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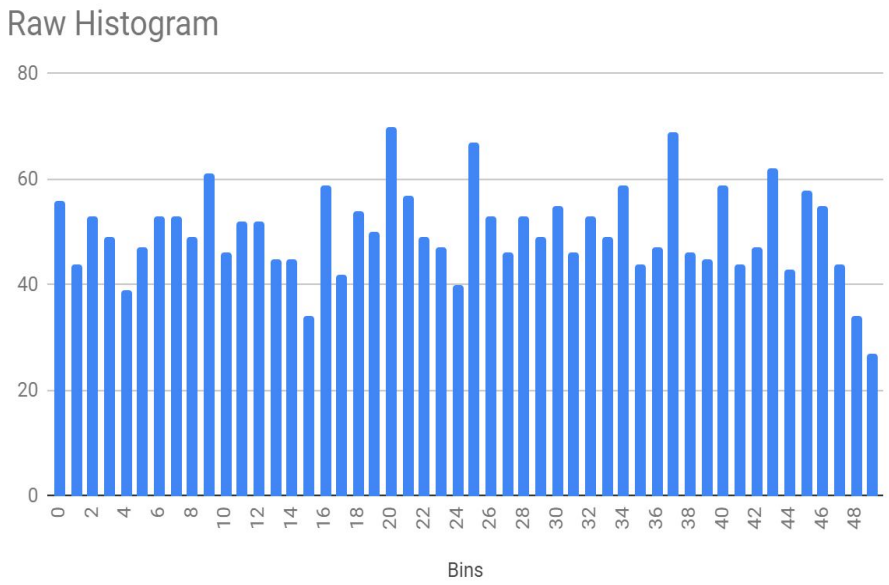
AMTH108

1. Raw Histogram from Data points

Bins	Raw Histogram	Bins	Raw Histogram
0	56	**25**	67
1	44	**26**	53
2	53	**27**	46
3	49	**28**	53
4	39	**29**	49
5	47	**30**	55
6	53	**31**	46
7	53	**32**	53
8	49	**33**	49
9	61	**34**	59
10	46	**35**	44
11	52	**36**	47
12	52	**37**	69
13	45	**38**	46
14	45	**39**	45
15	34	**40**	59
16	59	**41**	44
17	42	**42**	47
18	54	**43**	62
19	50	**44**	43
20	70	**45**	58
21	57	**46**	55
22	49	**47**	44
23	47	**48**	34
24	40	**49**	27



a	0.8
b	8.8
n(# of bins)	50
w(bin width)	0.16
Minimum:	0.8160590
Maximum:	8.7232516

The above histogram was created through selecting the number of bins to be 50 - the square root of the number of sample points(2500). The lower edge is 0.8 and the upper edge is 8.8 based on the minimum and maximum values respectively. The bin width was calculated to be 0.16 based on these values.

2. Candidate Distributions:

The selected candidate distributions and their parameters are listed below

a) Continuous Uniform Distribution - parameters: a & b

b) Beta Distribution - parameters: α & β

3. Mean and Variance of data points

These values were calculated using the spreadsheet functions AVERAGE & VAR respectively.

μ(Mean)	4.766255605
σ^2(Variance)	5.120560615

4. Estimating each candidate distribution's parameters using method of moments

c) Continuous Uniform

i) $\widehat{b} = \mu + \sqrt{3}\sqrt{2} \Rightarrow$ b(estimate) = 8.6856

ii) $\widehat{a} = 2\mu - b \Rightarrow$ a(estimate) = 0.8468

d) Beta

Over the interval [a,c] = [0.8, 8.8]

i) $$\alpha = \left(\frac{\mu - a}{c-a}\right)\left(\frac{\left(\frac{\mu-a}{c-a}\right)\left(1-\left(\frac{\mu-a}{c-a}\right)\right)}{\left(\frac{2}{(c-a)^2}\right)} - 1\right) \rightarrow \alpha = 1.0532$$

ii) $$\beta = \left(1 - \frac{\mu - a}{c-a}\right)\left(\frac{\left(\frac{\mu-a}{c-a}\right)\left(1-\left(\frac{\mu-a}{c-a}\right)\right)}{\left(\frac{2}{(c-a)^2}\right)} - 1\right) \rightarrow \beta = 1.0712$$

