48/88

Name Haley Cyr

CMSI 282 Final Exam - spring, 2013

Do ALL of the problems. Calculators (and similar) are NOT permitted.

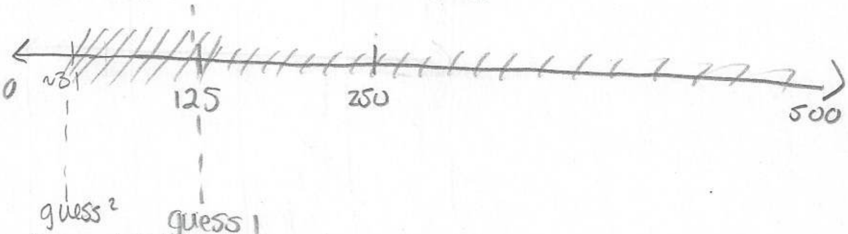
1) Answer all parts of all of the questions a-j, below:

a) Give approximate values[1] for each of these and **show all work:**

i) fourthRootOf (500):

$\sqrt[4]{500}$ ≈ 5

$2\frac{1}{2}$



ii) the probability of throwing exactly four 5's with six dice:

one way to throw exactly 4 fives

$$\frac{1 \cdot (15)(25)}{6^6}$$

total possibilities

3 ok.

1 1 1 1 5 5
1 1 1 5 1 5
1 1 1 5 5 1
1 1 5 1 5 1
1 1 5 5 1 1
1 1 5 1 1 5
1 5 1 1 1 5
1 5 1 1 5 1
1 5 1 5 1 1
1 5 5 1 1 1
5 1 1 1 1 5
5 1 1 1 5 1
5 1 1 5 1 1
5 1 5 1 1 1
5 5 1 1 1 1
15

iii) the number of bits in (the binary representation of) 10^{500}:

$\log_2 2 \cdot 10^{500}$

0 ?

iv) the hundredth Fibonacci number: Ugh...

- .... I feel like there is definitely a formula for this
- Fib numbers grow at $\cancel{n^{1.6}}$ so maybe $\boxed{\cancel{100}^{1.6}}$

1

1.6^n

1.6^{100}

[1] Better approximations earn more points.

b) Give precise values for each of these and **show all work**::

i) leastCommonMultiple (24, 60): $xy = gcd(xy) \cdot lcm(xy)$

$lcm(xy) = \frac{xy}{gcd(xy)}$

3

$60 = 2 \cdot 24 + 12$

$24 = 2 \cdot \boxed{12} + 0$

$x \cdot y = 24 \cdot 60 = 1440$

$\frac{1440}{12} = \boxed{120}$

ii) convertBaseFourToBaseThree (201321):

$1 \times 4^0 = 1$

$2 \times 4^1 = 8$

$3 \times 4^2 = 48$

$1 \times 4^3 = 64$

$0 \times 4^4 = 0$

$2 \times 4^5 = 1024$

1145_{10}

0

3 | 1145

3 | 381 rem 2

3 | 127 rem 0

3 | 42 rem 1

3 | 14 rem 2

3 | 4 rem 1

1

$\boxed{1112102}$

iii) the letter that occurs **least** often in English (state your justification):

Based upon the Huffman encoding tree we did for the frequency of letters given by the wikipedia article, the least occuring letter is z (I remember that x, q, j, and k have low frequencies too but I'm not sure which is the lowest).

3

c) Enumerate all of the Ramsey partitions of 5 with respect to 3,2:

3

{1, 1, 1, 1, 1}

{2, 1, 1, 1}

d) Define simple (aka proportional) fair division among *n* players:

0

Fair division/proportional division among n players can be defined as an algorithm, technique, or strategy to divide a good (say a cake) so that each person feels as though the piece they recieved is just as, ~~or more,~~ valuable than other players.

