

CSCE 580: Artificial Intelligence

Reinforcement Learning

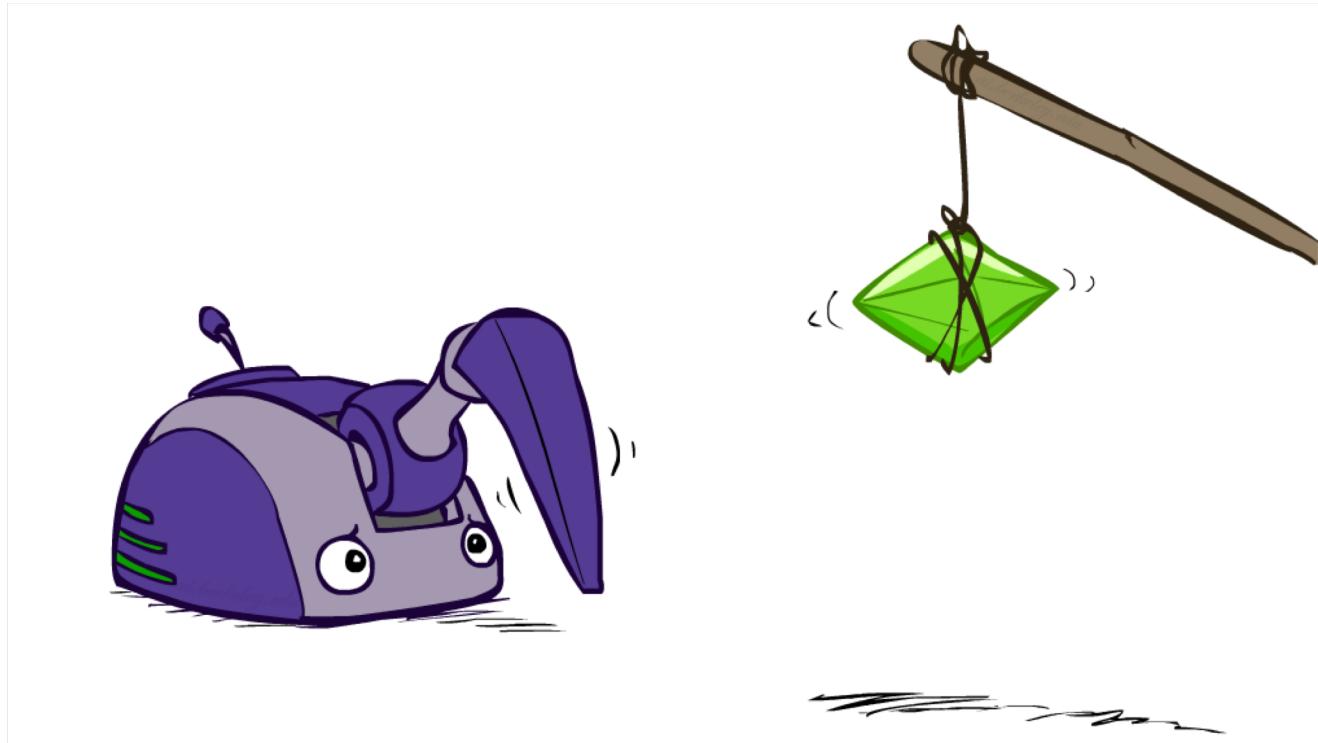


Instructor: Pooyan Jamshidi

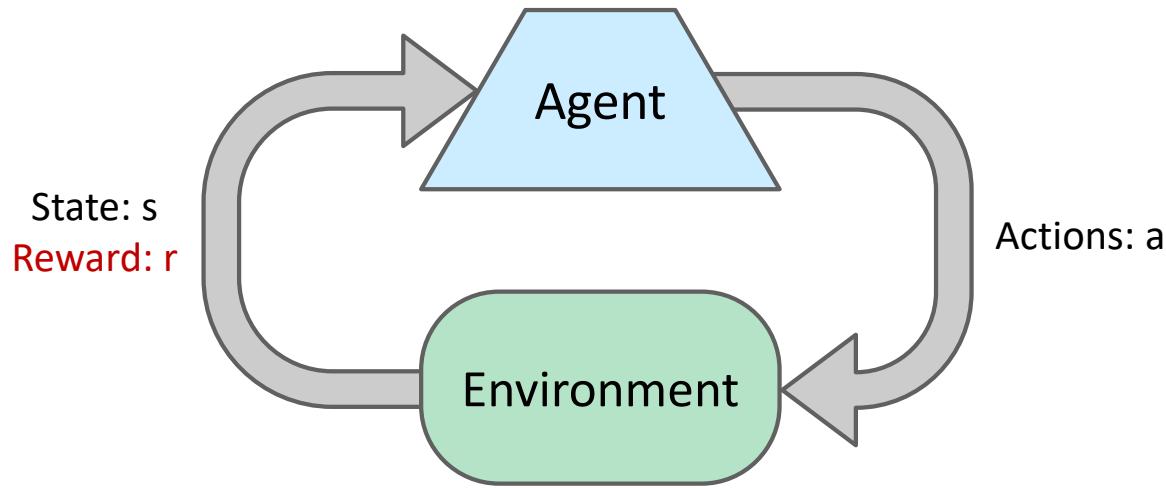
University of South Carolina

[These slides are mostly based on those of Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley, ai.berkeley.edu]

Reinforcement Learning



Reinforcement Learning



- Basic idea:
 - Receive feedback in the form of **rewards**
 - Agent's utility is defined by the reward function
 - Must (learn to) act so as to **maximize expected rewards**
 - All learning is based on observed samples of outcomes!

Example: Learning to Walk



Initial



A Learning Trial



After Learning [1K Trials]

[Kohl and Stone, ICRA 2004]

Example: Learning to Walk



Initial

[Kohl and Stone, ICRA 2004]

[Video: AIBO WALK – initial]

Example: Learning to Walk



Training

[Kohl and Stone, ICRA 2004]

[Video: AIBO WALK – training]

Example: Learning to Walk



Finished

[Kohl and Stone, ICRA 2004]

[Video: AIBO WALK – finished]

Example: Sidewinding



[Andrew Ng]

[Video: SNAKE – climbStep+sidewinding]

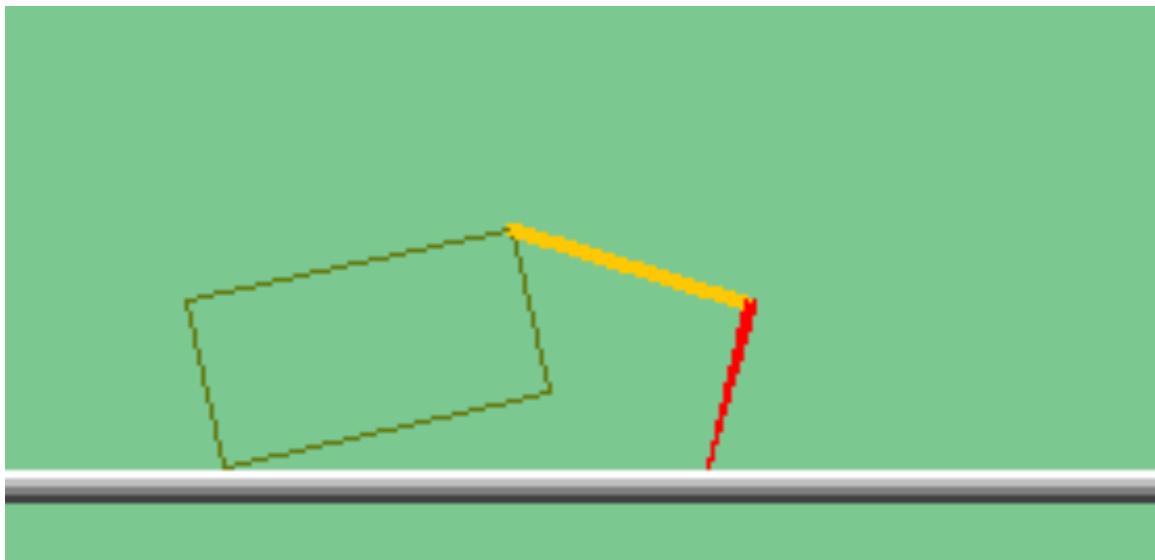
Example: Toddler Robot



[Tedrake, Zhang and Seung, 2005]

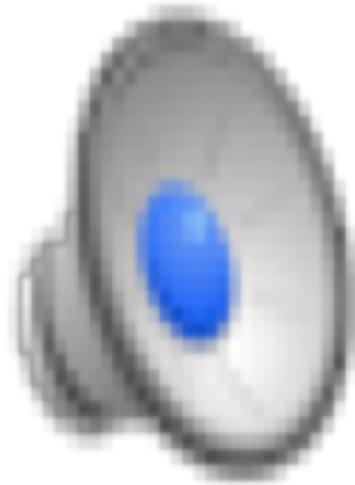
[Video: TODDLER – 40s]

The Crawler!



[Demo: Crawler Bot (L10D1)] [You, in Project 3]

Video of Demo Crawler Bot



Reinforcement Learning

- Still assume a Markov decision process (MDP):

- A set of states $s \in S$
- A set of actions (per state) A
- A model $T(s,a,s')$
- A reward function $R(s,a,s')$

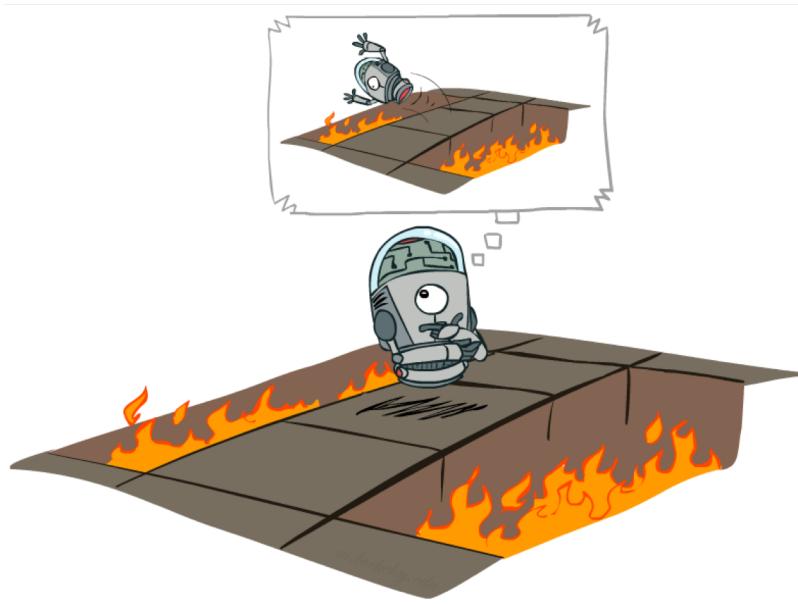


- Still looking for a policy $\pi(s)$

- New twist: don't know T or R

- I.e. we don't know which states are good or what the actions do
- Must actually try out actions and states to learn

Offline (MDPs) vs. Online (RL)

