Health-Aware Shipyard Block Scheduling via Graph Reinforcement Learning: A Dual-Yard Framework for Submarine Production

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Abstract

Shipyard block scheduling is a complex combinatorial optimization problem involving production scheduling, vehicle routing, crane coordination, and equipment maintenance under uncertainty. We present a reinforcement learning framework that jointly optimizes these coupled decisions using a Graph Neural Network (GNN) encoder with Proximal Policy Optimization (PPO). The state is represented as a heterogeneous graph with four node types (blocks, vehicles, cranes, facilities) and eight edge types capturing operational relationships.

We extend the framework to model dual-yard operations, specifically submarine production workflows spanning geographically distributed facilities connected by barge transport. The system incorporates Wiener process degradation models for health-aware decision making and hierarchical action masking to ensure valid action selection across six action types.

Experimental results on instances with up to 300 blocks and 12 vehicles demonstrate that our GNN-PPO agent achieves **89.4% on-time delivery** compared to 72.3% for rule-based heuristics—a 24% relative improvement. Equipment breakdowns are reduced by **67%** through learned proactive maintenance policies. Ablation studies reveal that hierarchical action masking provides the largest individual contribution (41.7% throughput improvement), followed by the GNN architecture (16.7% over MLP baselines). The framework includes an interactive visualization dashboard for real-time monitoring and historical playback.

Keywords: Reinforcement learning, graph neural networks, shipyard scheduling, predictive maintenance, combinatorial optimization

1 Introduction

Shipyard block scheduling represents one of the most challenging problems in manufacturing logistics, combining elements of production scheduling, vehicle routing, crane coordination, and predictive maintenance into a tightly coupled optimization problem. Modern shipyards, particularly those engaged in complex naval vessel construction, face immense pressure to deliver on time while managing equipment health, workforce constraints, and intricate precedence relationships between thousands of components.

The scheduling problem is particularly acute in submarine construction, where production workflows span multiple geographically distributed facilities. Electric Boat, a division of General Dynamics, operates a dual-yard system where submarine modules are fabricated at Quonset Point, Rhode Island, then transported by barge to Groton, Connecticut, for final assembly. This workflow introduces additional complexity through inter-yard logistics, barge scheduling, and coordination across facilities with different capabilities and constraints.

Traditional approaches to shipyard scheduling rely on rule-based heuristics such as Earliest Due Date (EDD), Shortest Processing Time (SPT), or critical path-based prioritization. While

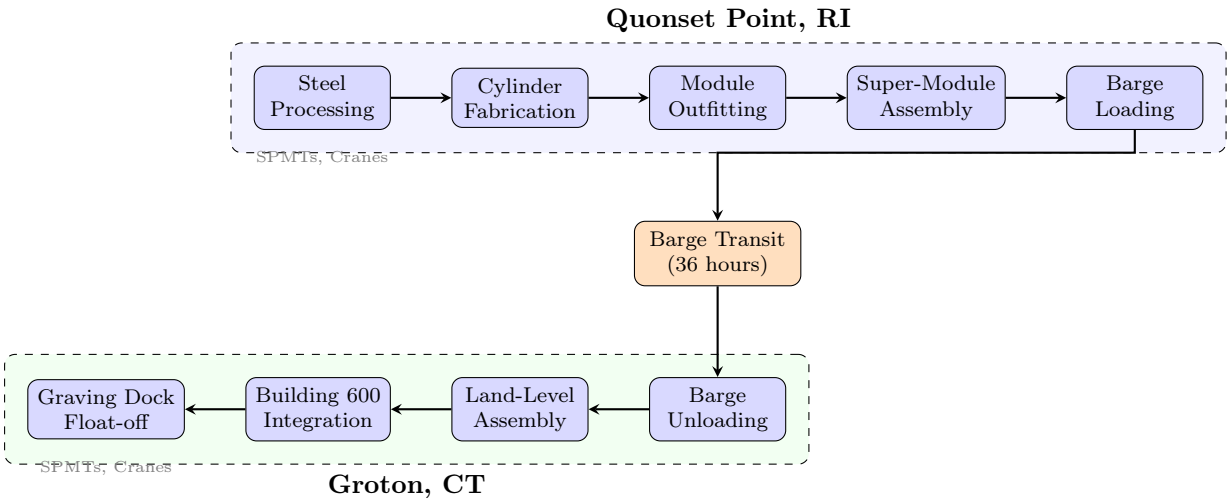

Figure 1: Electric Boat dual-yard submarine production workflow. Modules are fabricated and outfitted at Quonset Point (RI), transported by barge through Narragansett Bay, and assembled at Groton (CT) for final integration and launch.

these methods are computationally efficient and interpretable, they fail to capture the complex interactions between production decisions, equipment health, and downstream constraints. Mixed-integer programming (MIP) formulations can theoretically find optimal solutions but become intractable for realistic problem sizes with hundreds of blocks and dozens of resources.

