Leaving Certificate Honours Level 2002 Paper 1 Solutions

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Introduction

The following are the solutions to Paper 1 of the 2002 Leaving Certificate Honours course.

Question 1

(a)

$$\begin{aligned}
x &= \sqrt{x+2} \\
x^2 &= x+2 \\
x^2 - x - 2 &= 0 \\
(x-2)(x+1) &= 0 \\
x-2 &= 0 \\
x &= 2 \\
\\
x+1 &= 0 \\
x &= -1
\end{aligned}$$

(b)

Since one root is an integer try various values. $f(0) = -10 < 0$, $f(3) = 3^3 - 4(3)^2 + 9(3) - 10 = 8 > 0$,$f(2) = 2^3 - 4(2)^2 + 9(2) - 10 = 0$.Thus 2 is a root and $x-2$ is a factor. Now, dividing in

$$\begin{array}{r r}
 & x^2 - 2x \quad + 5 \\ \cline{2-2}
x-2) & x^3 - 4x^2 + 9x - 10 \\
 & -x^3 + 2x^2 \qquad\qquad \\ \cline{2-2}
 & -2x^2 + 9x \qquad \\
 & 2x^2 - 4x \qquad \\ \cline{2-2}
 & 5x - 10 \\
 & -5x + 10 \\ \cline{2-2}
 & 0
\end{array}$$

To get the roots of x^2-2x+5 use the '-b' formula. Thus

$$\begin{aligned}
x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(5)}}{2(1)} \\
&= \frac{2 \pm \sqrt{-16}}{2} \\
&= \frac{2 \pm 4\iota}{2} \\
&= 1 \pm 2\iota
\end{aligned}$$

Hence the roots are 2,$1+2\iota$ and $1-2\iota$.

(b)(i)

$$\underbrace{(p+r-t)}_{a}x^2 + \underbrace{2r}_{b}x + \underbrace{(t+r-p)}_{c} = 0$$

The roots of this equation are (using the '-b' formula)

$$\begin{aligned}
x &= \frac{-2r \pm \sqrt{(2r)^2 - 4(p+r-t)(t+r-p)}}{2(p+r-t)} \\
&= \frac{-2r \pm \sqrt{4r^2 - 4(pt + pr - p^2 + rt + r^2 - pr - t^2 - tr + tp)}}{2(p+r-t)} \\
&= \frac{-2r \pm \sqrt{4r^2 - 4(-p^2 + r^2 - t^2 + 2tp)}}{2(p+r-t)} \\
&= \frac{-2r \pm \sqrt{4r^2 + 4p^2 - 4r^2 + 4t^2 - 8tp}}{2(p+r-t)} \\
&= \frac{-2r \pm \sqrt{4p^2 + 4t^2 - 8tp}}{2(p+r-t)} \\
&= \frac{-2r \pm \sqrt{4(p^2 + t^2 - 2tp)}}{2(p+r-t)}
\end{aligned}$$

$$\begin{aligned}
&= \frac{-2r \pm \sqrt{4(p-t)^2}}{2(p+r-t)} \\
&= \frac{-2r \pm 2(p-t)}{2(p+r-t)} \\
&= \frac{-r \pm (p-t)}{p+r-t} \\
x &= \frac{-r + (p-t)}{p+r-t} \\
\text{Or } x &= \frac{-r - (p-t)}{p+r-t} \\
&= \frac{-r-p+t}{p+r-t}
\end{aligned}$$

As p,r and t are all Integers then both roots are of type Integer/Integer, i.e. rational.

