CIS 511 Homework 4

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Problem 1

We want to show that finding if a Turing Machine has a *useless state* is Turing Decidable. We formulate this as a language:

$$L = \{\langle M, q\rangle \mid q \in Q(M),\ \forall s \in \Sigma^* M(q)\, M \text{ does not enter } q\}$$

Now we reduce this to A_{TM} to show that it is undecidable. First as usual, we suppose toward contradiction that there was a decider of this language, which we will denote D. We build a Turing Machine in the following manner:

function N($\langle M, x\rangle$)
  Build the description for the following machine
  function A(y)
    Ignore input y
    Simulate $M(x)$, and output what it outputs
  end function
  Simulate $D(\langle A, q_A\rangle)$, where q_A is the accept state of D, and output what it outputs
end function

If $M(x)$ halts and accepts, then we eventually reach the accept state of A, by construction. Also by construction we never reach the accept state if $M(x)$ loops or if it rejects. Since D is a decider, and building the machine description takes finite time, N always halts. Therefore N accepts if and only if D accepts, which in turn accepts if and only if $\langle M, x\rangle \in A_{TM}$. Therefore, N decides A_{TM}, a contradiction.

Problem 2

Problem 3

We want to show that the intersection of two context free languages is undecidable using A_{TM}.

Problem 4

Problem 5

Show that the language

$$ISO = \{\langle G, H\rangle\} \mid G \text{ and } H \text{ are isomorphic graphs}$$

is in NP.

To do this we need to show a verifier V for ISO. As one might expect, the certificate for a member of the language $\langle G, H\rangle$ is the isomorphism $\phi : V_G \to V_H, v \to \phi(v)$ between them. The size of such an isomorphism would be about $2n$ where n is the number of nodes in G and H, so it polynomial in the size of G and H. We also need to check that the verifier V using this runs in polynomial time. A simple algorithm to use this is the following:

function V($\langle G, H\rangle$,ϕ)
  If the number of nodes or edges in G differ from the number of nodes or edges in H, reject
  for Nodes v in G **do**
    If the number of edges (v, u) in G differ from the number of edges $(\phi(v), u')$ in H, reject

for Edges (v, u) in G **do**
    If $(\phi(v), \phi(u))$ is not in H, reject
end for
end for
Accept
end function

By assering that the size of the graphs are the same, that each node v has the same degree as $\phi(v)$ and maps to the corresponding vertices, we have shown that the isomorphism is correct. If there is no isomorphism then at least one of these tests will fail. Therefore $V(\langle G, H\rangle, y)$ will accept for some input y if and only if there is an isomorphism between G and H

Problem 6

Problem 7