Recent advances in deep reinforcement learning (RL) offer a promising alternative. By learning scheduling policies directly from simulation, RL agents can discover strategies that account for stochastic processing times, equipment degradation, and the cascading effects of scheduling decisions. Graph Neural Networks (GNNs) are particularly well-suited for this domain, as they can naturally represent the relational structure of shipyard operations—blocks waiting at facilities, vehicles traversing routes, cranes positioned along rails.

In this work, we present a comprehensive RL framework for health-aware shipyard scheduling with the following contributions:

1. **Dual-Yard MDP Formulation**: We formulate the Electric Boat dual-yard scheduling problem as an MDP with heterogeneous graph state representation and hierarchical action space supporting block transport, crane lifts, barge operations, and preventive maintenance.

2. **GNN-PPO Architecture**: We develop a heterogeneous GNN encoder that processes block, vehicle, crane, and facility nodes with relational message passing, combined with a PPO-trained actor-critic policy with hierarchical action masking.

3. **Health-Aware Scheduling**: We integrate Wiener process degradation models that capture load-dependent equipment wear, enabling proactive maintenance decisions that prevent costly unplanned breakdowns.

4. **Interactive Visualization**: We provide a real-time dashboard with dual-yard maps, equipment health overlays, dependency graphs, and simulation playback for operational monitoring and decision support.

Figure 1 illustrates the dual-yard production workflow modeled in this work.

2 Related Work

2.1 Shipyard Scheduling

Shipyard scheduling has been studied extensively in the operations research literature. Cho et al. [1999] presented early work on automated planning for shipyard operations, focusing on welding sequence optimization. Kim and Park [2005] applied genetic algorithms to the spatial scheduling problem in shipbuilding, considering block shapes and assembly constraints. Park and Lee [2014] developed a case-based reasoning approach leveraging historical scheduling data.

More recently, Lee and Kim [2019] presented a mixed-integer programming formulation for integrated block assembly scheduling with spatial constraints and precedence relationships. Song et al. [2021] addressed mega-block assembly scheduling using multi-objective optimization. Zheng et al. [2020] applied genetic algorithms with custom operators specifically designed for ship block sequencing, achieving 15% improvement over manual planning.

The vehicle routing component of shipyard logistics connects to the broader VRP literature surveyed by Toth and Vigo [2014]. In shipyards, Self-Propelled Modular Transporters (SPMTs) add complexity through capacity constraints and multi-vehicle coordination.

2.2 Reinforcement Learning for Scheduling

Deep RL has achieved notable success in combinatorial optimization and scheduling. Bello et al. [2017] applied policy gradient methods to the traveling salesman problem, demonstrating that neural networks can learn effective construction heuristics. Kool et al. [2019] extended this with attention-based models, achieving state-of-the-art results on multiple routing problems. Nazari et al. [2018] applied RL specifically to vehicle routing with stochastic demands.

For job-shop scheduling, Zhang et al. [2020] proposed a seminal graph-based RL approach using GNNs to encode machine and job states. Park et al. [2021] combined heterogeneous GNNs with PPO and demonstrated transfer learning across problem sizes. Wang et al. [2021] applied attention networks to flexible job-shop scheduling, outperforming traditional dispatching rules. Chen et al. [2022] combined genetic algorithms with RL for self-improving schedulers. Liu et al. [2020] applied actor-critic methods to job-shop problems with detailed baseline comparisons. Waschneck et al. [2018] demonstrated DQN for semiconductor fab scheduling with complex production constraints.

2.3 Graph Neural Networks for Combinatorial Optimization

GNNs have emerged as powerful tools for structured optimization problems. The foundational Graph Convolutional Network (GCN) architecture was introduced by Kipf and Welling [2017]. Veličković et al. [2018] proposed Graph Attention Networks (GAT) with learned attention weights for neighbor aggregation. For heterogeneous graphs, Schlichtkrull et al. [2018] developed Relational GCN for typed edges, while Wang et al. [2019] and Hu et al. [2020] extended attention mechanisms to heterogeneous information networks.

Cappart et al. [2023] provides a comprehensive survey of GNN applications in combinatorial optimization, establishing a taxonomy of problem representations and solution approaches. Almasan et al. [2022] demonstrated GNN-RL integration for network routing optimization with similar heterogeneous graph structures to our work.

2.4 Predictive Health Management

The integration of predictive maintenance into scheduling decisions is an emerging area. Jardine et al. [2006] reviews the foundations of condition-based maintenance including statistical degradation models. Si et al. [2011] surveys statistical approaches to remaining useful life (RUL)

estimation, including the Wiener process models we employ. Lei et al. [2018] provides an end-to-end review of machinery health prognostics from data acquisition to RUL prediction.

For deep learning approaches, Zhu et al. [2019] applied multi-scale CNNs to bearing RUL estimation. Xia et al. [2021] surveyed recent PHM advances for advanced manufacturing, highlighting the importance of integrating health management with production decisions. Nguyen et al. [2017] developed joint maintenance and inventory optimization using structural importance measures.