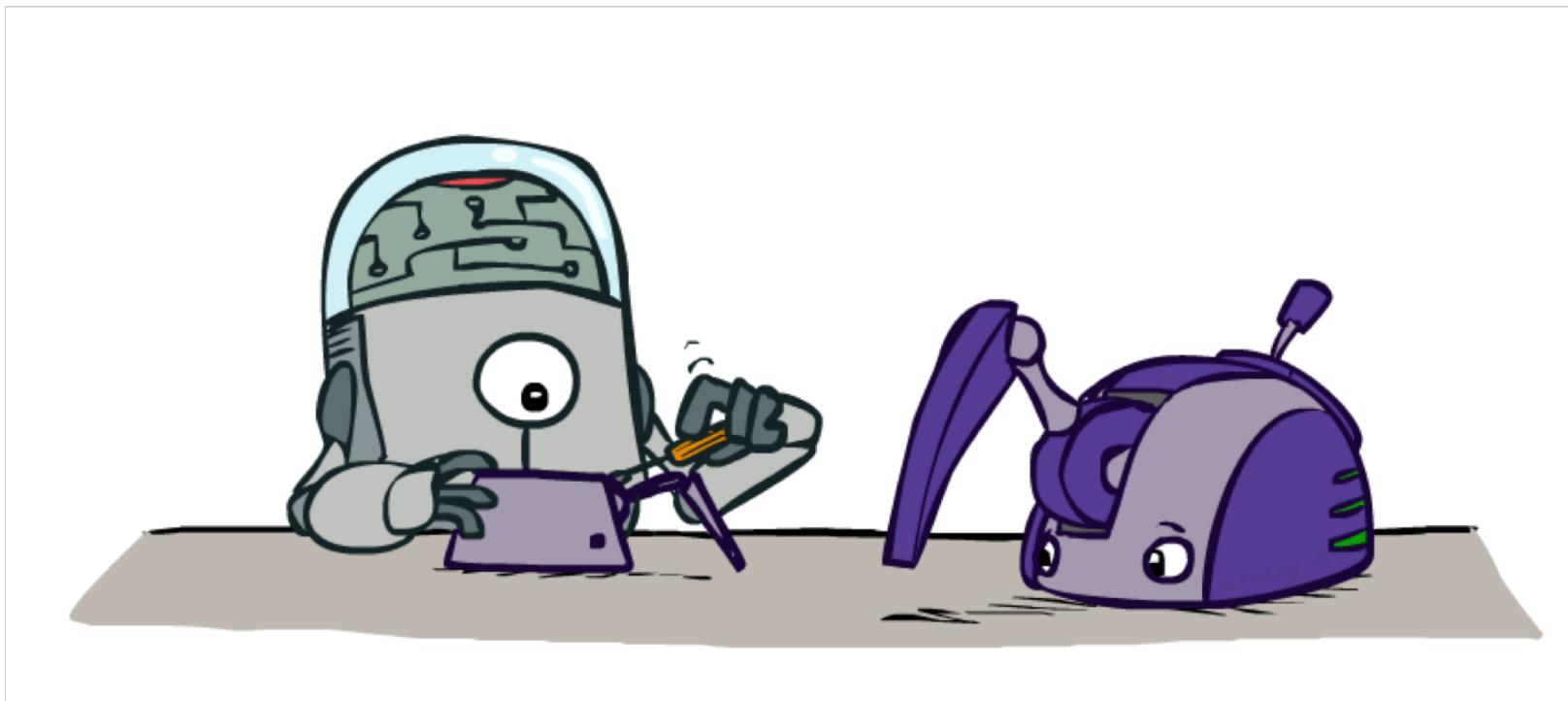


Offline Solution



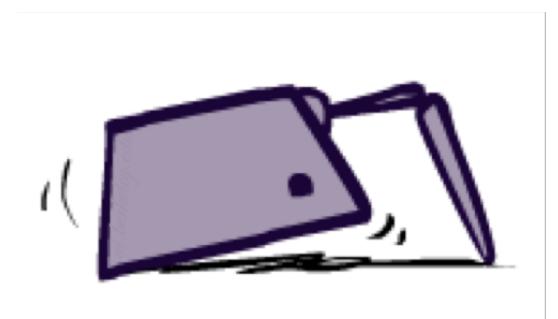
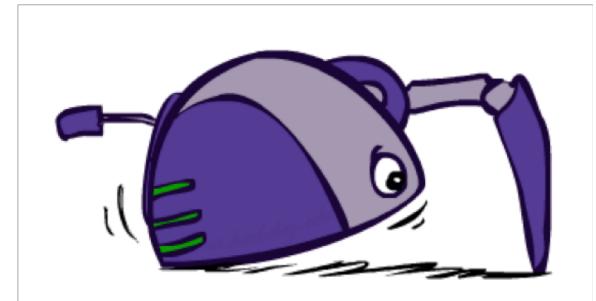
Online Learning

Model-Based Learning

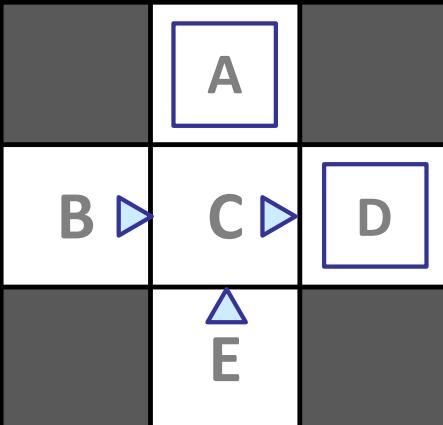


Model-Based Learning

- Model-Based Idea:
 - Learn an approximate model based on experiences
 - Solve for values as if the learned model were correct
- Step 1: Learn empirical MDP model
 - Count outcomes s' for each s, a
 - Normalize to give an estimate of $\hat{T}(s, a, s')$
 - Discover each $\hat{R}(s, a, s')$ when we experience (s, a, s')
- Step 2: Solve the learned MDP
 - For example, use value iteration, as before



Example: Model-Based Learning

Input Policy π	Observed Episodes (Training)	Learned Model
 <p>Assume: $\gamma = 1$</p>	<p>Episode 1</p> <div style="border: 1px solid blue; padding: 5px;">B, east, C, -1 C, east, D, -1 D, exit, x, +10</div> <p>Episode 2</p> <div style="border: 1px solid blue; padding: 5px;">B, east, C, -1 C, east, D, -1 D, exit, x, +10</div> <p>Episode 3</p> <div style="border: 1px solid blue; padding: 5px;">E, north, C, -1 C, east, D, -1 D, exit, x, +10</div> <p>Episode 4</p> <div style="border: 1px solid blue; padding: 5px;">E, north, C, -1 C, east, A, -1 A, exit, x, -10</div>	$\hat{T}(s, a, s')$ <div style="border: 1px solid black; padding: 5px;">$T(B, \text{east}, C) = 1.00$ $T(C, \text{east}, D) = 0.75$ $T(C, \text{east}, A) = 0.25$...</div> $\hat{R}(s, a, s')$ <div style="border: 1px solid black; padding: 5px;">$R(B, \text{east}, C) = -1$ $R(C, \text{east}, D) = -1$ $R(D, \text{exit}, x) = +10$...</div>

Example: Expected Age

Goal: Compute expected age of students in this class

Known $P(A)$

$$E[A] = \sum_a P(a) \cdot a = 0.35 \times 20 + \dots$$

Without $P(A)$, instead collect samples $[a_1, a_2, \dots, a_N]$

Unknown $P(A)$: “Model Based”

Why does this work? Because eventually you learn the right model.

$$\hat{P}(a) = \frac{\text{num}(a)}{N}$$

$$E[A] \approx \sum_a \hat{P}(a) \cdot a$$

Unknown $P(A)$: “Model Free”

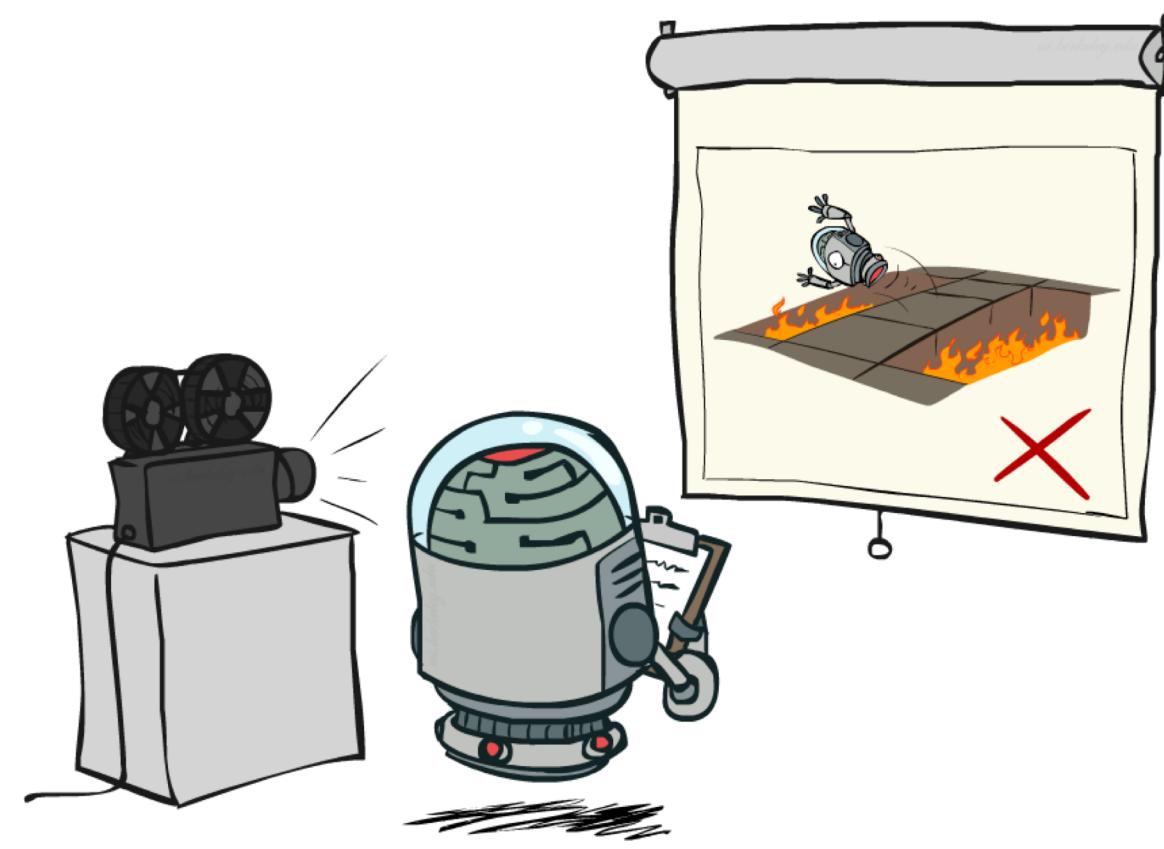
$$E[A] \approx \frac{1}{N} \sum_i a_i$$

Why does this work? Because samples appear with the right frequencies.