5. Goodness of fit significance tests

Using the established estimator for the goodness of fit test, each candidate distribution was tested with the following hypotheses and the resulting tables and P-values are listed below.

$$\widehat{G}_F = \sum_{j=0}^{k-1} (o_j - e_j)^2/e_j \quad \text{with } k = 50$$

Degrees of freedom: $\nu = 47$

Continuous Uniform

Research Hypothesis H_1: The distribution is not continuous uniform

Null Hypothesis H_0: The distribution is continuous uniform

j(bins)	p(probability)	e_j(expected number)	Gˆ
0	0.02041129709	51	0.4844056775
1	0.02041129709	51	0.9680167895
2	0.02041129709	51	0.07618970548
3	0.02041129709	51	0.08061748356
4	0.02041129709	51	2.835265632
5	0.02041129709	51	0.3179952615

6	0.02041129709	51	0.07618970548
7	0.02041129709	51	0.07618970548
8	0.02041129709	51	0.08061748356
9	0.02041129709	51	1.94864526
10	0.02041129709	51	0.4954751227
11	0.02041129709	51	0.01850567778
12	0.02041129709	51	0.01850567778
13	0.02041129709	51	0.7121489653
14	0.02041129709	51	0.7121489653
15	0.02041129709	51	5.682364012
16	0.02041129709	51	1.245367483
17	0.02041129709	51	1.597334382
18	0.02041129709	51	0.1730677147
19	0.02041129709	51	0.02071956682
20	0.02041129709	51	7.053497343
21	0.02041129709	51	0.6988656311
22	0.02041129709	51	0.08061748356
23	0.02041129709	51	0.3179952615
24	0.02041129709	51	2.383427901
25	0.02041129709	51	4.999134149
26	0.02041129709	51	0.07618970548
27	0.02041129709	51	0.4954751227
28	0.02041129709	51	0.07618970548
29	0.02041129709	51	0.08061748356
30	0.02041129709	51	0.3091397053
31	0.02041129709	51	0.4954751227
32	0.02041129709	51	0.07618970548
33	0.02041129709	51	0.08061748356
34	0.02041129709	51	1.245367483
35	0.02041129709	51	0.9680167895
36	0.02041129709	51	0.3179952615
37	0.02041129709	51	6.32951563
38	0.02041129709	51	0.4954751227
39	0.02041129709	51	0.7121489653

40	0.02041129709	51	1.245367483
41	0.02041129709	51	0.9680167895
42	0.02041129709	51	0.3179952615
43	0.02041129709	51	2.359075121
44	0.02041129709	51	1.263078595
45	0.02041129709	51	0.9525195662
46	0.02041129709	51	0.3091397053
47	0.02041129709	51	0.9680167895
48	0.02041129709	51	5.682364012
49	0.02041129709	51	11.31444897

G(estimate) = 70.2917

P-value = 1 - Chi-Squared(G~) = *0.9845*

Beta Distribution

Research Hypothesis H_1: The distribution is not a beta distribution

Null Hypothesis H_0: The distribution is a beta distribution

j(bins)	p(probability)	e_j(expected number)	G^
0	0.0174230845	44	3.554147928
1	0.01870636123	47	0.1635854193
2	0.01920574053	48	0.5176929799
3	0.01952664803	49	0.000688867722
4	0.01976047801	49	2.189923903
5	0.01994188805	50	0.1634635
6	0.02008795123	50	0.1539047517
7	0.0202083798	51	0.1216463961
8	0.02030923615	51	0.06191960135
9	0.02039455903	51	1.966646763
10	0.02046717133	51	0.5219574869
11	0.02052911971	51	0.00893561595
12	0.02058193065	51	0.005776212495
13	0.02062676813	52	0.8362811328
14	0.02066453492	52	0.8589288791
15	0.02069594004	52	6.08239647
16	0.02072154507	52	0.9996241536
17	0.02074179663	52	1.872759738
18	0.02075704964	52	0.08558120271