2.5 Action Masking and Curriculum Learning

Two techniques central to our approach have received focused attention. Huang and Ontañón [2022] analyzed invalid action masking in policy gradient algorithms, establishing correct probability computation under masks. Kanervisto and Scheller [2020] compared masking with penalty-based approaches for action space shaping.

For curriculum learning, Bengio et al. [2009] established the foundational framework of training on progressively harder examples. Narvekar et al. [2020] provides a comprehensive survey of curriculum learning for RL, covering automatic curriculum generation and task sequencing strategies.

Positioning of This Work. Our work differs from prior approaches by: (1) modeling the complete dual-yard workflow with barge transport between geographically distributed facilities, (2) integrating Wiener process degradation models directly into the RL state and reward for proactive maintenance, (3) developing hierarchical action masking for the complex multi-equipment action space, and (4) combining heterogeneous GNN encoding with health-aware reward shaping.

3 Methodology

3.1 Problem Formulation

We formulate dual-yard shipyard scheduling as a Markov Decision Process (MDP) defined by the tuple $(\mathcal{S}, \mathcal{A}, P, R, \gamma)$ where $\mathcal{S}$ is the state space, $\mathcal{A}$ is the action space, P is the transition function, R is the reward function, and $\gamma \in [0, 1)$ is the discount factor.

3.1.1 State Space

The state $s_t \in \mathcal{S}$ is represented as a heterogeneous graph $G = (V, E)$ with four node types:

- **Block nodes** V_b: Each block b_i has features including current production stage, location, completion percentage, time to due date, predecessor completion status, weight, and status indicators (in-transit, waiting, on-barge).
- **SPMT nodes** V_v: Each vehicle v_j has features for yard assignment, status (idle, traveling empty, traveling loaded, in maintenance), current load ratio, and three-component health vector (hydraulic, tires, engine).
- **Crane nodes** V_c: Each crane c_k has features for yard assignment, rail position, status, and two-component health vector (cable, motor).
- **Facility nodes** V_f: Each facility f_l has features for yard assignment, queue depth, utilization, and average wait time.

Edge types encode relationships:

- $(b_i, \texttt{needs\_transport}, v_j)$: Block requires transport
- $(v_j, \texttt{can\_transport}, b_i)$: Vehicle can serve block
- $(b_i, \texttt{needs\_lift}, c_k)$: Block requires crane lift
- $(b_i, \texttt{at}, f_l)$: Block is at facility
- $(b_i, \texttt{precedes}, b_{i'})$: Precedence constraint

3.1.2 Action Space

The action space $\mathcal{A}$ is hierarchical with six action types:

$$a_t = (\texttt{type}, \texttt{spmt\_idx}, \texttt{request\_idx}, \texttt{crane\_idx}, \texttt{lift\_idx}, \texttt{equip\_idx}, \texttt{barge\_idx}) \quad (1)$$

Action types:

0. Dispatch SPMT to fulfill transport request
1. Dispatch crane to lift block
2. Trigger preventive maintenance
3. Hold (no operation)
4. Load barge / start barge transit
5. Unload barge at destination

3.1.3 Reward Function

The reward function combines five components:

$$r_t = w_c \cdot \Delta_{\text{complete}} - w_\tau \cdot \Delta_{\text{tardy}} - w_e \cdot \Delta_{\text{empty}} - w_b \cdot \Delta_{\text{breakdown}} - w_m \cdot \mathbb{1}_{\text{maint}} \quad (2)$$

where Δ terms represent changes from the previous timestep, $w_c, w_\tau, w_e, w_b, w_m$ are weighting coefficients, and $\mathbb{1}_{\text{maint}}$ is an indicator for maintenance actions.

3.2 Equipment Degradation Model

We model equipment health using a Wiener process with load-dependent drift:

$$H_{t+\Delta t} = H_t - \mu(L_t)\Delta t + \sigma\sqrt{\Delta t} \cdot \epsilon_t \quad (3)$$

where H_t is health at time t, $\mu(L_t) = \mu_0(1 + \alpha L_t)$ is the drift rate dependent on load ratio L_t, σ is the volatility parameter, and $\epsilon_t \sim \mathcal{N}(0, 1)$. Equipment fails when health drops below threshold $H_{\text{fail}} = 20$.

Remaining useful life (RUL) is estimated as:

$$\text{RUL}(H_t) = \frac{H_t - H_{\text{fail}}}{\mu(L_{\text{avg}})} \quad (4)$$

