

**CIS 511: Spring 2015**  
**Problem Set 3: Due February 23, by 5 PM**

1. Provide the formal description of a Turing Machine that decides the following language:

$$L = \{x \in \{a, b, c\}^* \mid x \text{ contains more a's than b's and c's combined}\}$$

2. Problem 3.14 [**Hint:** Show how this automaton can simulate one step of a Turing Machine. You can describe this simulation in a high-level language,.]
3. Problem 3.16, part c.
4. Problem 3.18
5. Problem 4.14
6. A language  $L$  is co-Turing-recognizable if  $\bar{L}$  (the complement of  $L$ ) is Turing-recognizable. Prove that  $A = \{M : L(M) = \emptyset\}$  is co-Turing-recognizable. Here  $M$  is a description of a Turing Machine.
7. Problem 4.18
8. In class we gave the argument for why the language of Kolmogorov,  $L_{Kol}$  random strings is undecidable.
  - (a) Rigorously define this language and reproduce the argument from class showing that this language is undecidable.
  - (b) Define the language  $L = \{M \mid M \text{ halts on the empty input}\}$ . Show that if  $L$  were decidable, then  $L_{Kol}$  would also be decidable.
  - (c) What can you conclude about  $L$ ?