X

Best answer: brute-force!
Otherwise,

e) I need to know, with reasonable certainty, whether the number $p = 37542601$ is a prime? How would you recommend that I go about it?

I would use Fermat's Little Theorem and Rabin's primality/compositeness testing via randomization. So you would take a number $2 \rightarrow \sqrt{p}$, use Fermat's Little Theorem & since if the number is not a prime, half of the numbers you test should be "witnesses" to its compositeness. Run the test K times, picking a random value each time & depending on results, it's $\frac{1}{2^K}$% likely to be not prime.

OK

$2\frac{1}{2}$

f) What is the SUBSET-SUM problem? Give two instances of the problem- one yes-instance and one no-instance (and make sure to tell me which one is which!) What is the present state of our knowledge about the inherent complexity of this problem?

Given a subset of numbers, can you make a given sum only using each number in the set once.

Instance 1: Yes!
Sub-set: {2, 1, 3}, sum = 6; so 2+1+3 = 6

3

Instance 2: NO!
Subset: {3, 2, 5} sum = 6, impossible to do with given subset of numbers

The inherent complexity of the problem is NP-complete, meaning nondeterministic polynomial time.

g) Alice and her brother, Bob, want to divide a cake in the ratio 5:6, that is, Alice should end up with at least 5/11 of the cake by her valuation, etc. Accordingly, she instructs Bob to cut eleven equivalent pieces, from which Alice will attempt to choose five, with the rest going to Bob. However, Alice can only identify four pieces that she considers to be large enough! Can this situation be salvaged? If so, how; if not, why not?

I don't think this situation can be salvaged because based upon Bob's evaluation each piece is worth 1/11th of the cake. If Alice can't find a fifth piece she deems to be 1/11 then she is either stuck with 4/11 of the cake and is unhappy or she takes another/divides another piece so Bob would have less than 6/11 so he would be unhappy.

0

h) What's this? How does it work?

$$\frac{1}{1 + e^{[\,\text{eval}(c) - \text{eval}(n)\,]\,/\,T}}$$

$2\frac{1}{2}$

It's the "temperature" formula, I forget the guy's name who came up with it :(but I believe it's used to evaluate a point in comparison to another point, if the result is closer to one or zero (depending on what you're looking for) you choose the better point and continue with evaluations. ?

i) Is this set of clauses satisfiable? Prove it:

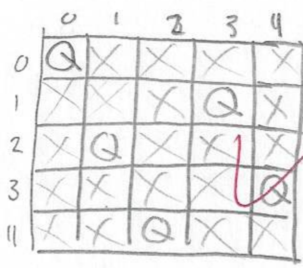
2

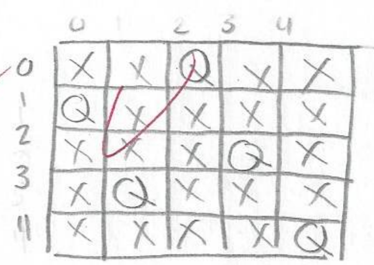
~b1 | ~b2 | ~b3   I'm assuming these values are "or'ed" together = we want all of these to be true ✓
~b1 | ~b2 | b3
b1 | ~b2 | ~b3
b1 | ~b2 | b3
b1 | b2 | ~b3   It is not based upon a truth table
b1 | b3
~b1 | b2

(work on next sheet) ✓

4

j) Illustrate the first two solutions that naturally arise from a backtracking approach to the Five Queens problem:





b_1	b_2	b_3	$\sim b_1$	$\sim b_2$	$\sim b_3$	$\sim b_1 + \sim b_2 + \sim b_3$	$\sim b_1 + \sim b_2 + b_3$	$b_1 + \sim b_2 + \sim b_3$	$b_1 + \sim b_2 + b_3$	$b_1 + b_2 + \sim b_3$	$b_1 + b_3$	$\sim b_1 + b_2$
T	T	T	F	F	F	F						
T	T	F	F	F	T	T	~~F~~	X	X	X	X	X
T	F	T	F	T	F	T	T	T	T	T	T	F
T	F	F	F	T	T	T	T	T	T	T	T	F
F	T	T	T	F	F	T	T	~~F~~	X	X	X	
F	T	F	T	F	T	T	T	T	~~F~~	X	X	
F	F	T	T	T	F	T	T	T	T	~~F~~	X	
F	F	F	T	T	T	T	T	T	T	T	~~F~~	

So, what's the answer?