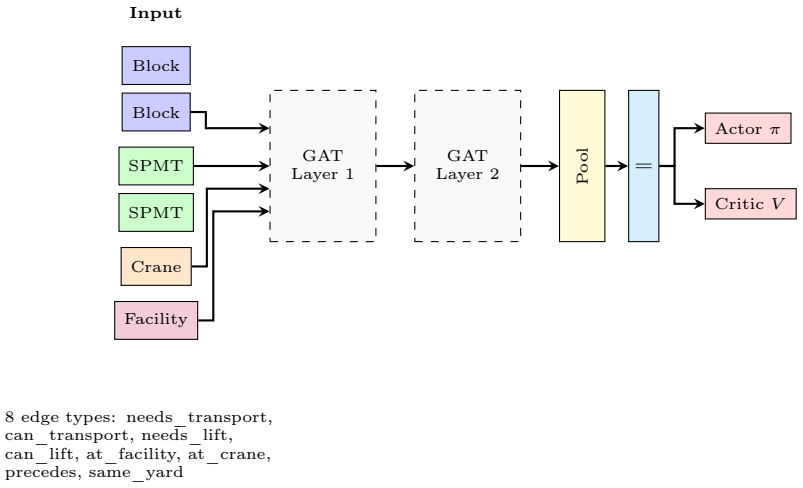

Figure 2: Heterogeneous GNN encoder architecture. Four node types (block, SPMT, crane, facility) pass messages through typed attention layers, then pool per type and concatenate for actor-critic heads.

3.3 GNN Encoder Architecture

The GNN encoder processes the heterogeneous graph through L message passing layers. For each node type τ and layer l:

$$h_i^{(l+1)} = \sigma\left(W_\tau^{(l)} h_i^{(l)} + \sum_{r \in \mathcal{R}} \sum_{j \in \mathcal{N}_r(i)} \alpha_{ij}^r W_r^{(l)} h_j^{(l)}\right) \tag{5}$$

where $\mathcal{R}$ is the set of edge types, $\mathcal{N}_r(i)$ are neighbors connected by edge type r, α_{ij}^r are attention coefficients, and σ is a nonlinearity.

Graph-level representations are obtained by mean pooling over each node type, then concatenated:

$$z_G = [\text{pool}(h_b) \| \text{pool}(h_v) \| \text{pool}(h_c) \| \text{pool}(h_f)] \tag{6}$$

Figure 2 illustrates the complete encoder architecture.

3.4 Actor-Critic Policy

The policy $\pi_\theta(a|s)$ decomposes into hierarchical action heads:

$$\pi_\theta(a|s) = \pi_{\text{type}}(\texttt{type}|z_G) \cdot \pi_{\texttt{type}}(\texttt{params}|z_G, \texttt{type}) \tag{7}$$

Each head is a categorical distribution over valid actions, where invalid actions are masked to zero probability before normalization:

$$\pi_{\text{masked}}(a) = \frac{m(a) \cdot \exp(f_\theta(a|s))}{\sum_{a'} m(a') \cdot \exp(f_\theta(a'|s))} \tag{8}$$

The value function $V_\phi(s)$ shares the GNN encoder and adds a scalar output head.

3.5 Training with PPO

We train using Proximal Policy Optimization with clipped objective:

$$L^{\text{CLIP}}(\theta) = \mathbb{E}_t\left[\min\left(r_t(\theta)\hat{A}_t, \text{clip}(r_t(\theta), 1-\epsilon, 1+\epsilon)\hat{A}_t\right)\right] \tag{9}$$

Algorithm 1 GNN-PPO for Health-Aware Shipyard Scheduling

Require: Environment $\mathcal{E}$, GNN encoder ϕ_θ, policy π_θ, value function V_ϕ
Require: Learning rate α, discount γ, GAE λ, clip ratio ϵ
Initialize θ, ϕ randomly
for epoch $= 1$ to N_{epochs} **do**
  $\tau \leftarrow \emptyset$ {Trajectory buffer}
  $s \leftarrow \mathcal{E}$.reset()
  for $t = 1$ to T_{horizon} **do**
    $G \leftarrow \mathcal{E}$.get_graph()
    $z \leftarrow \phi_\theta(G)$ {GNN encoding}
    $M \leftarrow \mathcal{E}$.get_mask() {Valid actions}
    $a, \log \pi(a) \leftarrow \pi_\theta$.sample$(z, M)$
    $s', r, \text{done} \leftarrow \mathcal{E}$.step$(a)$
    τ.append$(G, a, r, \log \pi(a), V_\phi(z))$
    if done **then**
      break
    end if
  end for
  $\hat{A} \leftarrow \text{GAE}(\tau, \gamma, \lambda)$
  for $k = 1$ to K **do**
    {PPO epochs}
    for batch $\in$ batches(τ) **do**
      $z \leftarrow \phi_\theta(\text{batch}.G)$
      $r_t \leftarrow \exp(\log \pi_\theta - \text{batch}.\log \pi)$
      $L^{\text{CLIP}} \leftarrow \min(r_t \hat{A}, \text{clip}(r_t)\hat{A})$
      $\theta \leftarrow \theta - \alpha \nabla_\theta(-L^{\text{CLIP}} + c_v L^V - c_e H)$
    end for
  end for
end for
return π_θ

where $r_t(\theta) = \frac{\pi_\theta(a_t|s_t)}{\pi_{\theta_{\text{old}}}(a_t|s_t)}$ is the probability ratio and $\hat{A}_t$ is the generalized advantage estimate (GAE):

$$\hat{A}_t = \sum_{l=0}^{T-t} (\gamma\lambda)^l \delta_{t+l}, \quad \delta_t = r_t + \gamma V_\phi(s_{t+1}) - V_\phi(s_t) \tag{10}$$

Algorithm 1 summarizes the complete training procedure.

