix in the parentheses.



Use **leveneTest()** from the **car** package to run Levene's test for homogeneity of variance.

- For your code, the first thing you will type in the parentheses is your dependent variable by (~) your independent variable:
 data_ratings ~ data_group. You will then use a comma and type the name of your data matrix to indicate to the function which data to run the code.
- Levene's Test is similar to a t-test in that you have a null hypothesis [that the variances for each of your samples are the same] and an alternative hypothesis [that the variances for each of your samples is different].
- To meet the assumption criteria, you need to fail to reject the null—stating that there are no differences.
- For your output, you will pay attention to the value for **Pr(>F)** otherwise known as a *p*-value.
- If your *p*-value is greater than 0.05, then you fail to reject the null meaning the variances for each of your samples are statistically equivalent (good!). If your *p*-value is less than 0.05, you reject the null meaning the variances for each of your samples are statistically different (bad!).



Step 2: State Hypotheses

There is **no significant difference** in means
between the two samples.
Implies the samples
represent the **same population** of scores.

$$H_0$$
: $\mu_1 = \mu_2$

$$H_0$$
: $\mu_1 - \mu_2 = 0$

$$H_a$$
: $\mu_1 \neq \mu_2$

$$H_a$$
: $\mu_1 - \mu_2 \neq 0$

There is a **significant difference** between the two
sample means. Implies the
means from each sample
represent **different populations** of scores

Step 3: Degrees of Freedom and T-Critical

- ° We are still going to use our T-Table to determine our *t*-critical values.
- Formula for the degrees of freedom is a bit different for an independent-samples *t*-test because we now take into consideration **two samples**.

$$df = (N_1 - 1) + (N_2 - 1)$$
$$df = (4 - 1) + (4 - 1)$$
$$df = (3) + (3) = 6$$

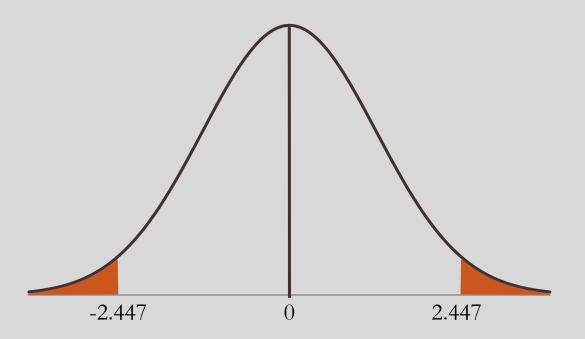
Step 3: Degrees of Freedom and T Critical

 $total\ df = 6$

t-critical (two-tailed test at $\alpha = .05$) = ± 2.447

cum. prob one-tail two-tails	t _{.50} 0.50 1.00	t _{.75} 0.25 0.50	t _{.80} 0.20 0.40	t _{.85} 0.15 0.30	t _{.90} 0.10 0.20	t _{.95} 0.05 0.10	t _{.975} 0.025 0.05	t _{.99} 0.01 0.02	t _{.995} 0.005 0.01	t _{.999} 0.001 0.002	t _{.9995} 0.0005 0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1 156	1.476	2 015	2 571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
4	0.000	0.711	0.896	1.119	1.415	1.895	2.300	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073

Step 3: Degrees of Freedom and T-Critical



Step 4: Calculate T-Statistic

• Calculate the mean as usual (sum all scores and then divide by sample size)

Filtered Photo Group					
Subject	Subject Attractiveness				
1	9				
2	8				
3	6				
4	9				
	8.00				

	Unfiltered Photo Group				
Subject	Subject Attractiveness				
5	5				
6	6				
7	5				
8	4				
	5.00				

Step 4: Calculate T-Statistic

Then use the raw scores for each sample to calculate an **estimated population variance** for each group.

