CIS 511 Homework 7

Stephen Phillips, Dagaen Golomb

April 15, 2015

Problem 1

Show that $STRONGLY - CONNECTED = \{\langle G \rangle \mid G \text{ is a graph that is strongly connected}\}$ is NL-complete

Since $NL = \text{co-}NL$, and we know that $\overline{STRONGLY - CONNECTED}$ is in NL, then $STRONGLY - CONNECTED$ is in NL

Problem 2

Show that $2SAT$ is NL-complete.

Problem 3

Give an example of an NL-complete context free language.

Problem 4

Define $pad : \Sigma^* \times \mathcal{N} \to \Sigma^*\#^*$ as $pad(s, l) = s\#^l$. Define the language $pad(A, f)$ for language A and function $f : \mathcal{N} \to \mathcal{N}$ as

$$pad(A, f) = \{pad(s, f(|s|)) \mid s \in A\}$$

Show that if $A \in \text{TIME}(n^6)$ then $pad(A, n^2) \in \text{TIME}(n^3)$

Is this not just that padding extends the length of the string? Now if you run the TM for A on the first part of the string, it will run in time $O(|s|^6)$, and since the length of our input is $n = |s|^2 + |s| = O(|s|^2)$ this becomes $O(|s|^6) = O((|s|^2)^3) = O(n^3)$ (?)

Problem 5

Prove using *pad* from previous problem that if $NEXPTIME \neq EXPTIME$ then $P \neq NP$.

Problem 6

Show that for an n variable polynomial P, with degree at most d, and total degree of t. We showed in class that if you pick $r_1, \ldots, r_n$ uniformly and independently at random in a set S then

$$\mathbf{Pr}[P(r_1, r_2, \ldots, r_n) = 0] \leq \frac{nd}{|S|}$$

We now want to strengthen this result to:

$$\mathbf{Pr}[P(r_1, r_2, \ldots, r_n) = 0] \leq \frac{t}{|S|}$$

Problem 7

Show if $NP \subseteq BPP$, then $NP = RP$

Problem 8

Define a ZPP-machine as a probabilistic Turing Machine that can output 3 things: accept, reject, and ?. A ZPP-machine M decided a langauge A if for every $x \in L$ it accepts with probability at least $2/3$, and rejects with probability 0, for $x \notin L$ it rejects with probability $2/3$ and accepts with probability 0, and it outputs ? on any input with probability at most $1/3$.