Model-Free Learning

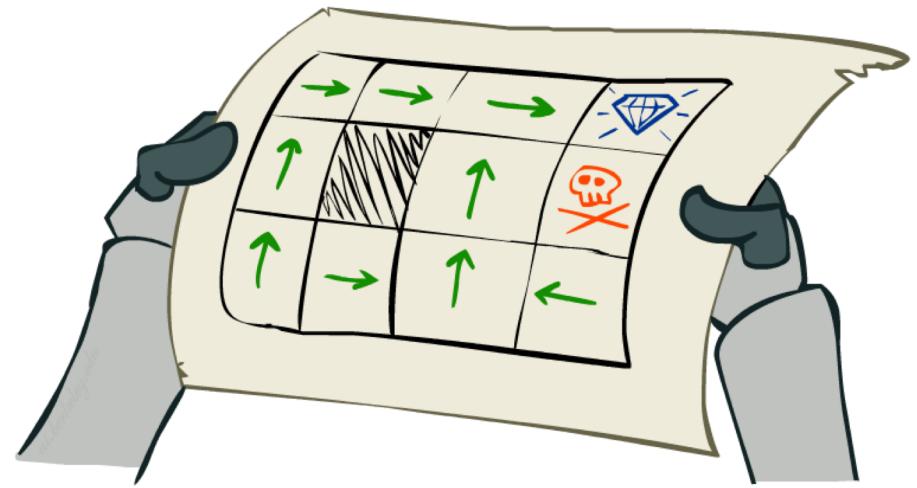


Passive Reinforcement Learning



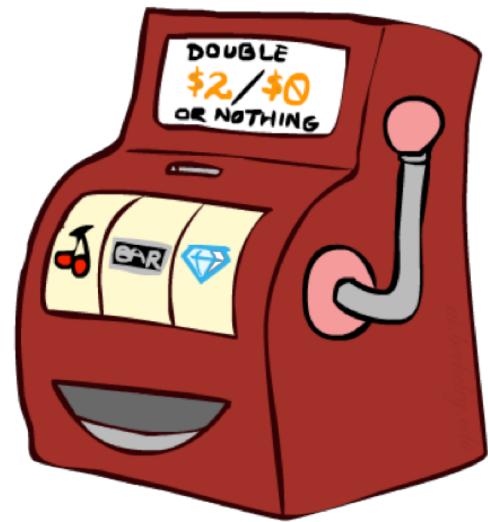
Passive Reinforcement Learning

- Simplified task: policy evaluation
 - Input: a fixed policy $\pi(s)$
 - You don't know the transitions $T(s,a,s')$
 - You don't know the rewards $R(s,a,s')$
 - Goal: learn the state values
- In this case:
 - Learner is “along for the ride”
 - No choice about what actions to take
 - Just execute the policy and learn from experience
 - This is NOT offline planning! You actually take actions in the world.



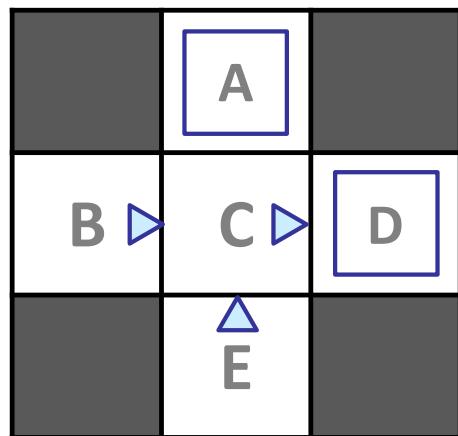
Direct Evaluation

- Goal: Compute values for each state under π
- Idea: Average together observed sample values
 - Act according to π
 - Every time you visit a state, write down what the sum of discounted rewards turned out to be
 - Average those samples
- This is called direct evaluation



Example: Direct Evaluation

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Output Values

	-10	
A	+4	+10
B	+8	D
E	-2	

Problems with Direct Evaluation

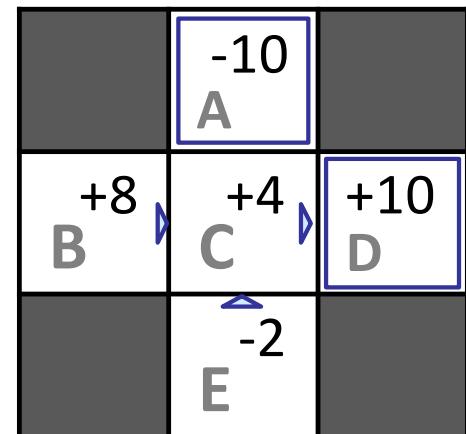
- What's good about direct evaluation?

- It's easy to understand
- It doesn't require any knowledge of T, R
- It eventually computes the correct average values, using just sample transitions

- What bad about it?

- It wastes information about state connections
- Each state must be learned separately
- So, it takes a long time to learn

Output Values



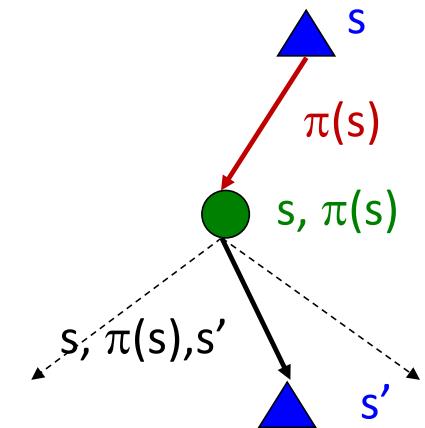
If B and E both go to C under this policy, how can their values be different?

Why Not Use Policy Evaluation?

- Simplified Bellman updates calculate V for a fixed policy:
 - Each round, replace V with a one-step-look-ahead layer over V

$$V_0^\pi(s) = 0$$

$$V_{k+1}^\pi(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_k^\pi(s')]$$



- This approach fully exploited the connections between the states
- Unfortunately, we need T and R to do it!
- Key question: how can we do this update to V without knowing T and R ?
 - In other words, how to we take a weighted average without knowing the weights?

Sample-Based Policy Evaluation?

- We want to improve our estimate of V by computing these averages:

$$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} T(s, \pi(s), s') [R(s, \pi(s), s') + \gamma V_k^{\pi}(s')]$$

- Idea: Take samples of outcomes s' (by doing the action!) and average

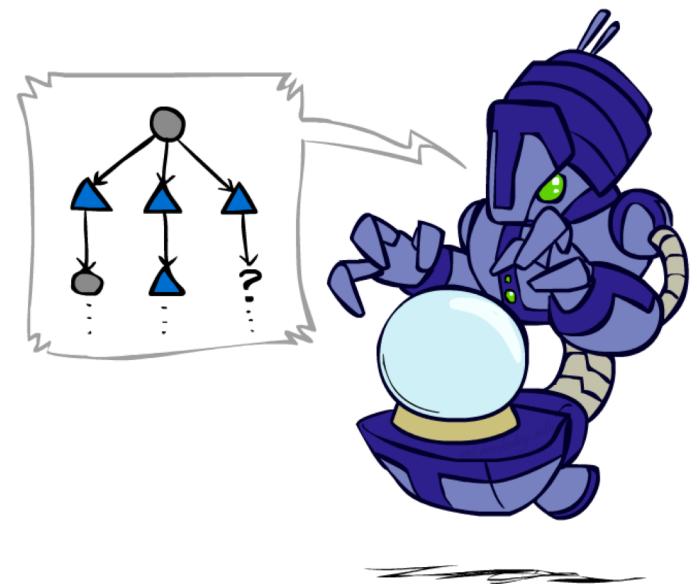
$$\text{sample}_1 = R(s, \pi(s), s'_1) + \gamma V_k^{\pi}(s'_1)$$

$$\text{sample}_2 = R(s, \pi(s), s'_2) + \gamma V_k^{\pi}(s'_2)$$

...

$$\text{sample}_n = R(s, \pi(s), s'_n) + \gamma V_k^{\pi}(s'_n)$$

$$V_{k+1}^{\pi}(s) \leftarrow \frac{1}{n} \sum_i \text{sample}_i$$

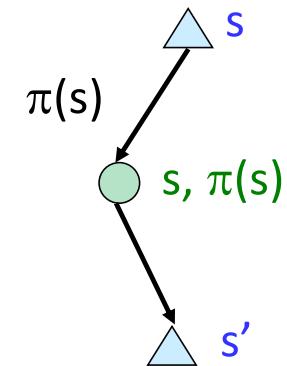


Temporal Difference Learning



Temporal Difference Learning

- Big idea: learn from every experience!
 - Update $V(s)$ each time we experience a transition (s, a, s', r)
 - Likely outcomes s' will contribute updates more often
- Temporal difference learning of values
 - Policy still fixed, still doing evaluation!
 - Move values toward value of whatever successor occurs: running average



Sample of $V(s)$: $\text{sample} = R(s, \pi(s), s') + \gamma V^\pi(s')$

Update to $V(s)$: $V^\pi(s) \leftarrow (1 - \alpha)V^\pi(s) + (\alpha)\text{sample}$

Same update: $V^\pi(s) \leftarrow V^\pi(s) + \alpha(\text{sample} - V^\pi(s))$

Exponential Moving Average

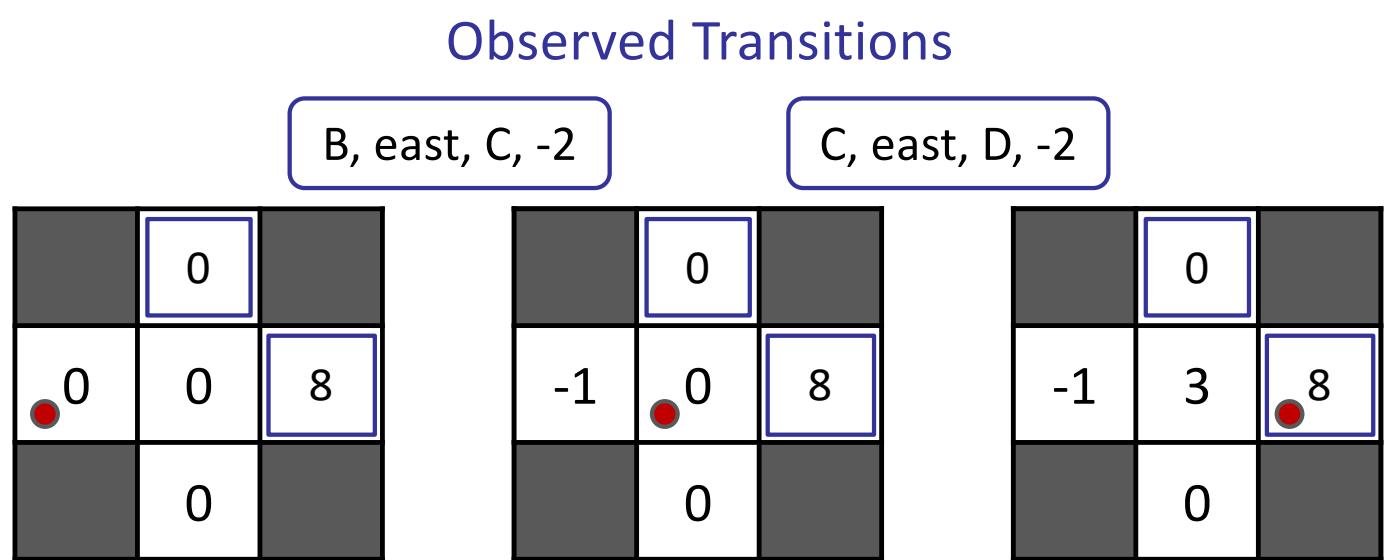
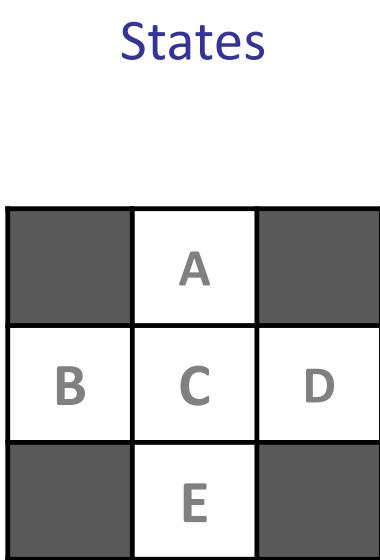
- Exponential moving average

- The running interpolation update: $\bar{x}_n = (1 - \alpha) \cdot \bar{x}_{n-1} + \alpha \cdot x_n$
 - Makes recent samples more important:

$$\bar{x}_n = \frac{x_n + (1 - \alpha) \cdot x_{n-1} + (1 - \alpha)^2 \cdot x_{n-2} + \dots}{1 + (1 - \alpha) + (1 - \alpha)^2 + \dots}$$

- Forgets about the past (distant past values were wrong anyway)
- Decreasing learning rate (alpha) can give converging averages