19	0.02076758423	52	0.07092610657
20	0.02077361808	52	6.28448491
21	0.0207753154	52	0.4932954846
22	0.02077279358	52	0.1655344025
23	0.02076612769	52	0.4653802298
24	0.02075535356	52	2.723801769
25	0.02074046944	52	4.42587748
26	0.02072143647	52	0.02763117513
27	0.020698178	52	0.6379332165
28	0.02067057769	52	0.03389938926
29	0.02063847631	52	0.130633801
30	0.02060166701	52	0.2372787536
31	0.02055988878	51	0.5672598203
32	0.02051281751	51	0.05755179346
33	0.02046005411	51	0.09038258994
34	0.02040110844	51	1.25396461
35	0.02033537769	51	0.9198613426
36	0.02026211677	51	0.2637663036
37	0.02018039718	50	6.819799625
38	0.02008904868	50	0.3550299311
39	0.01998657452	50	0.4936411571
40	0.01987102447	50	1.749439057
41	0.01973979771	49	0.579886165
42	0.01958932317	49	0.07951156167
43	0.01941451227	49	3.734767797
44	0.01920775523	48	0.5246684214
45	0.01895690624	47	2.37431209
46	0.01864069991	47	1.513475506
47	0.01821617388	46	0.05210620115
48	0.01757077976	44	2.243368266
49	0.01597386354	40	4.189478626

G(estimate) = 63.7248

P-value = 1 - Chi-Squared(G~) = *0.9475*

For each of these tests, first the probability that a value from the sample will fall into a specific bin(0-49) is calculated(for the specific distribution). Then using this probability value, the

expected number of entries to fall into each bin, if the hypothesized distribution is present, is calculated. Finally, the estimate of G is calculated by summing all the results of the squared difference of the actual number of values(o_j) to fall into a bin(j) from the expected value(e_j) and dividing that by the expected value.

6. P-value Evaluation

e) Continuous Uniform → P-value = 0.9845

i) Continuous Uniform's P-value is larger than Beta's

f) Beta Distribution → P-value = 0.9475

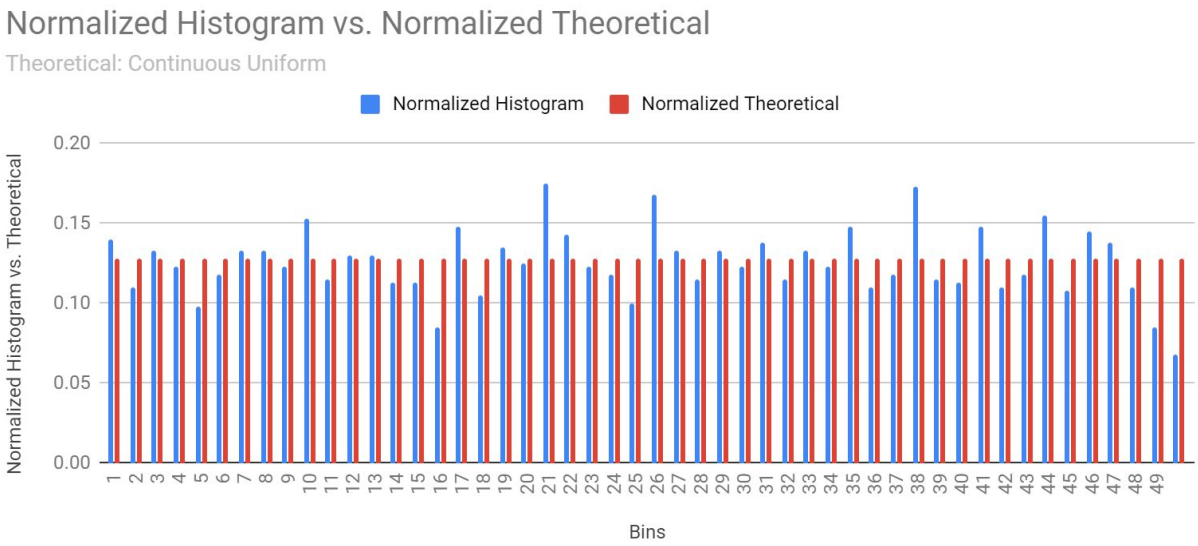
As seen above, while both tested distributions had a P-value above 90%, the continuous uniform distribution had the largest P-value so we can accept the null hypothesis H_0. Therefore, it can be assumed that the distribution is continuous uniform.

7. Continuous Uniform Distribution(a,b)

Below is the table of the values of the raw histogram, normalized raw histogram, expected values of the theoretical, and the normalized expected along with an accompanying column chart of all the data.