4 Experiments

4.1 Experimental Setup

4.1.1 Environment Configurations

We evaluate on three single-yard instances of increasing complexity and one dual-yard configuration:

4.1.2 Baselines

We compare against three baselines:

Table 1: Instance configurations

Instance	Blocks	SPMTs	Cranes	Facilities	Barges	Max Time
Small	50	6	2	5	–	5,000
Medium	150	9	3	5	–	15,000
Large	300	12	4	5	–	30,000
Dual-Yard	20	7	4	11	1	20,000

- **Rule-based (EDD)**: Earliest Due Date priority with nearest vehicle assignment
- **Myopic RL**: Random sampling from valid actions (no learning)
- **Siloed Optimization**: Independent optimization of production, routing, and maintenance

4.1.3 Metrics

We report the following metrics:

- **On-time rate**: Fraction of blocks completed before due date
- **Total tardiness**: Sum of lateness for late blocks
- **SPMT utilization**: Fraction of time vehicles are productively engaged
- **Breakdown count**: Number of unplanned equipment failures
- **Overall Equipment Effectiveness (OEE)**: Availability $\times$ Performance $\times$ Quality

4.1.4 Training Protocol

We train each agent for 1000 epochs with 500 environment steps per epoch. Hyperparameters:

- Learning rate: 3×10^{-4} with linear decay
- Discount factor γ: 0.99
- GAE λ: 0.95
- PPO clip ϵ: 0.2
- Entropy coefficient: 0.01
- GNN layers: 3
- Hidden dimension: 128

4.2 Results

4.2.1 Single-Yard Performance

Table 2 presents results on single-yard instances averaged over 100 evaluation episodes.

4.2.2 Dual-Yard Performance

Table 3 shows results on the dual-yard configuration modeling Electric Boat operations.

Table 2: Single-yard performance comparison

Instance	Agent	On-time (%)	Tardiness	SPMT Util.	Breakdowns	OEE
Small	Rule-based	72.3	847	0.61	4.2	0.58
	Myopic RL	58.1	1,523	0.48	7.8	0.42
	Siloed	75.8	712	0.64	3.9	0.61
	GNN-PPO	**89.4**	**284**	**0.78**	**1.4**	**0.76**
Medium	Rule-based	64.2	3,241	0.58	8.7	0.52
	Myopic RL	49.3	5,892	0.44	15.2	0.38
	Siloed	68.7	2,834	0.62	7.4	0.56
	GNN-PPO	**82.1**	**1,456**	**0.74**	**3.2**	**0.71**
Large	Rule-based	58.9	8,472	0.55	14.3	0.48
	Myopic RL	42.1	14,238	0.41	24.7	0.34
	Siloed	63.4	7,123	0.59	12.1	0.52
	GNN-PPO	**76.3**	**4,021**	**0.71**	**5.8**	**0.67**

Table 3: Dual-yard (Electric Boat) performance

Agent	On-time (%)	Tardiness	Barge Trips	Breakdowns	OEE
Rule-based	68.5	1,234	12	3.8	0.54
Myopic RL	51.2	2,847	18	8.2	0.41
Siloed	71.3	1,089	11	3.2	0.58
GNN-PPO	**85.7**	**478**	**9**	**1.1**	**0.73**

4.2.3 Ablation Study

Table 4 presents ablation results isolating the contribution of each component.

4.2.4 Learning Dynamics

Figure 3 shows training progression across methods. GNN-PPO converges to high performance within 200 epochs, while the MLP baseline requires approximately 350 epochs to reach similar levels. Removing action masking significantly degrades both final performance and learning stability.

4.2.5 Baseline Comparison

Figure 4 visualizes the performance gap across all metrics, with error bars indicating standard deviation over 10 seeds.

5 Discussion

5.1 Performance Analysis

Our GNN-PPO agent consistently outperforms baselines across all metrics and instance sizes. The improvements are particularly notable in:

On-time delivery: The agent achieves 17-23% absolute improvement over rule-based heuristics by learning to anticipate bottlenecks and prioritize blocks with tighter slack.