(b)(i)

From (b)(i) above, one of the roots is

$$\begin{aligned}
x &= \frac{-r-p+t}{p+r-t} \\
&= -\left(\frac{r+p-t}{p+r-t}\right) \\
&= -1
\end{aligned}$$

Question 2

(a)

$$\begin{aligned}
x + 2y + 4z &= 7 \text{ (Eqn 1)} \\
x + 3y + 2z &= 1 \text{ (Eqn 2)} \\
-y + 3z &= 8 \text{ (Eqn 3)}
\end{aligned}$$

$$\begin{aligned}
y - 2z &= -6 \text{ (Eqn 4 = Eqn 2 - Eqn 1)} \\
z &= 2 \text{ (Eqn 3 + Eqn 4)} \\
y &= -6 + 2z \\
y &= -2 \\
x &= 7 - 2y - 4z \\
&= 7 - 2(-2) - 4(2) \\
x &= 3
\end{aligned}$$

(b)(i)

First find where $x^2 + x - 20 = 0$.

$$\begin{aligned} x^2 + x - 20 &= 0 \\ (x+5)(x-4) &= 0 \\ x + 5 &= 0 \\ \therefore \quad x &= -5 \\ \text{Or } x - 4 &= 0 \\ x &= 4 \end{aligned}$$

Now test a value between -5 and 4, say 0. $0^2 + 0 - 20 < 0$ and thus $-5 \leq x \leq 4$.

(b)(ii)

$$\begin{aligned} g(x) &= x^n + 3 \\ g(-x) &= (-x)^n + 3 \\ &= (-1)^n x^n + 3 \\ &= -x^n + 3 \quad \text{(if n is odd)} \\ \therefore \quad g(x) + g(-x) &= x^n + 3 - x^n + 3 \\ &= 6 \end{aligned}$$

(c)(i)

The roots of the equation are $(-b \pm \sqrt{b^2 - 4c})/2$. Thus, if the difference is 1

$$\begin{aligned} \left(\frac{-b + \sqrt{b^2 - 4c}}{2}\right) - \left(\frac{-b - \sqrt{b^2 - 4c}}{2}\right) &= 1 \\ \frac{2\sqrt{b^2 - 4c}}{2} &= 1 \\ \sqrt{b^2 - 4c} &= 1 \\ \therefore \quad b^2 - 4c &= 1 \end{aligned}$$

(c)(ii)

If the roots are consecutive integers then from (c)(i)

$$\begin{aligned} (4k-5)^2 - 4(1)(k) &= 1 \\ 16k^2 - 40k + 25 - 4k &= 1 \\ 16k^2 - 44k + 24 &= 0 \\ 4k^2 - 11k + 6 &= 0 \\ (4k-3)(k-2) &= 0 \end{aligned}$$

$$
\begin{aligned}
4k-3 &= 0 \\
k &= \frac{3}{4}
\end{aligned}
$$

$$
\begin{aligned}
\text{Or} \qquad k-2 &= 0 \\
k &= 2
\end{aligned}
$$

To find out which value of k yields consecutive integers remember that the roots are (from the '-b') formula

$$
\begin{aligned}
x &= \frac{-(4k-5)\pm 1}{2} \\
&= \frac{-\left(4(\frac{3}{4})-5\right)\pm 1}{2} \\
&= \frac{2\pm 1}{2} \\
&= \frac{3}{2}, \frac{1}{2}
\end{aligned}
$$

$$
\begin{aligned}
\text{Or} \qquad x &= \frac{-\left(4(2)-5\right)\pm 1}{2} \\
&= \frac{-3\pm 1}{2} \\
&= -2, -1
\end{aligned}
$$

Thus $k=2$ and the roots are -2 and -1.