Example: Temporal Difference Learning



Assume: $\gamma = 1, \alpha = 1/2$

$$V^\pi(s) \leftarrow (1 - \alpha)V^\pi(s) + \alpha [R(s, \pi(s), s') + \gamma V^\pi(s')]$$

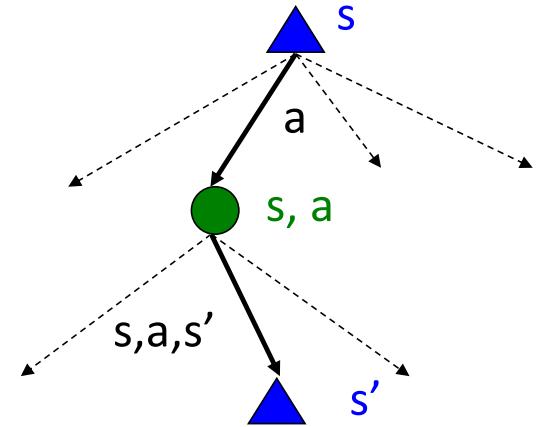
Problems with TD Value Learning

- TD value learning is a model-free way to do policy evaluation, mimicking Bellman updates with running sample averages
- However, if we want to turn values into a (new) policy, we're sunk:

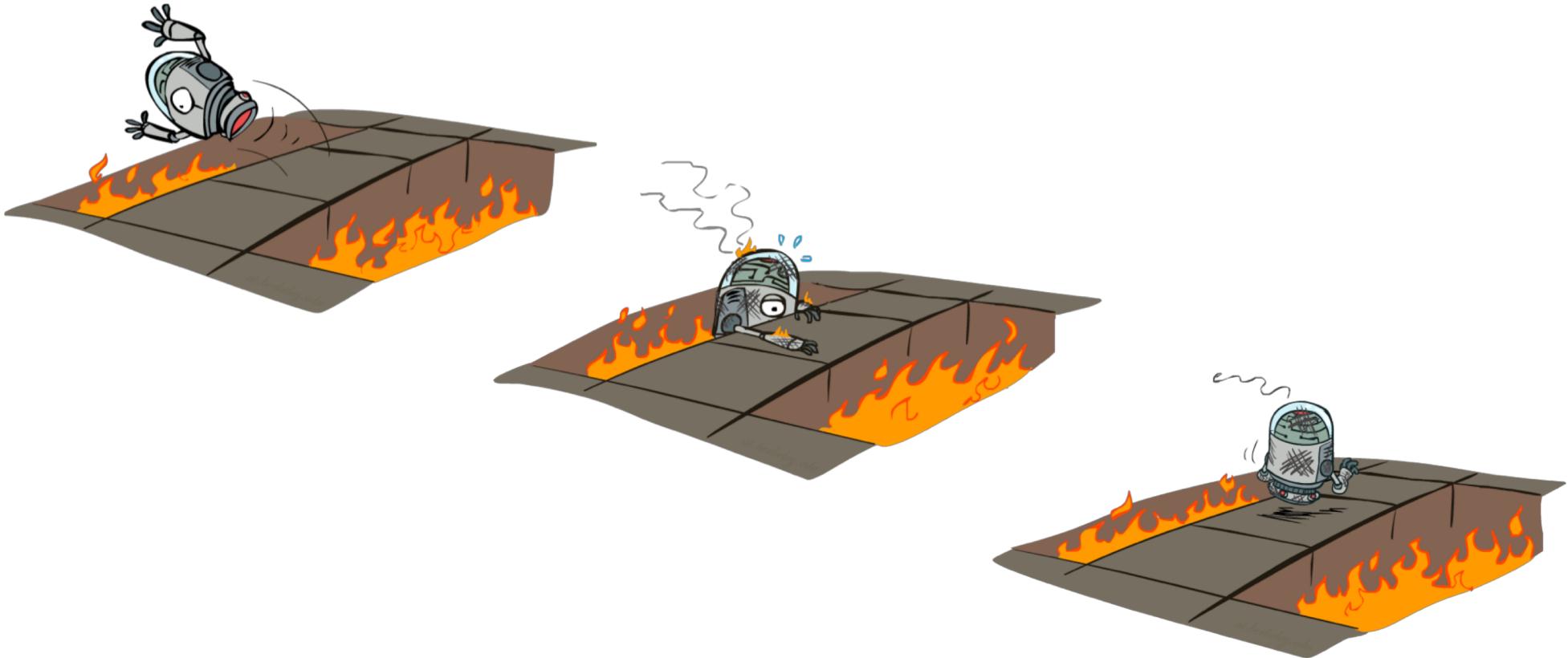
$$\pi(s) = \arg \max_a Q(s, a)$$

$$Q(s, a) = \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V(s')]$$

- Idea: learn Q-values, not values
- Makes action selection model-free too!

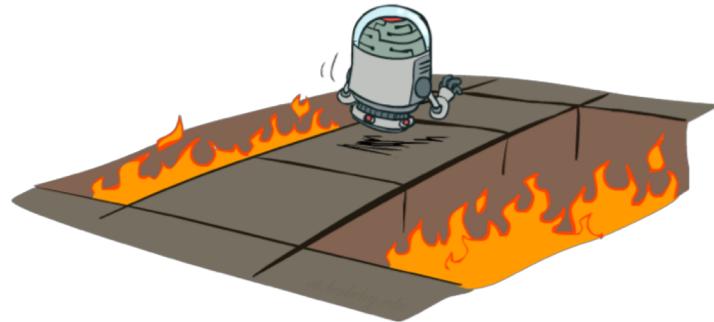


Active Reinforcement Learning



Active Reinforcement Learning

- Full reinforcement learning: optimal policies (like value iteration)
 - You don't know the transitions $T(s,a,s')$
 - You don't know the rewards $R(s,a,s')$
 - You choose the actions now
 - Goal: learn the optimal policy / values
- In this case:
 - Learner makes choices!
 - Fundamental tradeoff: exploration vs. exploitation
 - This is NOT offline planning! You actually take actions in the world and find out what happens...



Detour: Q-Value Iteration

- Value iteration: find successive (depth-limited) values
 - Start with $V_0(s) = 0$, which we know is right
 - Given V_k , calculate the depth $k+1$ values for all states:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V_k(s')]$$

- But Q-values are more useful, so compute them instead
 - Start with $Q_0(s, a) = 0$, which we know is right
 - Given Q_k , calculate the depth $k+1$ q-values for all q-states:

$$Q_{k+1}(s, a) \leftarrow \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma \max_{a'} Q_k(s', a')]$$

Q-Learning

- Q-Learning: sample-based Q-value iteration

$$Q_{k+1}(s, a) \leftarrow \sum_{s'} T(s, a, s') \left[R(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right]$$

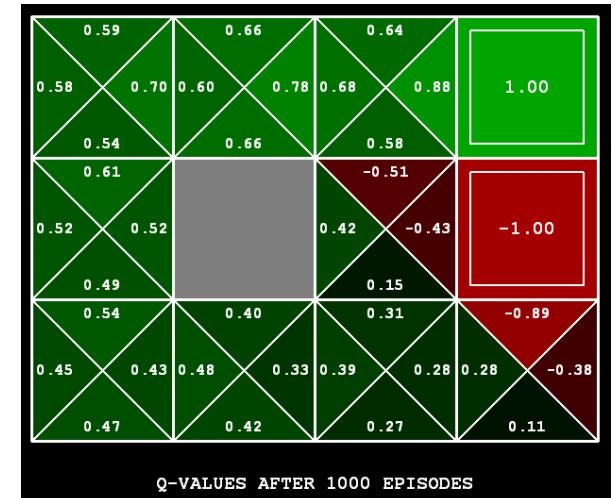
- Learn $Q(s, a)$ values as you go

- Receive a sample (s, a, s', r)
- Consider your old estimate: $Q(s, a)$
- Consider your new sample estimate:

$$\text{sample} = R(s, a, s') + \gamma \max_{a'} Q(s', a')$$

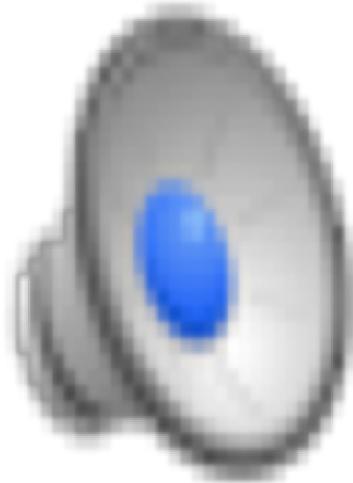
- Incorporate the new estimate into a running average:

$$Q(s, a) \leftarrow (1 - \alpha)Q(s, a) + (\alpha) [\text{sample}]$$

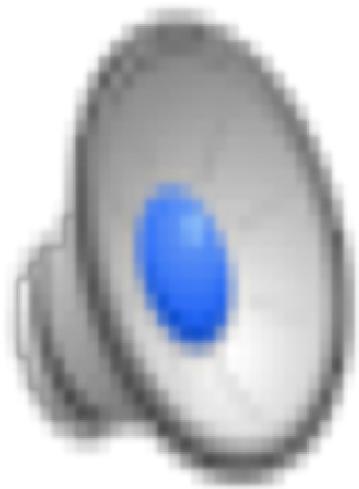


[Demo: Q-learning – gridworld (L10D2)]
[Demo: Q-learning – crawler (L10D3)]

Video of Demo Q-Learning -- Gridworld

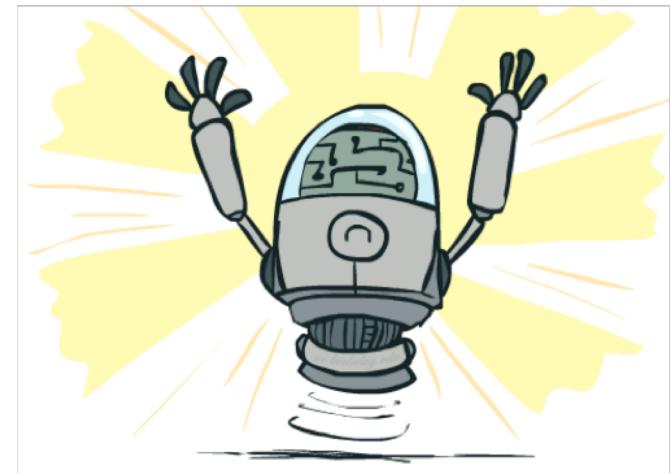


Video of Demo Q-Learning -- Crawler



Q-Learning Properties

- Amazing result: Q-learning converges to optimal policy -- even if you're acting suboptimally!
- This is called **off-policy learning**
- Caveats:
 - You have to explore enough
 - You have to eventually make the learning rate small enough
 - ... but not decrease it too quickly
 - Basically, in the limit, it doesn't matter how you select actions (!)