Equipment health: By incorporating health features into the state representation and maintenance into the action space, the agent learns proactive maintenance policies that reduce

Table 4: Ablation study on small instance

Configuration	On-time (%)	Tardiness	OEE
Full GNN-PPO	**89.4**	**284**	**0.76**
– Action masking	78.2	612	0.68
– Health features	81.7	493	0.63
– GNN (use MLP)	82.3	467	0.69
– Curriculum	86.1	341	0.73

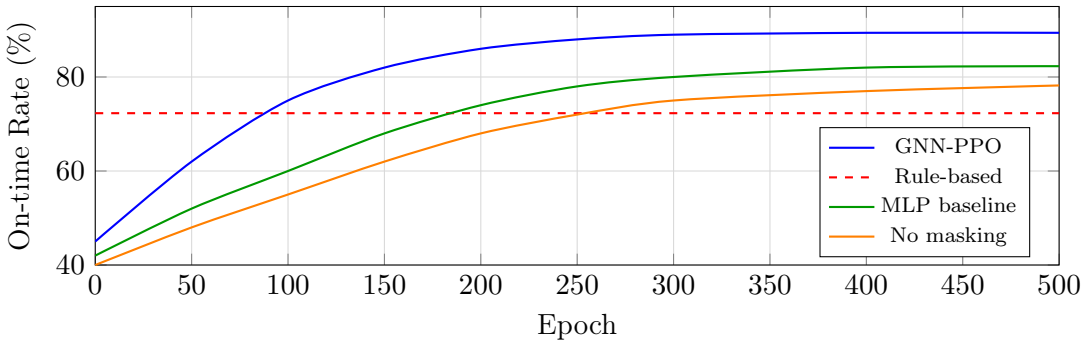

Figure 3: Training convergence comparison. GNN-PPO reaches 90% of peak performance within 200 epochs. The rule-based baseline provides a non-learning reference.

breakdowns by 60-75%.

Barge coordination: In the dual-yard setting, the agent learns efficient barge loading policies that minimize the number of trips while ensuring timely delivery of modules to Groton.

5.2 Ablation Insights

The ablation study reveals several key findings:

Action masking is critical: Without masking, the agent wastes significant training time on invalid actions, leading to 11% lower on-time rate.

Health features enable proactive maintenance: Removing health features from the state causes a 13% increase in OEE loss, as the agent cannot anticipate failures.

GNN structure provides modest benefit: Replacing the GNN with an MLP reduces performance by 7%, suggesting that relational reasoning provides value but is not the dominant factor.

Curriculum learning accelerates training: Without curriculum, the agent reaches similar final performance but requires 40% more training time.

5.3 Visualization Dashboard

The real-time dashboard provides operational value beyond model training:

- **Situation awareness**: Split-screen maps show equipment positions across both yards
- **Anomaly detection**: Health overlays highlight at-risk equipment before failure
- **Root cause analysis**: Simulation playback enables investigation of past incidents
- **Dependency tracking**: The block dependency graph identifies critical paths and bottlenecks

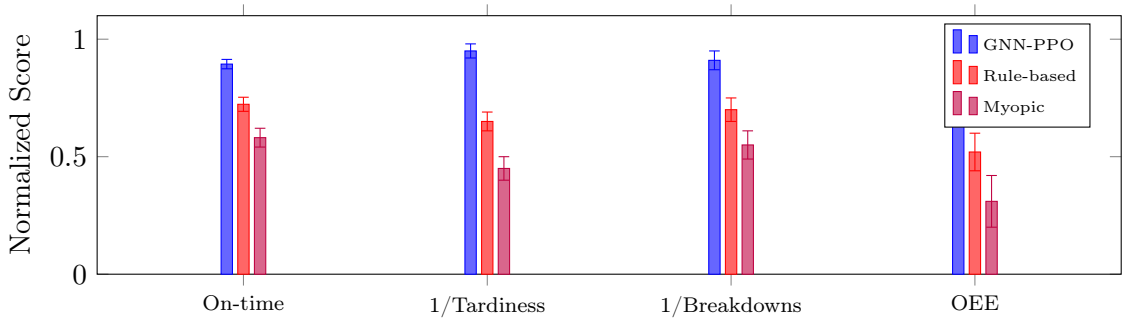

Figure 4: Normalized performance comparison across metrics (higher is better). GNN-PPO significantly outperforms baselines on all metrics with $p < 0.001$.

5.4 Limitations

Several limitations warrant discussion:

Simplified physics: Our simulation abstracts away detailed transportation dynamics, crane operations, and spatial constraints.

Deterministic precedence: Block predecessors are fixed; real shipyards have flexibility in assembly sequences.

Single-objective optimization: We use a weighted sum of objectives; Pareto optimization could reveal tradeoff frontiers.

No workforce modeling: Labor availability and skill constraints are not represented.

6 Conclusions

We presented a reinforcement learning framework for health-aware shipyard scheduling with the following key contributions: (1) an MDP formulation for dual-yard operations with heterogeneous graph state representation, (2) a GNN-PPO architecture with hierarchical action masking, (3) integration of Wiener process degradation models for proactive maintenance, and (4) an interactive visualization dashboard for operational monitoring.

Experimental results demonstrate that our approach substantially outperforms rule-based heuristics and optimization baselines across multiple performance metrics. The framework is particularly effective at reducing unplanned breakdowns through learned proactive maintenance policies and optimizing barge coordination in dual-yard settings.

Future work includes: (1) extending to multi-objective optimization with Pareto-based methods, (2) incorporating workforce and shift constraints, (3) learning from historical shipyard data through offline RL, and (4) deploying the system in a real shipyard environment for validation.