$4\overline{)500}$ = 125
100

$4\overline{)125}$ = 31
12
05
4
1

31
31
31
930

$4\overline{)31}$ = 7

7 · 7 · 7
49 · 7
49 × 7 = 343
40 × 7 = 280
9 × 7 = 63
280 + 63 = 343
343 · 7

4^4
4 · 4
16 · 4
64 · 4 = 256
60 × 4 = 240
240 + 24 = 256

5 · 5 · 5
125 · 5

125 + 125 + 125 + 125 = 500

$\log_b a < d = n^d$
$\log_b a = d = n^d \log n$
$\log_b a > d = n^{\log_b a}$

2·2·2·2
4
8
16
32
64
128
256
512
1024

2) Classify these five recurrences using big-theta notation:

a) $T_a(n)$ = **if** n = 1 **then** 1 **else** 1024 $T_a(n/2)$ + 2048 n^{10}

$2^x = 1024$

$\log_2 1024 = 10$

3

$n^d \log n = \Theta(n^{10} \log n)$

b) $T_b(n)$ = **if** n = 1 **then** 1 **else** 32 $T_b(n/2)$ + n^4

$\log_2 32 ? 4$
$5 > 4$

2

$n^{\log_b a} = \Theta(n^{\log_2 32}) = \Theta(n^5)$

c) $T_c(n)$ = **if** n = 1 **then** 1 **else** 300 $T_c(n/15)$ + (n+2)(3n+4)(5n+6) + 7 ⇒ really?

$15^x \overset{?}{=} 300$

$15 \cdot 15$ 15^2
$125 \cdot 15$

$(n+2)(3n+4)(5n+6)$
$(3n^2+4n+6n+8)(5n+6)$
$15n^3+18n^2+20n^2+24n+ \ldots$

3

$\log_{15} 300 ? 3$
$\log_{15} 300 < 3$
$n^d = \Theta(n^3)$

d) $T_d(n)$ = **if** n = 1 **then** 1 **else** $T_d(n-1) + 1$ $T_d(1) = 1$

0

$\Theta($ $)$

e) $T_e(n)$ = **if** n = 1 **then** 1 **else** $T_e(n-1) + n$

0.

$\Theta($ $)$

3) Four questions about your program for Kirkman's Schoolgirl problem. => Mine & Ed's isn't turned in yet because we're awful... ?

a) In what order does your program attempt to grow a solution? More specifically, after your program successfully places girl g in the first (leftmost) column of row # 3 on day # 5, what will it do next?

It attempts to grow the solution from the top left corner to the bottom right. After placing girl g in row 3, column 1 on day five, it will check G's previous neighbors and try to place the next girl in row 3, column 2, on day 5 accordingly.

ok.

b) What *major* pruning techniques does your algorithm employ?

- we kept track of who the girls previous neighbors were,
- we started a method that checks to see which rows have already been made & backtracks if the program starts to make another row that has already existed,

c) Would a genetic program be a good choice for solving this problem? Explain:

I don't believe so. A genetic program would find the best solutions, ("the fittest") and then combine those partial solutions and discard some others that weren't as good, but those discarded solutions could be used later on (possibly)

1½

d) Answer this part **only** if you were part of a programming pair:
Who was your partner and, in your opinion, what percentage of the overall effort was yours vs. your partner's? Feel free to elaborate:

ok. My partner was Ed and I believe we both gave equal amounts of effort. We took turns writing the code & "logicing" things out and correcting each others mistakes. There were some disagreements about how a method should be written which caused some confusion later on.