Question 3

(a)

$$
\begin{aligned}
-1+\sqrt{3}\iota &= r(\cos\theta + \iota\sin\theta) \\
r &= |-1+\sqrt{3}\iota| \\
&= \sqrt{(-1)^2+(\sqrt{3})^2} \\
&= 2 \\
\tan\theta &= \frac{\sqrt{3}}{-1} \\
&= -\sqrt{3} \\
\theta &= \frac{2\pi}{3} \\
\therefore \quad -1+\sqrt{3}\iota &= 2\left(\cos\frac{2\pi}{3} + \iota\sin\frac{2\pi}{3}\right)
\end{aligned}
$$

(b)(i)

$$\begin{aligned} z &= 2-\sqrt{3}\iota \\ \therefore z^2+tz &= (2-\sqrt{3}\iota)^2+t(2-\sqrt{3}\iota) \\ &= 4-4\sqrt{3}\iota-3+2t-\sqrt{3}t\iota \\ &= 1+2t-\iota\sqrt{3}(4+t) \end{aligned}$$

If this is real then the complex part of the number must be zero, i.e. $t=-4$.

(b)(ii)
Let $w=a+b\iota$. Then $\overline{w}=a-b\iota$ and $w\overline{w}=a^2+b^2$. Thus

$$\begin{aligned} w\overline{w}-2w\iota &= 7-4\iota \\ a^2+b^2-2\iota(a+b\iota) &= 7-4\iota \\ a^2+b^2+2b-2a\iota &= 7-4\iota \\ -2a &= -4 \\ a &= 2 \end{aligned}$$

$$\begin{aligned} 4+b^2+2b &= 7 \\ b^2+2b-3 &= 0 \\ (b+3)(b-1) &= 0 \\ b+3 &= 0 \\ b+3 &= 0 \\ b &= -3 \\ b-1 &= 0 \\ b &= 1 \end{aligned}$$

So $w=2-3\iota$ or $w=2+\iota$.

(c)

$$\begin{aligned} x+y &= \begin{pmatrix} 3 & -1 \\ -6 & 2 \end{pmatrix} + \begin{pmatrix} 2 & 3 \\ 6 & 9 \end{pmatrix} \\ &= \begin{pmatrix} 5 & 2 \\ 0 & 11 \end{pmatrix} \\ y+x &= \begin{pmatrix} 2 & 3 \\ 6 & 9 \end{pmatrix} + \begin{pmatrix} 3 & -1 \\ -6 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 5 & 2 \\ 0 & 11 \end{pmatrix} \\ &= x+y \end{aligned}$$

$$\begin{aligned}
xy &= \begin{pmatrix} 3 & -1 \\ -6 & 2 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 6 & 9 \end{pmatrix} \\
&= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\
yx &= \begin{pmatrix} 2 & 3 \\ 6 & 9 \end{pmatrix} + \begin{pmatrix} 3 & -1 \\ -6 & 2 \end{pmatrix} \\
&= \begin{pmatrix} -12 & 4 \\ -36 & 12 \end{pmatrix} \\
&\neq xy
\end{aligned}$$

In the above $xy = 0$ but $x \neq 0$ and $y \neq 0$. Thus the statement is false.

Question 4

(a)

$$\begin{aligned}
S_n &\triangleq \frac{a(1-r^n)}{1-r} \\
&= \frac{3\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}} \\
&= 6\left(1-\frac{1}{2^n}\right)
\end{aligned}$$

(b)(i)

$$\begin{aligned}
\frac{1}{k} - \frac{1}{k+2} &= \frac{k+2-k}{k(k+2)} \\
&= \frac{2}{k(k+2)}
\end{aligned}$$

(b)(ii)

$$\sum_{k=1}^{n} \frac{2}{k(k+2)} = \sum_{k=1}^{n} \frac{1}{k} - \sum_{k=1}^{n} \frac{1}{k+2}$$

