CIS 511: Spring 2015
Problem Set 4: Due March 25, 2015

1. Problem 5.13.

2. Problem 5.20.

3. Recall in a previous homework that we showed that the class of context-free languages is not closed under intersection.

   Let $L = \{\langle M, w\rangle \mid M \text{ has an accepting computational path on input } w\}$. Note that $L = A_{TM}$, and A_{TM} is undecidable.

   Let $S = S_{M,w}$ be the set of accepting computational paths for M on input w. Then S is non-empty only if M accepts w. Carefully construct a format for computational paths, so that S can be written as $S = L_1 \cap L_2$, where both L_1 and L_2 are context-free. Conclude that determining whether the intersection of two context-free languages is empty, is undecidable. [**Hint.** Recall the proof of the undecidability of ALL_{CFG}. There we were interested in invalid computational paths, but that proof should give a hint about what to do with L.]

4. Suppose that M is a DFA over the alphabet $\{0, 1\}$, and let n be a natural number. We will count the number of strings of length n which are accepted by M.

   (a) Let $M = (Q, \Sigma, \delta, q_0, F)$. Take $q \in Q$, and let $T[q, i]$ denote the number of strings s of length i for which M ends in state q after reading s. Give an algorithm for computing $T[q, i]$ for $q \in Q$ and $0 \leq i \leq n$.

   (b) Using $T[q, i]$ computed in the previous part, answer the original question: how many strings are there of length n which are accepted by M?

   (c) This algorithm does not, in general, run in polynomial time. Explain why.

5. Exercise 7.12.

6. Problem 7.13.

7. Problem 7.18.