Reinforcement Learning

- We still assume an MDP:
 - A set of states $s \in S$
 - A set of actions (per state) A
 - A model $T(s,a,s')$
 - A reward function $R(s,a,s')$
- Still looking for a policy $\pi(s)$
- New twist: don't know T or R , so must try out actions
- Big idea: Compute all averages over T using sample outcomes



The Story So Far: MDPs and RL

Known MDP: Offline Solution

Goal	Technique
Compute V^* , Q^* , π^*	Value / policy iteration
Evaluate a fixed policy π	Policy evaluation

Unknown MDP: Model-Based

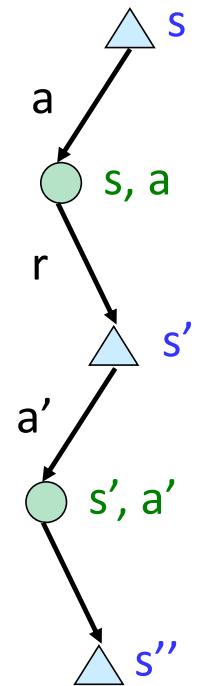
Goal	Technique
Compute V^* , Q^* , π^*	VI/PI on approx. MDP
Evaluate a fixed policy π	PE on approx. MDP

Unknown MDP: Model-Free

Goal	Technique
Compute V^* , Q^* , π^*	Q-learning
Evaluate a fixed policy π	Value Learning

Model-Free Learning

- Model-free (temporal difference) learning
 - Experience world through episodes
$$(s, a, r, s', a', r', s'', a'', r'', s''', \dots)$$
 - Update estimates each transition (s, a, r, s')
 - Over time, updates will mimic Bellman updates



Q-Learning

- We'd like to do Q-value updates to each Q-state:

$$Q_{k+1}(s, a) \leftarrow \sum_{s'} T(s, a, s') \left[R(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right]$$

- But can't compute this update without knowing T, R

- Instead, compute average as we go

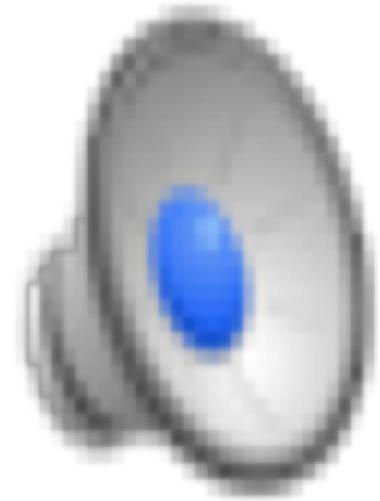
- Receive a sample transition (s, a, r, s')
- This sample suggests

$$Q(s, a) \approx r + \gamma \max_{a'} Q(s', a')$$

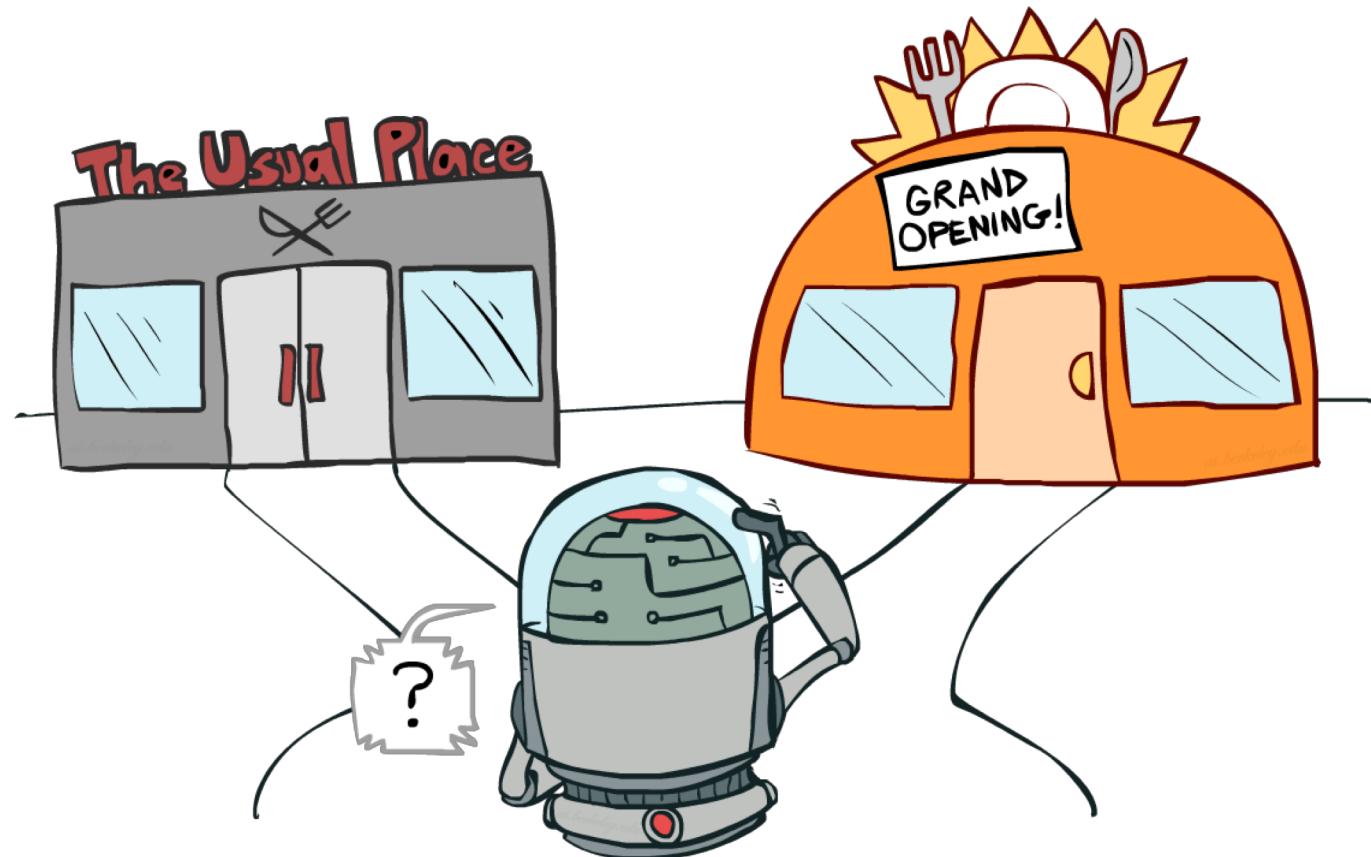
- But we want to average over results from (s, a) (Why?)
- So keep a running average

$$Q(s, a) \leftarrow (1 - \alpha)Q(s, a) + (\alpha) \left[r + \gamma \max_{a'} Q(s', a') \right]$$

Video of Demo Q-Learning Auto Cliff Grid



Exploration vs. Exploitation



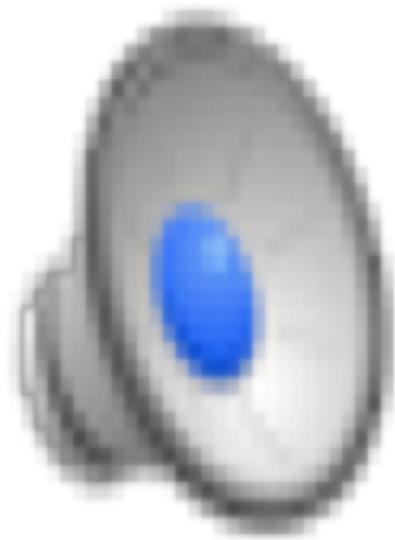
How to Explore?

- Several schemes for forcing exploration
 - Simplest: random actions (ε -greedy)
 - Every time step, flip a coin
 - With (small) probability ε , act randomly
 - With (large) probability $1-\varepsilon$, act on current policy
 - Problems with random actions?
 - You do eventually explore the space, but keep thrashing around once learning is done
 - One solution: lower ε over time
 - Another solution: exploration functions



[Demo: Q-learning – manual exploration – bridge grid (L11D2)]
[Demo: Q-learning – epsilon-greedy -- crawler (L11D3)]

Video of Demo Q-learning – Epsilon-Greedy – Crawler



Exploration Functions

- When to explore?

- Random actions: explore a fixed amount
- Better idea: explore areas whose badness is not (yet) established, eventually stop exploring

- Exploration function

- Takes a value estimate u and a visit count n , and returns an optimistic utility, e.g. $f(u, n) = u + k/n$

Regular Q-Update: $Q(s, a) \leftarrow_{\alpha} R(s, a, s') + \gamma \max_{a'} Q(s', a')$

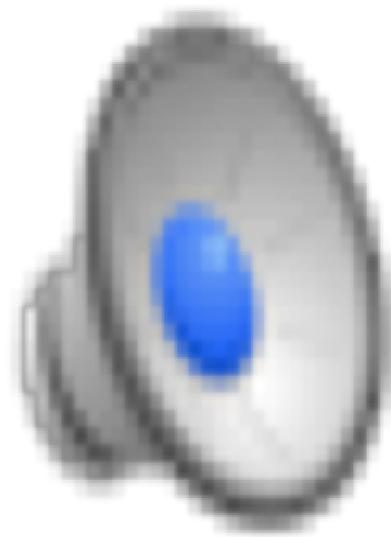
Modified Q-Update: $Q(s, a) \leftarrow_{\alpha} R(s, a, s') + \gamma \max_{a'} f(Q(s', a'), N(s', a'))$

- Note: this propagates the “bonus” back to states that lead to unknown states as well!

[Demo: exploration – Q-learning – crawler – exploration function (L11D4)]