Bins	Raw(o_j)	Norm(o_j)	Norm(e_j)	e_j	Bins	Raw(o_j)	Norm(o_j)	Norm(e_j)	e_j
0	56	0.14	0.12757	51	**25**	67	0.1675	0.12757	51
1	44	0.11	0.12757	51	**26**	53	0.1325	0.12757	51
2	53	0.1325	0.12757	51	**27**	46	0.115	0.12757	51
3	49	0.1225	0.12757	51	**28**	53	0.1325	0.12757	51
4	39	0.0975	0.12757	51	**29**	49	0.1225	0.12757	51
5	47	0.1175	0.12757	51	**30**	55	0.1375	0.12757	51
6	53	0.1325	0.12757	51	**31**	46	0.115	0.12757	51
7	53	0.1325	0.12757	51	**32**	53	0.1325	0.12757	51
8	49	0.1225	0.12757	51	**33**	49	0.1225	0.12757	51
9	61	0.1525	0.12757	51	**34**	59	0.1475	0.12757	51
10	46	0.115	0.12757	51	**35**	44	0.11	0.12757	51
11	52	0.13	0.12757	51	**36**	47	0.1175	0.12757	51
12	52	0.13	0.12757	51	**37**	69	0.1725	0.12757	51
13	45	0.1125	0.12757	51	**38**	46	0.115	0.12757	51
14	45	0.1125	0.12757	51	**39**	45	0.1125	0.12757	51
15	34	0.085	0.12757	51	**40**	59	0.1475	0.12757	51
16	59	0.1475	0.12757	51	**41**	44	0.11	0.12757	51
17	42	0.105	0.12757	51	**42**	47	0.1175	0.12757	51
18	54	0.135	0.12757	51	**43**	62	0.155	0.12757	51
19	50	0.125	0.12757	51	**44**	43	0.1075	0.12757	51

20	70	0.175	0.12757	51	**45**	58	0.145	0.12757	51
21	57	0.1425	0.12757	51	**46**	55	0.1375	0.12757	51
22	49	0.1225	0.12757	51	**47**	44	0.11	0.12757	51
23	47	0.1175	0.12757	51	**48**	34	0.085	0.12757	51
24	40	0.1	0.12757	51	**49**	27	0.0675	0.12757	51



As shown here on the chart, the theoretical density function appears to match up with the given sample of the population and therefore it's assumed to be a continuous uniform distribution.

8. Confidence Intervals @ 96%

Below are the calculations for the confidence intervals of the mean and the parameters of the continuous uniform distribution at a confidence level of 96%

Mean

$\alpha = 0.04$

$P[Y >= R] = 0.04/2 = 0.02$

$P[Y >= L] = 1 - 0.02 = 0.98$

$L = Z_{0.98} = -2.05$

$R = Z_{0.02} = 2.05$

$P[-2.05 < Y < 2.05] \rightarrow P[-2.05 < (X - \mu)/ (\sigma/\sqrt{N}) < 2.05]$

$\rightarrow X - 2.05(\sigma/\sqrt{N}) <= \mu <= X + 2.05(\sigma/\sqrt{N}) \rightarrow$ $4.6735 <= \mu <= 4.8590$

Parameters

$\hat{b} = \mu + \sqrt{3}\sqrt{2}$

$\rightarrow \mu = 4.6735$: $b = 8.5929$

$\rightarrow \mu = 4.8590$: $b = 8.7784$

$8.5929 <= b <= 8.7784$

$\hat{a} = 2\mu - b$

→ μ = 4.6735: a = 0.7541

→ μ = 4.8590: a = 0.9396

0.7541 <= a <= 0.9396

In calculating the confidence interval estimates, the estimators, a & b, for the continuous uniform distribution were used as was the form for the standard normal and the central limit theorem. Within these resulting intervals is the true value of the mean and parameters of the distribution from the population that the given sample of 2500 values is taken from.

The Z-tables below were used for standard normal calculations.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06
–3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001
–3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002
–3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003
–3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004
–3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006
–3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008
–3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011
–2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015
–2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021
–2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029
–2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039
–2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052
–2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069
–2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091
–2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119
–2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154
–2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197
–1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250
–1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314
–1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392
–1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485
–1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594
–1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721
–1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869
–1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038
–1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230
–1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446
–0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685
–0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949
–0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236
–0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546
–0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877
–0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228
–0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594
–0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974
–0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364
–0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999