So

$$\begin{aligned}
u_1 &= \frac{1}{1} - \cancel{\frac{1}{3}} \\
u_2 &= \frac{1}{2} - \cancel{\frac{1}{4}} \\
u_3 &= \cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}
\end{aligned}$$

$$
\begin{aligned}
u_4 &= \cancel{\frac{1}{4}} - \cancel{\frac{1}{6}} \\
\vdots &\quad\ \vdots \quad \vdots \\
u_{n-2} &= \cancel{\frac{1}{n-2}} - \cancel{\frac{1}{n}} \\
u_{n-1} &= \cancel{\frac{1}{n-1}} - \frac{1}{n+1} \\
u_n &= \cancel{\frac{1}{n}} - \frac{1}{n+2}
\end{aligned}
$$

$$
S_n = 1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}
$$

(b)(iii)

$$
\begin{aligned}
S_\infty &= \lim_{n\to\infty} S_n \\
&= \lim_{n\to\infty}\left(\frac{3}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right) \\
&= \frac{3}{2} - 0 - 0 \\
&= \frac{3}{2}
\end{aligned}
$$

(c)(i)

Let a, $a+d$ and $a+2d$ be the numbers. Then

$$
\begin{aligned}
a + (a+d) + (a+2d) &= 27 \\
3a + 3d &= 27 \\
a + d &= 9 \\
d &= 9 - a \\
\text{and} \quad a(a+d)(a+2d) &= 704 \\
a(a+9-a)(a+18-2a) &= 704 \\
9a(18-a) &= 704 \\
-9a^2 + 162a &= 704 \\
9a^2 - 162a + 704 &= 0 \\
a &= \frac{162 \pm \sqrt{162^2 - 4(9)(704)}}{18} \\
&= \frac{162 \pm \sqrt{900}}{18}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{162 \pm 30}{18} \\
&= \frac{32}{3}, \frac{22}{3} \\
d &= 9 - a \\
&= -\frac{5}{3}, \frac{5}{3}
\end{aligned}
$$

So $d = 5/3$ and the numbers are 22/3, 9 and 32/3.

Question 5

(a)(i)

$$
\begin{aligned}
\frac{8}{2^x} &= 32 \\
\frac{2^3}{2^x} &= 2^5 \\
2^{3-x} &= 2^5 \\
3 - x &= 5 \\
x &= -2
\end{aligned}
$$

(a)(ii)

$$
\begin{aligned}
\log_9 x &= \frac{3}{2} \\
x &= 9^{\frac{3}{2}} \\
&= \left(3^2\right)^{\frac{3}{2}} \\
&= 3^3 \\
x &= 27
\end{aligned}
$$

(b)(i)

$$
(1 + ax)^n \quad \triangleq \quad 1 + n(ax) + \frac{n(n-1)}{2!}(ax)^2 + \dots
$$

Thus

$$
\begin{aligned}
1 + 2x + \frac{7}{4}x^2 &\equiv 1 + nax + \frac{n(n-1)}{2}(ax)^2 \\
na &= 2 \\
\text{Or} \quad a &= \frac{2}{n}
\end{aligned}
$$

$$
\begin{aligned}
\frac{n(n-1)}{2}a^2 &= \frac{7}{4} \\
\frac{n(n-1)}{2}\left(\frac{2}{n}\right)^2 &= \frac{7}{4} \\
\frac{2(n-1)}{n} &= \frac{7}{4} \\
8n-8 &= 7n \\
n &= 8 \\
a &= \frac{2}{n} \\
&= \frac{2}{8} \\
a &= \frac{1}{4}
\end{aligned}
$$

(b)(ii)
Since $n = 8$ the middle term is T_5. This term is

$$
\begin{aligned}
\binom{8}{5} 1^4 \left(\frac{1}{4}x\right)^4 &= \frac{8!}{4!4!}\left(\frac{1}{4}\right)^4 x^4 \\
&= \frac{35}{128}x^4
\end{aligned}
$$

(c)
To prove

$$
x + x^2 + x^3 + \ldots + x^n = \frac{x(x^n - 1)}{x-1} \qquad n \in N \qquad x \neq 1
$$