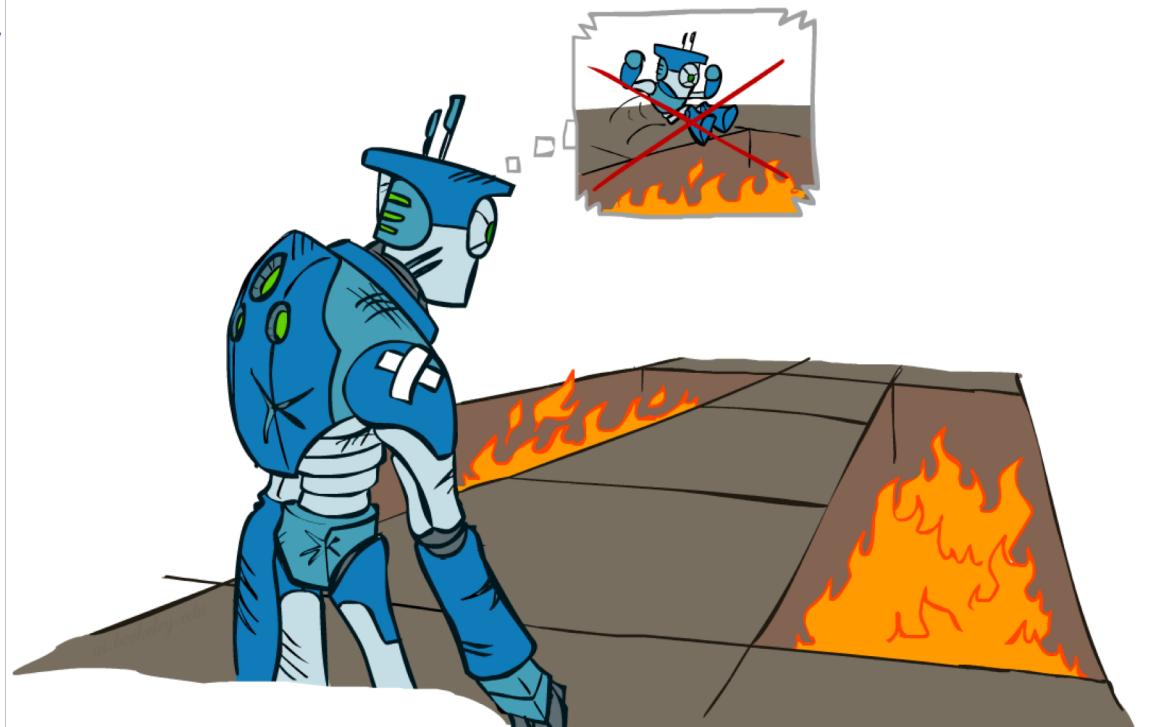


Video of Demo Q-learning – Exploration Function – Crawler

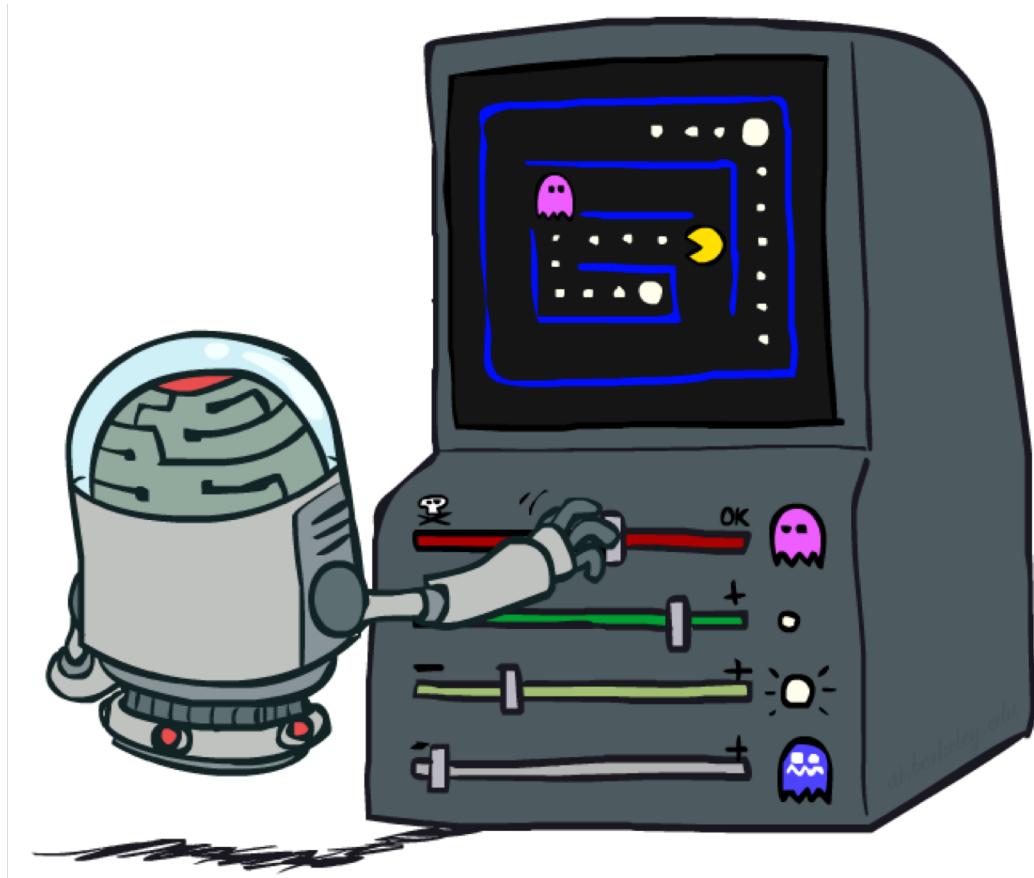


Regret

- Even if you learn the optimal policy, you still make mistakes along the way
- Regret is a measure of your total mistake cost: the difference between your (expected) rewards, including youthful suboptimality, and optimal (expected) rewards
- Minimizing regret goes beyond learning to be optimal – it requires optimally learning to be optimal
- Example: random exploration and exploration functions both end up optimal, but random exploration has higher regret

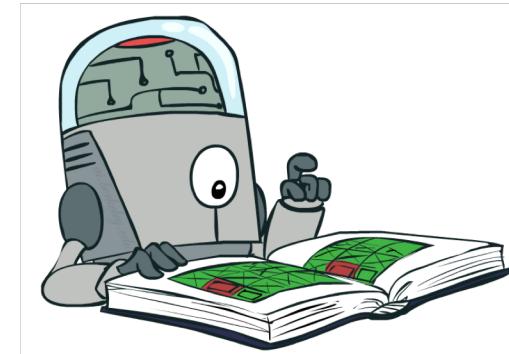


Approximate Q-Learning



Generalizing Across States

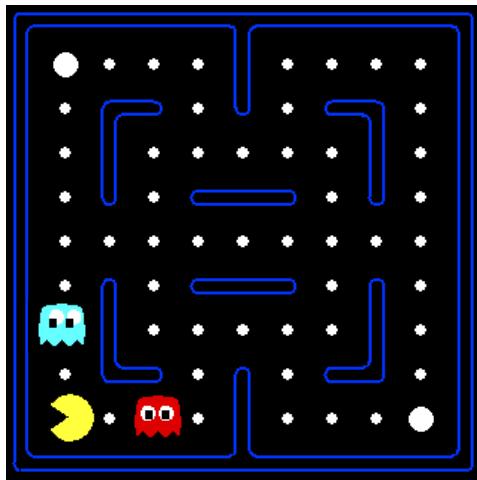
- Basic Q-Learning keeps a table of all q-values
- In realistic situations, we cannot possibly learn about every single state!
 - Too many states to visit them all in training
 - Too many states to hold the q-tables in memory
- Instead, we want to generalize:
 - Learn about some small number of training states from experience
 - Generalize that experience to new, similar situations
 - This is a fundamental idea in machine learning, and we'll see it over and over again



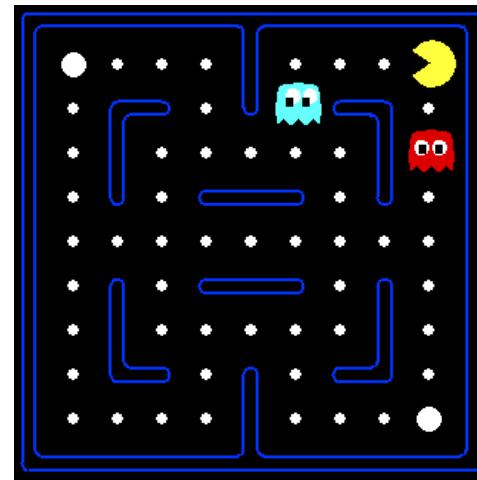
[demo – RL pacman]

Example: Pacman

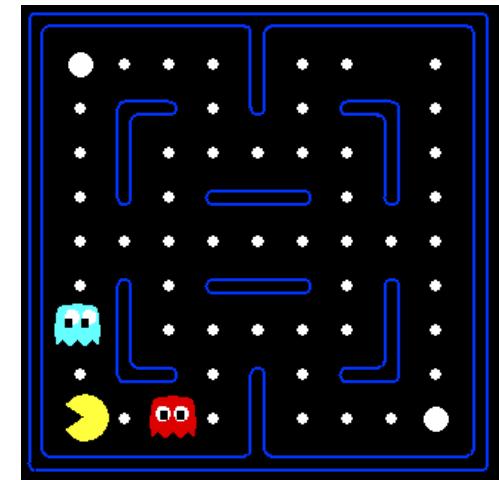
Let's say we discover through experience that this state is bad:



In naïve q-learning, we know nothing about this state:

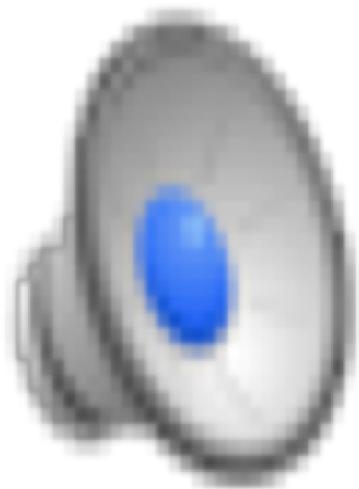


Or even this one!

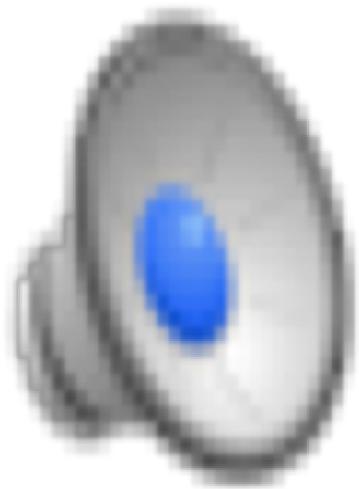


[Demo: Q-learning – pacman – tiny – watch all (L11D5)], [Demo: Q-learning – pacman – tiny – silent train (L11D6)], [Demo: Q-learning – pacman – tricky – watch all (L11D7)]

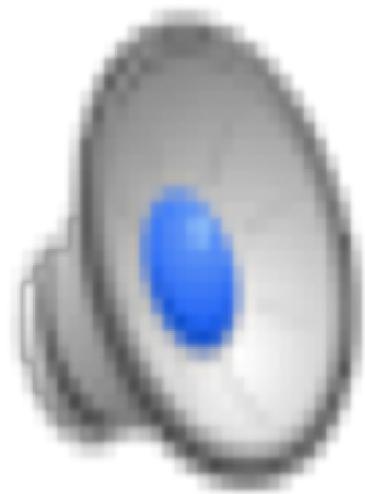
Video of Demo Q-Learning Pacman – Tiny – Watch All



Video of Demo Q-Learning Pacman – Tiny – Silent Train



Video of Demo Q-Learning Pacman – Tricky – Watch All



Feature-Based Representations

- Solution: describe a state using a vector of features (properties)
 - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
 - Example features:
 - Distance to closest ghost
 - Distance to closest dot
 - Number of ghosts
 - $1 / (\text{dist to dot})^2$
 - Is Pacman in a tunnel? (0/1)
 - etc.
 - Is it the exact state on this slide?
 - Can also describe a q-state (s, a) with features (e.g. action moves closer to food)



Linear Value Functions

- Using a feature representation, we can write a q function (or value function) for any state using a few weights:

$$V(s) = w_1 f_1(s) + w_2 f_2(s) + \dots + w_n f_n(s)$$

$$Q(s, a) = w_1 f_1(s, a) + w_2 f_2(s, a) + \dots + w_n f_n(s, a)$$

- Advantage: our experience is summed up in a few powerful numbers
- Disadvantage: states may share features but actually be very different in value!

Approximate Q-Learning

$$Q(s, a) = w_1 f_1(s, a) + w_2 f_2(s, a) + \dots + w_n f_n(s, a)$$

- Q-learning with linear Q-functions:

transition = (s, a, r, s')

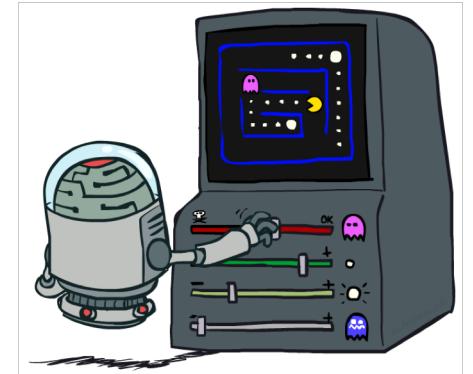
difference = $[r + \gamma \max_{a'} Q(s', a')] - Q(s, a)$