- If you remember from earlier in the semester, the variance formula is basically the same thing as the SD formula.
 - The only difference is that the variance is **missing** the square root sign.

$$\hat{\sigma}^2 = \frac{\sum X^2 - \frac{(\sum X)^2}{N}}{N-1}$$

Calculating the Estimated Population Variance

Filtered Photo)
Group	

Subject	Attractiveness
1	9
2	8
3	6
4	9
	8.00

$$\hat{\sigma}_{filter}^2 = \frac{262 - \frac{(32)^2}{4}}{4 - 1}$$

Unfiltered Photo Group

Subject	Attractiveness
1	5
2	6
3	4
4	4
	5.00

$$\hat{\sigma}_{unfiltered}^2 = \frac{102 - \frac{(20)^2}{4}}{4 - 1}$$

$$\hat{\sigma}_{unfiltered}^2 = .6667$$

$$\hat{\sigma}_{unfiltered}^2 = .6667$$

Pooling the Variances

When we "pool" the variances, we are basically just adding them together and finding the average between them.

$$\sigma_{pool}^{2} = \frac{\left(df_{1}\right)\mathring{\sigma}_{1}^{2} + (df_{2})\mathring{\sigma}_{2}^{2}}{\left(df_{1}\right) + (df_{2})}$$

Pooling the Variances

$$\sigma_{pool}^2 = \frac{\left(df_1\right)\mathring{\sigma}_1^2 + (df_2)\mathring{\sigma}_2^2}{\left(df_1\right) + (df_2)}$$

$$\sigma_{pool}^2 = \frac{(3)2.00 + (3).6667}{(3) + (3)} \qquad \sigma_{pool}^2 = \frac{8.0001}{6} = 1.3334$$

Step 4: Calculate T-Statistic

• After calculating the two sample means and the estimated population variance, we can plug those values into our *t*-statistic formula.

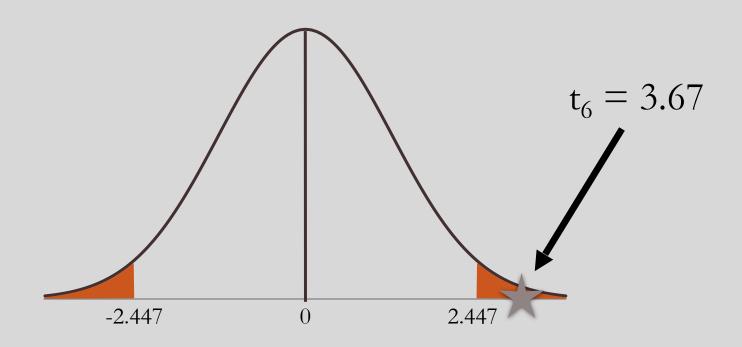
 $ar{X}_1$ = sample 1 mean $ar{X}_2$ = sample 2 mean σ_{pool}^2 = pooled estimated population variance \mathbf{N} = sample size (for each sample 1 and 2)

$$t_{df} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_{pool}^2}{N_1} + \frac{\sigma_{pool}^2}{N_2}}}$$

Step 4: Calculate T-Statistic

$$t_{df} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_{pool}^2}{N_1} + \frac{\sigma_{pool}^2}{N_2}}} \qquad t_6 = \frac{8 - 5}{\sqrt{\frac{1.3334}{4} + \frac{1.3334}{4}}} \qquad t_6 = 3.674$$

Step 5: Compare T-Statistic to Rejection Region



Step 6: Calculate 95% Confidence Interval

• For a two-tailed test, we are going to want to calculate both an upper and lower confidence interval limit.

$$CI_{upper} = t \ critical \left(\sqrt{\frac{\sigma_{pool}^2}{N_1} + \frac{\sigma_{pool}^2}{N_2}} \right) + (\bar{X}_1 - \bar{X}_2)$$

$$\text{CI}_{\text{lower}} = -t \ critical \left(\sqrt{\frac{\sigma_{pool}^2}{N_1} + \frac{\sigma_{pool}^2}{N_2}} \right) + (\bar{X}_1 - \bar{X}_2)$$

Step 6: Calculate 95% Confidence Interval

Upper Confidence Interval Limit

$$CI_{upper} = 2.447 \left(\sqrt{\frac{1.3334}{4} + \frac{1.3334}{4}} \right) + (8 - 5)$$

$$CI_{upper} = 2.447 \left(\sqrt{.6667} \right) + (3)$$

$$CI_{upper} = 1.9980 + 3$$

$$CI_{upper} = 4.9980$$

Lower Confidence Interval Limit

$$CI_{lower} = -2.447 \left(\sqrt{\frac{1.3334}{4} + \frac{1.3334}{4}} \right) + (8 - 5)$$