Step 1: Check $P(1), n = 1$

$$
\begin{aligned}
x &= \frac{x(x^1 - 1)}{x-1} \\
&= x
\end{aligned}
$$

So it is true for $n = 1$. Now, for $P(k), n = k$,

$$
x + x^2 + x^3 + \ldots + x^k = \frac{x(x^k - 1)}{x-1}
$$

For $P(k+1), n = k+1$,

$$
\begin{aligned}
x + x^2 + x^3 + \ldots + x^k + x^{k+1} &= \frac{x(x^k-1)}{x-1} + x^{k+1} \\
&= \frac{x(x^k-1) + (x-1)x^{k+1}}{x-1}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x^{k+1} - x + x^{k+2} - x^{k+1}}{x-1} \\
&= \frac{-x + x^{k+2}}{x-1} \\
&= \frac{x(x^{k+1} - 1)}{x-1}
\end{aligned}
$$

Thus the proposition is true for $n = k + 1$. However, if it is true for $n = 1$ then it is true for $n = 1 + 1 = 2$ and true for $3, 4 \ldots$ etc.

Question 6

(a)

$$
\begin{aligned}
\frac{d}{dx}\left(x^4+1\right)^5 &= 5\left(x^4+1\right)^4 \times 4x^3 \\
&= 20x^3(x^4+1)^4
\end{aligned}
$$

(b)(i)

$$
\begin{aligned}
f(x) &= u(x) + v(x) \\
\frac{d(f(x))}{dx} &= \lim_{h\to 0}\left[\frac{u(x+h) + v(x+h) - (u(x)+v(x))}{h}\right] \\
&= \lim_{h\to 0}\left[\frac{u(x+h) - u(x)}{h}\right] + \lim_{h\to 0}\left[\frac{v(x+h) - v(x)}{h}\right] \\
&= \frac{d(u(x))}{dx} + \frac{d(v(x))}{dx}
\end{aligned}
$$

(b)(ii)

$$
\begin{aligned}
y &= 2x - \sin 2x \\
\frac{dy}{dx} &= 2 - 2\cos 2x \\
&= 2(1 - \cos 2x) \\
&= 4 \times \frac{1}{2}(1 - \cos 2x) \\
&= 4\sin^2 x
\end{aligned}
$$

(c)

$$
f(x) = ax^3 + bx^2 + cx + d
$$

$$
\begin{aligned}
f(0) &= a(0)^3 + b(0)^2 + c(0) + d \\
4 &= d \\
f(1) &= a(1)^3 + b(1)^2 + c(1) + d \\
0 &= a + b + c + 4 \\
-4 &= a + b + c \\
f'(x) &= 3ax^2 + 2bx + c \\
0 &= 3a(0)^2 + 2b(0) + c \quad \text{(Maximum at } x = 0\text{)} \\
0 &= c \\
f''(x) &= 6ax + 2b \\
f''(1) &= 6a + 2b \\
0 &= 6a + 2b \\
-4 &= a + b \\
-8 &= 2a + 2b
\end{aligned}
$$

$$
\begin{aligned}
8 &= 4a \\
a &= 2 \\
b &= -6 \\
c &= 0 \\
d &= 4
\end{aligned}
$$

Question 7

(a)(i)
dy/dx is the slope of the tangent to the curve at ANY point on the curve. Thus

$$
\begin{aligned}
9x^2 + 4y^2 &= 40 \\
18x + 8y\frac{dy}{dx} &= 0 \\
\frac{dy}{dx} &= -\frac{18x}{8y} \\
&= -\frac{9x}{4y} \\
&= -\frac{9(2)}{4(1)} \quad \text{at the point } (2, 1)
\end{aligned}
$$

$$= -\frac{9}{2}$$

(b)(i)

$$\begin{aligned}
y &= \sin^{-1} 10x \\
\frac{dy}{dx} &= \frac{1}{\sqrt{1-(10x)^2}} \\
&= \frac{1}{\sqrt{1-(10(\frac{1}{20}))^2}} \quad \text{when } x = 1/20 \\
&= \frac{10}{\sqrt{\frac{3}{4}}} \\
&= \frac{20}{\sqrt{3}}
\end{aligned}$$