$Q(s, a) \leftarrow Q(s, a) + \alpha \text{ [difference]}$

$w_i \leftarrow w_i + \alpha \text{ [difference]} f_i(s, a)$

Exact Q's

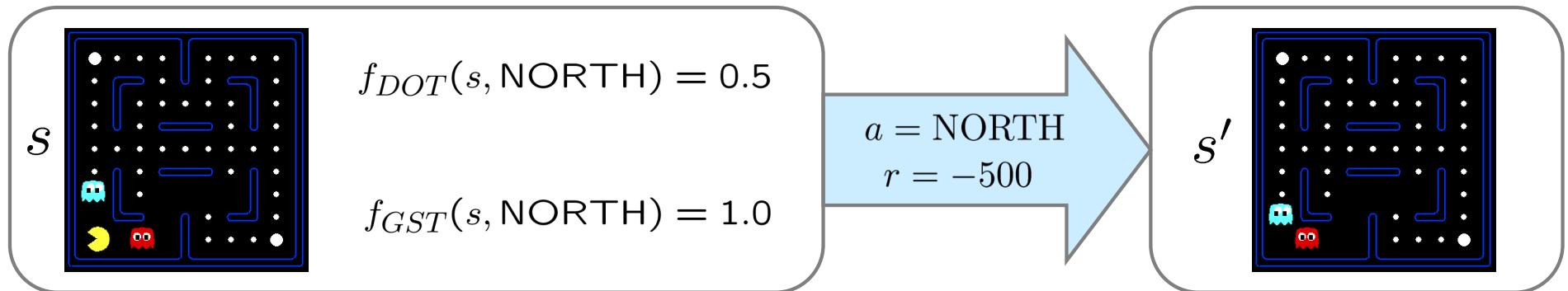
Approximate Q's



- Intuitive interpretation:
 - Adjust weights of active features
 - E.g., if something unexpectedly bad happens, blame the features that were on: disprefer all states with that state's features
- Formal justification: online least squares

Example: Q-Pacman

$$Q(s, a) = 4.0f_{DOT}(s, a) - 1.0f_{GST}(s, a)$$



$$Q(s, \text{NORTH}) = +1$$

$$Q(s', \cdot) = 0$$

$$r + \gamma \max_{a'} Q(s', a') = -500 + 0$$

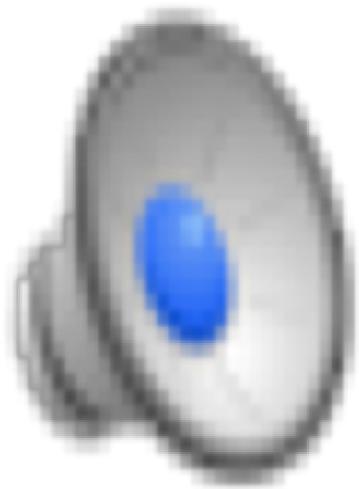
$$\text{difference} = -501$$

$$w_{DOT} \leftarrow 4.0 + \alpha [-501] 0.5$$
$$w_{GST} \leftarrow -1.0 + \alpha [-501] 1.0$$

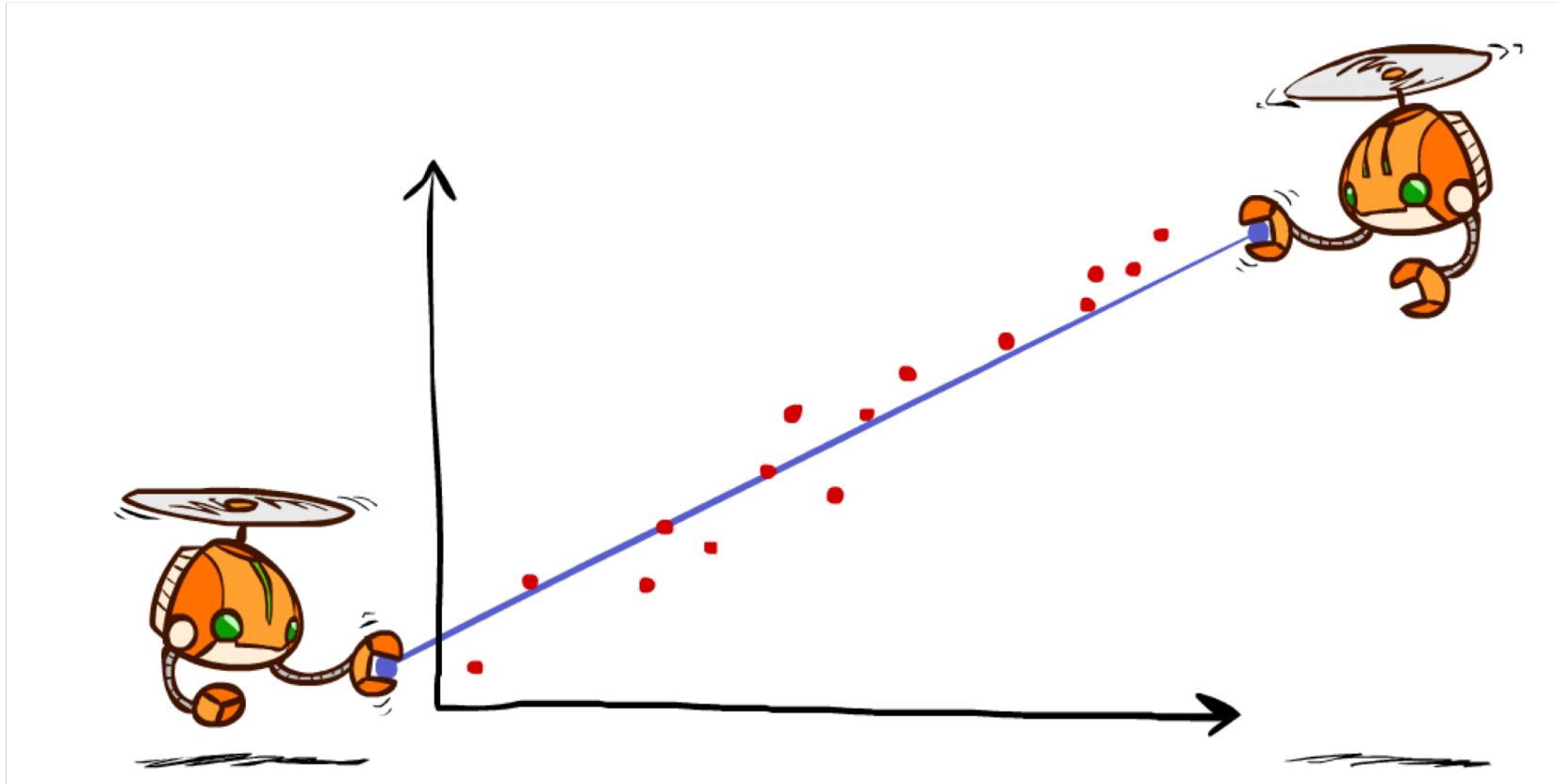
$$Q(s, a) = 3.0f_{DOT}(s, a) - 3.0f_{GST}(s, a)$$

[Demo: approximate Q-learning pacman (L11D10)]

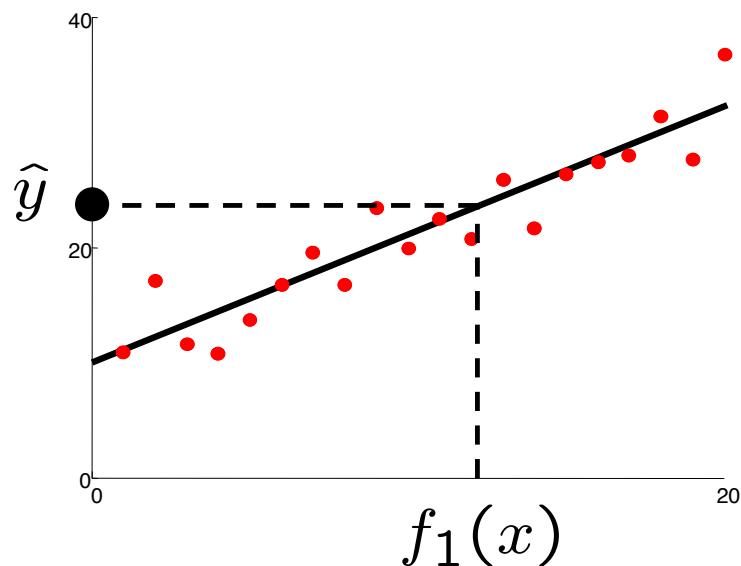
Video of Demo Approximate Q-Learning -- Pacman



Q-Learning and Least Squares

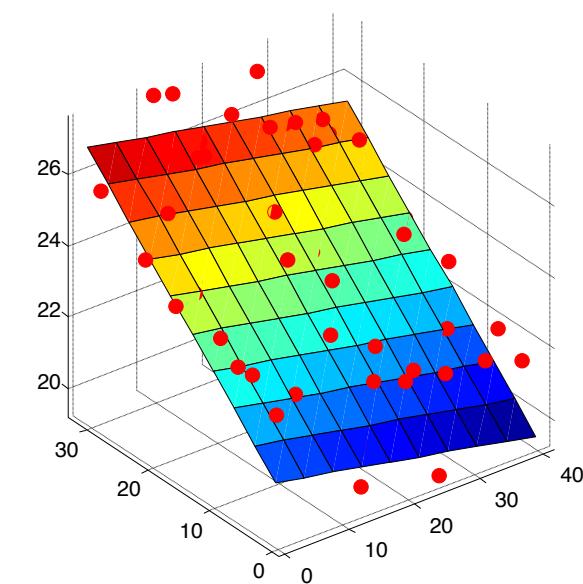


Linear Approximation: Regression*



Prediction:

$$\hat{y} = w_0 + w_1 f_1(x)$$

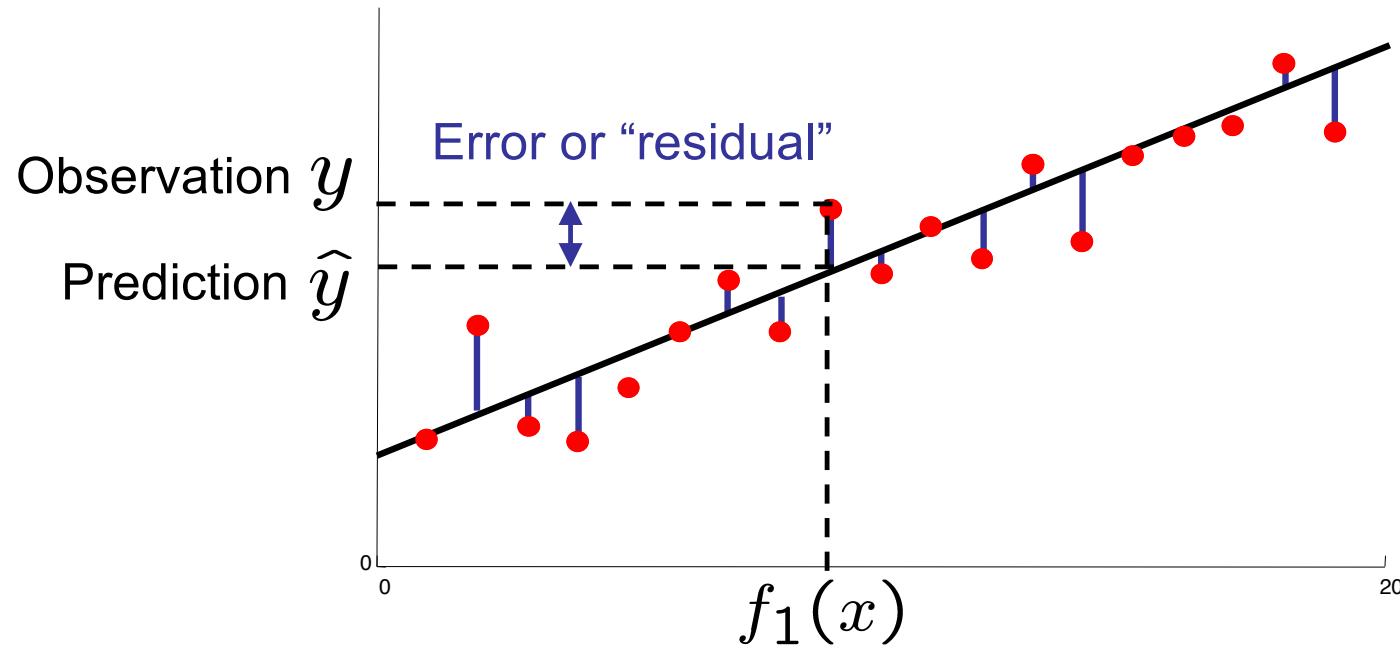


Prediction:

$$\hat{y}_i = w_0 + w_1 f_1(x) + w_2 f_2(x)$$

Optimization: Least Squares*

$$\text{total error} = \sum_i (y_i - \hat{y}_i)^2 = \sum_i \left(y_i - \sum_k w_k f_k(x_i) \right)^2$$