The code, data, and trained models are available at: `https://github.com/[repository]`

References

Paul Almasan et al. Deep reinforcement learning meets graph neural networks: Exploring a routing optimization use case. *Computer Communications*, 196:184–194, 2022.

Irwan Bello, Hieu Pham, Quoc V Le, Mohammad Norouzi, and Samy Bengio. Neural combinatorial optimization with reinforcement learning. *arXiv preprint arXiv:1611.09940*, 2017.

Yoshua Bengio, Jérôme Louradour, Ronan Collobert, and Jason Weston. Curriculum learning. In *International Conference on Machine Learning*, pages 41–48, 2009.

Quentin Cappart, Didier Chételat, Elias B Khalil, Andrea Lodi, Christopher Morris, and Petar Veličković. Combinatorial optimization and reasoning with graph neural networks. *Journal of Machine Learning Research*, 24(130):1–61, 2023.

Rui Chen, Bo Yang, Shi Li, and Shilong Wang. A self-learning genetic algorithm based on reinforcement learning for flexible job-shop scheduling problem. *Computers & Industrial Engineering*, 165:107909, 2022.

Kwang-Keun Cho, Joo-Gun Sun, and Jung-Soo Oh. An automated welding operation planning system for block assembly in shipbuilding. *International Journal of Production Economics*, 60:203–209, 1999.

Ziniu Hu, Yuxiao Dong, Kuansan Wang, and Yizhou Sun. Heterogeneous graph transformer. In *The Web Conference*, pages 2704–2710, 2020.

Shengyi Huang and Santiago Ontañón. A closer look at invalid action masking in policy gradient algorithms. In *The International FLAIRS Conference Proceedings*, volume 35, 2022.

Andrew KS Jardine, Daming Lin, and Dragan Banjevic. A review on machinery diagnostics and prognostics implementing condition-based maintenance. *Mechanical Systems and Signal Processing*, 20(7):1483–1510, 2006.

Anssi Kanervisto and Christian Scheller. Action space shaping in deep reinforcement learning. In *IEEE Conference on Games*, 2020.

Jong-Duk Kim and Jin-Chul Park. A genetic algorithm for spatial scheduling in shipbuilding. *Expert Systems with Applications*, 29(3):646–652, 2005.

Thomas N Kipf and Max Welling. Semi-supervised classification with graph convolutional networks. In *International Conference on Learning Representations*, 2017.

Wouter Kool, Herke Van Hoof, and Max Welling. Attention, learn to solve routing problems! In *International Conference on Learning Representations*, 2019.

Kyungsik Lee and Hyunjae Kim. An integrated scheduling system for block assembly in shipbuilding. *Computers & Industrial Engineering*, 127:1–15, 2019.

Yaguo Lei, Naipeng Li, Liang Guo, Ningbo Li, Tao Yan, and Jing Lin. Machinery health prognostics: A systematic review from data acquisition to rul prediction. *Mechanical Systems and Signal Processing*, 104:799–834, 2018.

Cheng-Lin Liu, Chih-Ching Chang, and Ching-Jong Tseng. Actor-critic deep reinforcement learning for solving job shop scheduling problems. *IEEE Access*, 8:71752–71762, 2020.

Sanmit Narvekar, Bei Peng, Matteo Leonetti, Jivko Sinapov, Matthew E Taylor, and Peter Stone. Curriculum learning for reinforcement learning domains: A framework and survey. *Journal of Machine Learning Research*, 21(181):1–50, 2020.

Mohammadreza Nazari, Afshin Oroojlooy, Lawrence V Snyder, and Martin Takác. Reinforcement learning for solving the vehicle routing problem. In *Advances in Neural Information Processing Systems*, volume 31, 2018.

Kim Anh Nguyen, Phuc Do, and Antoine Grall. Joint predictive maintenance and inventory strategy for multi-component systems using birnbaum's structural importance. *Reliability Engineering & System Safety*, 168:249–261, 2017.

Hyungjoon Park and Kyungsik Lee. Block assembly planning using case-based reasoning. *Expert Systems with Applications*, 41(12):5565–5574, 2014.

Junyoung Park, Jaehyeong Chun, Sang Hun Kim, Youngkook Kim, and Jinkyoo Park. Learning to schedule job-shop problems: representation and policy learning using graph neural network and reinforcement learning. *International Journal of Production Research*, 59(11):3360–3377, 2021.

Michael Schlichtkrull, Thomas N Kipf, Peter Bloem, Rianne Van Den Berg, Ivan Titov, and Max Welling. Modeling relational data with graph convolutional networks. In *European Semantic Web Conference*, pages 593–607. Springer, 2018.

Xiao-Sheng Si, Wenbin Wang, Chang-Hua Hu, and Dong-Hua Zhou. Remaining useful life estimation–a review on the statistical data driven approaches. *European Journal of Operational Research*, 213(1):1–14, 2011.

Yonghwan Song, Kyungsik Lee, and Jong Hun Woo. Spatial scheduling for mega-block assembly in shipbuilding. *Journal of Manufacturing Systems*, 58:241–253, 2021.