4) Do parts a-c, below:

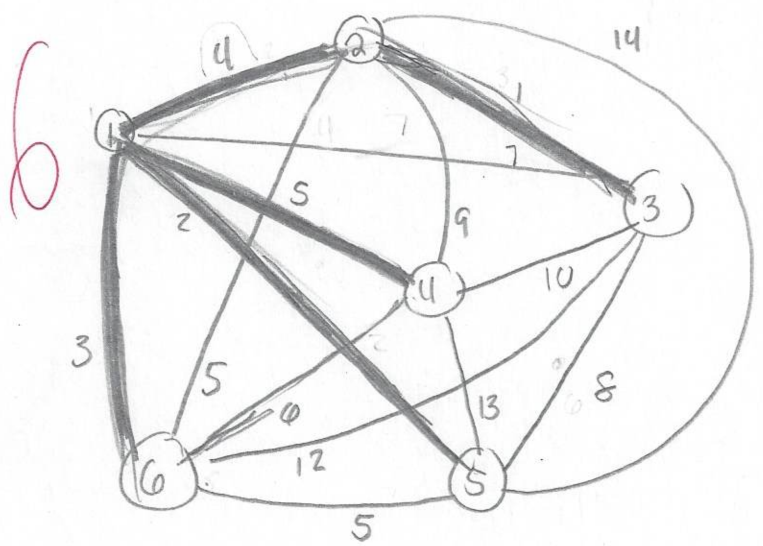
a) A certain undirected graph, *G*, has vertices {*1,2,3,4,5,6*} and edge weights given by this table:

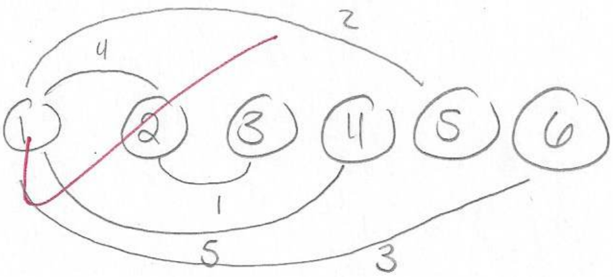
	1	***2***	***3***	***4***	***5***	***6***
1		4	7	5	2	3
2	4		1	9	14	5
3	7	1		10	8	12
4	5	9	10		13	6
5	2	14	8	13		7
6	3	5	12	6	7	

bold= lines we use

Find a minimum spanning tree for *G*. Show your work in a way that (somehow) indicates the correct use of Kruskal's algorithm:

6





-no circuits, no abandoned nodes, they all connect to one structure using least cost, I call it good !!

3

b) ~~For the same graph, G, find the shortest path from vertex 1 to vertex 6.~~ Again, show your work in a way that indicates the correct use of Dijkstra's algorithm:

I'm going to look at my graph from part a

vertex	path length
1	0
2	~~∞~~ $\boxed{4}$ ~~6~~ ~~8~~
3	~~∞~~ ~~7~~ ~~10~~ ~~X~~ $\boxed{5}$ ~~X~~
4	~~∞~~ $\boxed{5}$ ~~X~~ ~~X~~ ~~X~~
5	~~∞~~ $\boxed{2}$
6	~~∞~~ $\boxed{3}$ ~~X~~

?

The shortest path from vertex "1" to vertex "6" is of length 3.

c) Matrices *A*, *B*, *C*, and *D* have dimensions *4 x 6*, *6 x 5*, *5 x 3*, and *3 x 7*, respectively. If we wish to compute the matrix product *ABCD*, what's the optimum order for multiplying them? Show all work:

notes: Matrix multiplication is the domino puzzle in disguise sort of.

	A	B	C	D
A		AB	ABC	ABCD
B			BC	BCD
C				CD
D				

0