(b)(ii)

$$\begin{aligned}
x &= \ln(1+t^2) \\
\frac{dx}{dt} &= \frac{1}{(1+t^2)} \times 2t \\
\frac{dt}{dx} &= \frac{(1+t^2)}{2t} \\
y &= \ln(2t) \\
\frac{dy}{dt} &= \frac{1}{2t} \times 2 \\
&= \frac{1}{t} \\
\frac{dy}{dx} &= \frac{dy}{dt} \times \frac{dt}{dx} \\
\frac{dy}{dx} &= \frac{1}{t} \times \frac{(1+t^2)}{2t} \\
&= \frac{(1+t^2)}{2t^2} \\
&= \frac{(1+(\sqrt{5})^2)}{2(\sqrt{5})^2} \\
&= \frac{(1+5)}{10} \\
&= \frac{3}{5}
\end{aligned}$$

(c)(i)

$$\begin{aligned}
f(x) &= \frac{e^x + e^{-x}}{2} \\
f'(x) &= \frac{e^x + e^{-x}(-1)}{2} \\
&= \frac{e^x - e^{-x}}{2} \\
f''(x) &= \frac{e^x - e^{-x}(-1)}{2} \\
&= \frac{e^x + e^{-x}}{2} \\
&= f(x)
\end{aligned}$$

(c)(ii)

$$\begin{aligned}
f(2x) &= \frac{e^{2x} + e^{-2x}}{2} \\
f'(2x) &= \frac{e^{2x}(2) + e^{-2x}(-2)}{2} \\
&= e^{2x} - e^{-2x} \\
f'(x) &= \frac{e^x - e^{-x}}{2} \\
\frac{f'(2x)}{f'(x)} &= \frac{e^{2x} - e^{-2x}}{\frac{e^x - e^{-x}}{2}} \\
&= 2\frac{e^{2x} - e^{-2x}}{e^x - e^{-x}} \\
&= 2\frac{(e^x - e^{-x})(e^x + e^{-x})}{e^x - e^{-x}} \\
&= 2(e^x + e^{-x}) \\
&= 2f(x)
\end{aligned}$$

Question 8

(a)(i)

$$\begin{aligned}
\int (x^3 + \sqrt{x} + 2)dx &= x^4 + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2x + C \\
&= x^4 + \frac{2x^{\frac{3}{2}}}{3} + 2x + C
\end{aligned}$$

(b)(i)

$$\int_2^7 \frac{2x-4}{x^2-4x+29}dx$$

This is done by letting $u = x^2 - 4x + 29$. Then $du = (2x-4)dx$. Next the integral range must be changed. When $x = 2 \quad u = 2^2 - 4(2) + 29 = 25$ and similarly when $x = 7 \quad u = 7^2 - 4(7) + 29 = 50$. Thus the integral becomes

$$\begin{aligned}
\int_{25}^{50} \frac{du}{u} &= \ln|u| \Big|_{25}^{50} \\
&= \ln|50| - \ln|25| \\
&= \ln\left|\frac{50}{25}\right| \\
&= \ln 2
\end{aligned}$$

(b)(ii)

$$\begin{aligned}
\int_2^7 \frac{1}{x^2-4x+29}dx &= \int_2^7 \frac{1}{(x-2)^2+5^2}dx \\
&= \frac{1}{5}\tan^{-1}\left(\frac{x-2}{5}\right) \Big|_2^7 \\
&= \frac{1}{5}\left(\tan^{-1}1 - \tan^{-1}0\right) \\
&= \frac{1}{5}\left(\frac{\pi}{4} - 0\right) \\
&= \frac{\pi}{20}
\end{aligned}$$

(c)