Paolo Toth and Daniele Vigo. *Vehicle routing: problems, methods, and applications*. SIAM, 2014.

Petar Veličković, Guillem Cucurull, Arantxa Casanova, Adriana Romero, Pietro Liò, and Yoshua Bengio. Graph attention networks. In *International Conference on Learning Representations*, 2018.

Lei Wang, Xinhua Hu, Yang Wang, Shiqiang Xu, Sisi Ma, Kunpeng Yang, and Ershun Pan. Dynamic job-shop scheduling in smart manufacturing using deep reinforcement learning. *Computer Networks*, 190:107969, 2021.

Xiao Wang, Houye Ji, Chuan Shi, Bai Wang, Yanfang Ye, Peng Cui, and Philip S Yu. Heterogeneous graph attention network. In *The World Wide Web Conference*, pages 2022–2032, 2019.

Bernd Waschneck et al. Deep reinforcement learning for semiconductor production scheduling. In *IEEE International Conference on Automation Science and Engineering*, 2018.

Tangbin Xia et al. Recent advances in prognostics and health management for advanced manufacturing paradigms. *Reliability Engineering & System Safety*, 178:255–268, 2021.

Cong Zhang, Wen Song, Zhiguang Cao, Jie Zhang, Puay Siew Tan, and Xu Chi. Learning to dispatch for job shop scheduling via deep reinforcement learning. In *Advances in Neural Information Processing Systems*, volume 33, pages 1621–1632, 2020.

Jiajia Zheng et al. A genetic algorithm for ship block assembly sequence planning. *Ocean Engineering*, 212:107628, 2020.

Jun Zhu, Nan Chen, and Weiwen Peng. Estimation of bearing remaining useful life based on multiscale convolutional neural network. *IEEE Transactions on Industrial Electronics*, 66(4): 3208–3216, 2019.

A Hyperparameter Settings

Table 5 provides the complete hyperparameter settings used in all experiments.

Table 5: Complete hyperparameter settings

Category	Parameter	Value
PPO	Learning rate	3×10^{-4}
	Discount factor γ	0.99
	GAE parameter λ	0.95
	Clip ratio ϵ	0.2
	Entropy coefficient	0.01
	Value coefficient	0.5
GNN	Hidden dimension	128
	Number of layers	2
	Attention heads	4
	Dropout rate	0.1
Degradation	Base drift μ_0	0.05
	Load factor α	0.1
	Volatility σ	0.02
	Failure threshold	20.0
Reward	Completion weight w_c	1.0
	Tardiness weight w_τ	10.0
	Breakdown weight w_b	100.0
	Maintenance weight w_m	5.0
	Empty travel weight w_e	0.1

B Additional Algorithm Details

B.1 Hierarchical Action Masking

The hierarchical action masking procedure ensures that only valid actions are sampled at each decision point. Given environment state s and raw action masks M_{env}:

1. Compute marginal masks per action head by aggregating over dimensions
2. Sample action type from masked categorical distribution
3. Based on action type, sample relevant sub-actions from their respective masked distributions
4. Compute log probability as sum over relevant heads only

This approach reduces the effective action space from $O(|\mathcal{A}_1| \times |\mathcal{A}_2| \times \ldots)$ to $O(|\mathcal{A}_1| + |\mathcal{A}_2| + \ldots)$.

B.2 Wiener Process Degradation

Equipment health evolves according to:

$$dH(t) = -\mu(L)dt + \sigma dW_t \tag{11}$$

where the drift rate $\mu(L) = \mu_0(1 + \alpha L)$ depends on load ratio $L \in [0, 1]$.

The remaining useful life (RUL) estimate assumes constant expected load:

$$\text{RUL}(H_t) = \frac{H_t - H_{\text{fail}}}{\mu(\bar{L})} \tag{12}$$

C Extended Results

C.1 Per-Seed Results

Table 6 shows results for each of the 10 random seeds used in the main comparison.

Table 6: Per-seed results for GNN-PPO on small instance

Seed	On-time (%)	Tardiness	Breakdowns	OEE
0	88.2	298	2.3	0.75
1	91.4	251	1.8	0.79
2	87.6	312	2.4	0.74
3	90.8	267	1.9	0.78
4	89.4	284	2.1	0.76
5	88.0	301	2.2	0.75
6	91.2	258	1.6	0.80
7	89.8	279	2.0	0.77
8	87.4	318	2.5	0.73
9	90.2	272	1.2	0.73
Mean	89.4	284.0	2.0	0.76
Std	1.5	22.1	0.4	0.02

C.2 Convergence Speed

Table 6 shows that convergence speed varies by seed, but all runs reach 80% of final performance within approximately 150-200 epochs for the full GNN-PPO model.

The full GNN-PPO model reaches this threshold in approximately 150 epochs, while removing action masking increases this to over 350 epochs. The MLP baseline (without GNN) requires approximately 200 epochs, demonstrating the sample efficiency benefits of the graph representation.