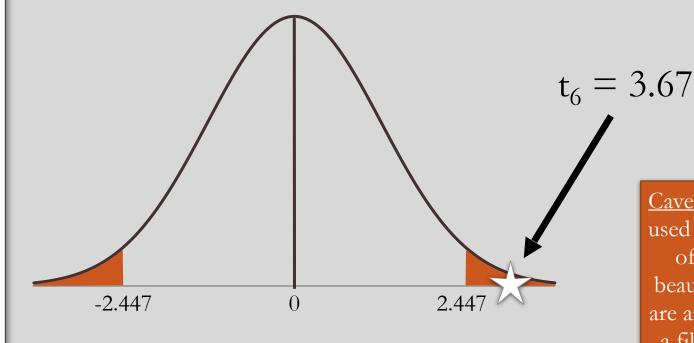
$$CI_{lower} = -2.447 \left(\sqrt{.6667} \right) + (3)$$

$$CI_{lower} = -1.9980 + 3$$

$$CI_{lower} = 1.0020$$

Step 7: Interpretation/Conclusion

Reject the null hypothesis: Participants who viewed filtered photos ($\bar{X} = 8$) differed significantly in their attractiveness rating compared to participants who saw unfiltered photos $(\bar{X} = 5), t(6) = 3.67, p < .05.$ Participants who saw filtered photos rated them as more attractive. We can be confident the true mean difference in attractiveness rating between filtered and unfiltered photos is between 1.00 and 5.00.



Caveat: This is FAKE data used only for the purposes of this class. You are beautiful just the way you are and you do NOT need a filter to be a 10 out of 10 holy hot! level of attractiveness.

```
Console Jobs >
> t.test(data_ratings ~ data_group, data = ex.data, var.equal =
TRUE, na.rm = TRUE)
       Two Sample t-test
data: data_ratings by data_group
t = 3.6742, df = 6, p-value = 0.0104
alternative hypothesis: true difference in means between group
fil and group unf is not equal to 0
95 percent confidence interval:
1.002105 4.997895
sample estimates:
mean in group fil mean in group unf
```

Use **t.test()** to conduct your analysis.

- The first thing you will write in your parentheses is the formula: dependent variable ~ independent variable.
- Then, you will include a comma and type in the name of your dataset. You will write **data** = and then the name of your data.
- Then, you will include another comma and fill in the argument var.equal = with either a TRUE (if you met the homogeneity of variance assumption) or FALSE (if you did NOT meet the assumption).
- Then, you will include a final comma and fill in the argument **na.rm** = with either a **TRUE** or **FALSE**. When na.rm is TRUE, the function skips over any NA values (not applicable; this means there is a missing data point). However, when na.rm is FALSE, then it returns NA from the calculation being done on the entire row or column.

REMEMBER: If you are doing a one-tailed test, you need to include the **alternative** = argument (!!!!!!!)

```
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t = *t*-statistic that we calculate to compare to the *t*-critical value

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t = *t*-statistic that we calculate to compare to the *t*-critical value

df = degrees of freedom for an independent-samples *t*-test

```
Console Jobs >
> t.test(data_ratings ~ data_group, data = ex.data, var.equal =
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       Two Sample t-test
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t = *t*-statistic that we calculate to compare to the *t*-critical value

df = degrees of freedom for an independent-samples *t*-test

p-value = p-value that we compare to 0.05; if our p-value is less than 0.05, we reject the null hypothesis; if our p-value is greater than or equal to 0.05, we fail to reject the null hypothesis

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df = degrees of freedom for an independent-samples *t*-test

p-value = p-value that we compare to 0.05; if our p-value is less than 0.05, we reject the null hypothesis; if our p-value is greater than or equal to 0.05, we fail to reject the null hypothesis

95% confidence interval = if our CI includes 0 (zero), then we fail to reject the null hypothesis; if our CI does not include zero, we reject the null hypothesis