Figure 1 shows the curve $f(x) = x^3 - 3x^2 + 5$ and the shaded region is the area that is required. To find the bounding points it is is first necessary to find line L which is a tangent at the local maximum. Since dy/dx is a tangent to the curve at any point we can then easily find the equation of L. So

$$\begin{aligned}
f(x) &= x^3 - 3x^2 + 5 \\
f'(x) &= 3x^2 - 6x
\end{aligned}$$

$$\begin{aligned}
3x^2 - 6x &= 0 \quad \text{at local max and min} \\
x(3x-6) &= 0 \\
x &= 0 \\
x &= 2
\end{aligned}$$

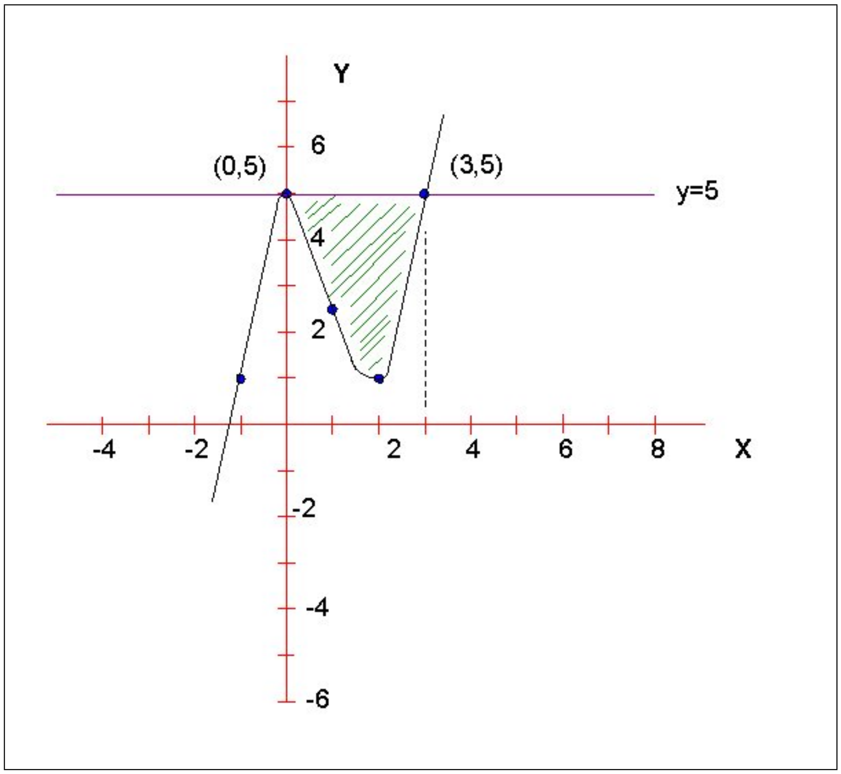

Figure 1: $f(x) = x^3 - 3x^2 + 5$

At $x = 0, f(x) = 5$ and at $x = 2, f(x) = 1$. Thus the tangent is at $x = 0$,the maximum point. Now the slope of the tangent at $x = 0$ is $f'(0) = 3(0)^2 - 6(0) = 0$. Thus the equation of line L is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 5 &= 0(x - 0) \\ y &= 5 \end{aligned}$$

To find the boundary points it is necessary to solve $x^2 - 3x + 5 = 5$ which yields $x(x-3) = 0$ or $x = 0$ (which we knew already) and $x = 3$. So the boundary points are $(0, 5)$ and $(3, 5)$ as can be seen from figure 1.

The shaded area A is equal to the Area of the Rectangle minus the area between the curve and the x-axis., i.e

$$\begin{aligned} A &= A_{\square} - \int_0^3 (x^3 - 3x^2 + 5)dx \\ A &= 5 \times 3 - \left(\frac{x^4}{4} - x^3 + 5x \right) \Big|_2^7 \\ &= 15 - \left(15 - \frac{81}{12} \right) \\ &= \frac{27}{4} \end{aligned}$$