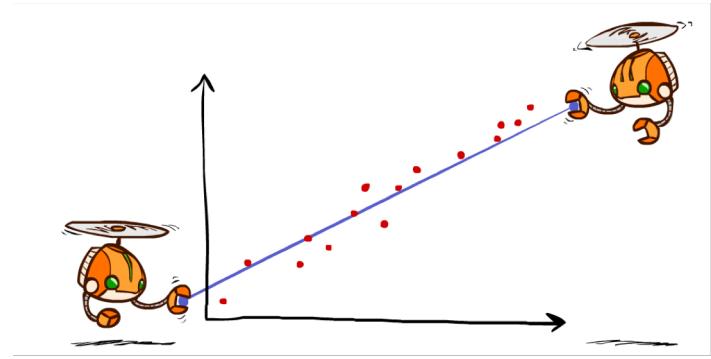
Minimizing Error*

Imagine we had only one point x , with features $f(x)$, target value y , and weights w :

$$\text{error}(w) = \frac{1}{2} \left(y - \sum_k w_k f_k(x) \right)^2$$

$$\frac{\partial \text{error}(w)}{\partial w_m} = - \left(y - \sum_k w_k f_k(x) \right) f_m(x)$$

$$w_m \leftarrow w_m + \alpha \left(y - \sum_k w_k f_k(x) \right) f_m(x)$$

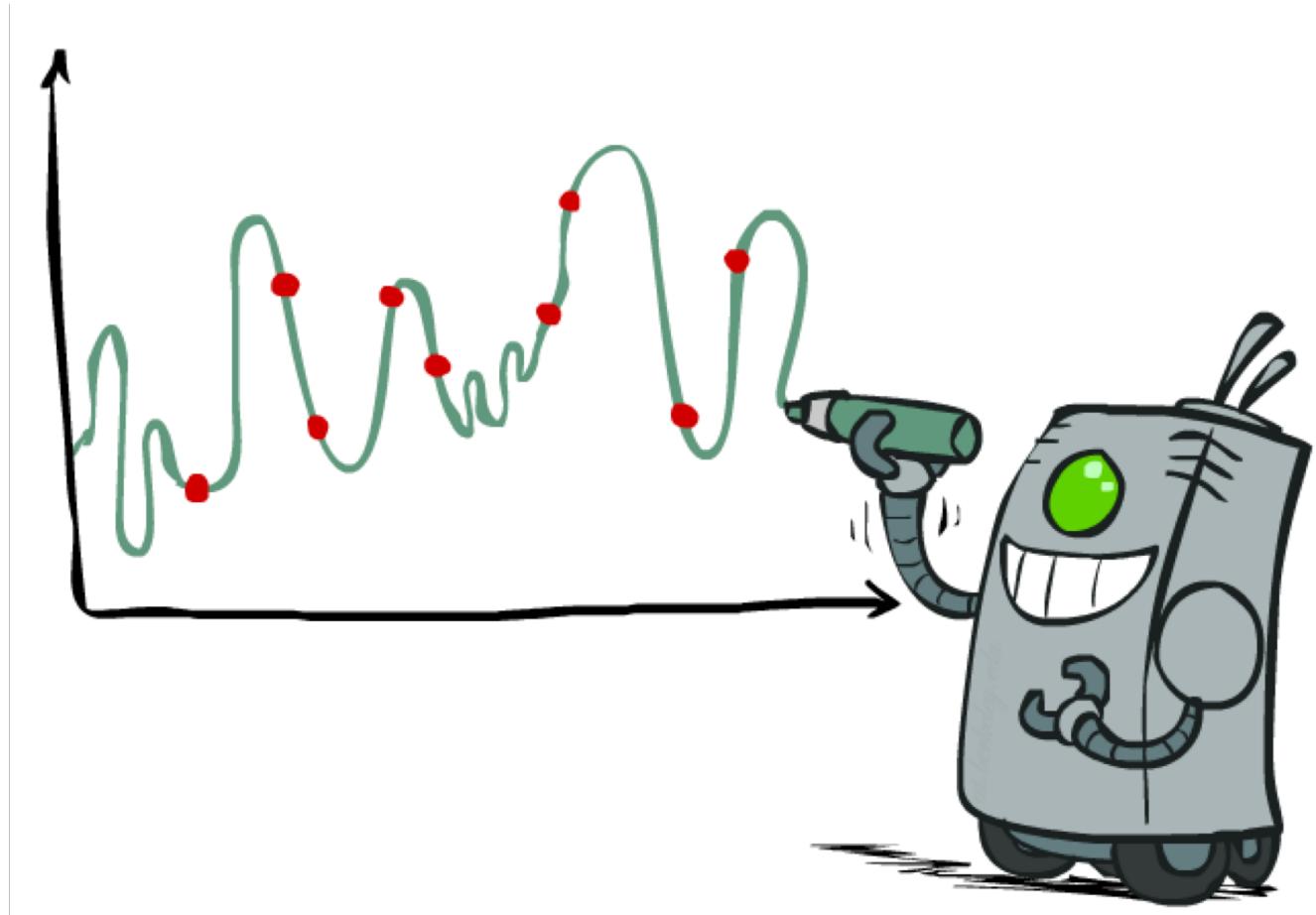


Approximate q update explained:

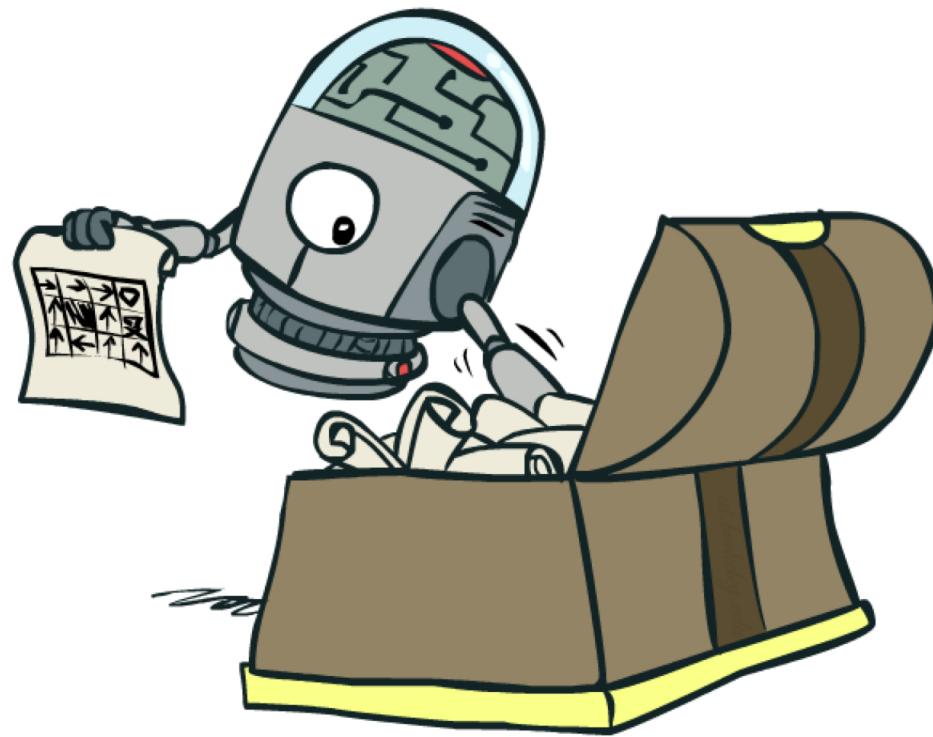
$$w_m \leftarrow w_m + \alpha \left[r + \gamma \max_a Q(s', a') - Q(s, a) \right] f_m(s, a)$$

“target” “prediction”

Overfitting: Why Limiting Capacity Can Help*



Policy Search



Policy Search

- Problem: often the feature-based policies that work well (win games, maximize utilities) aren't the ones that approximate V / Q best
 - E.g. your value functions from project 2 were probably horrible estimates of future rewards, but they still produced good decisions
 - Q-learning's priority: get Q -values close (modeling)
 - Action selection priority: get ordering of Q -values right (prediction)
 - We'll see this distinction between modeling and prediction again later in the course
- Solution: learn policies that maximize rewards, not the values that predict them
- Policy search: start with an ok solution (e.g. Q-learning) then fine-tune by hill climbing on feature weights

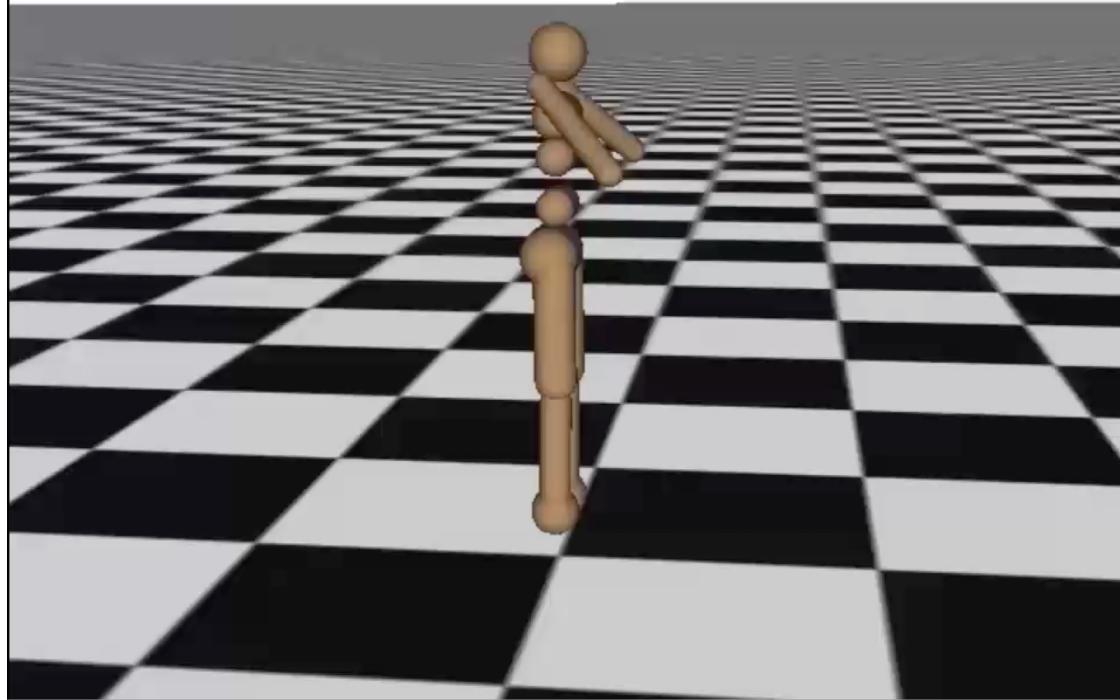
Policy Search

- Simplest policy search:
 - Start with an initial linear value function or Q-function
 - Nudge each feature weight up and down and see if your policy is better than before
- Problems:
 - How do we tell the policy got better?
 - Need to run many sample episodes!
 - If there are a lot of features, this can be impractical
- Better methods exploit lookahead structure, sample wisely, change multiple parameters...

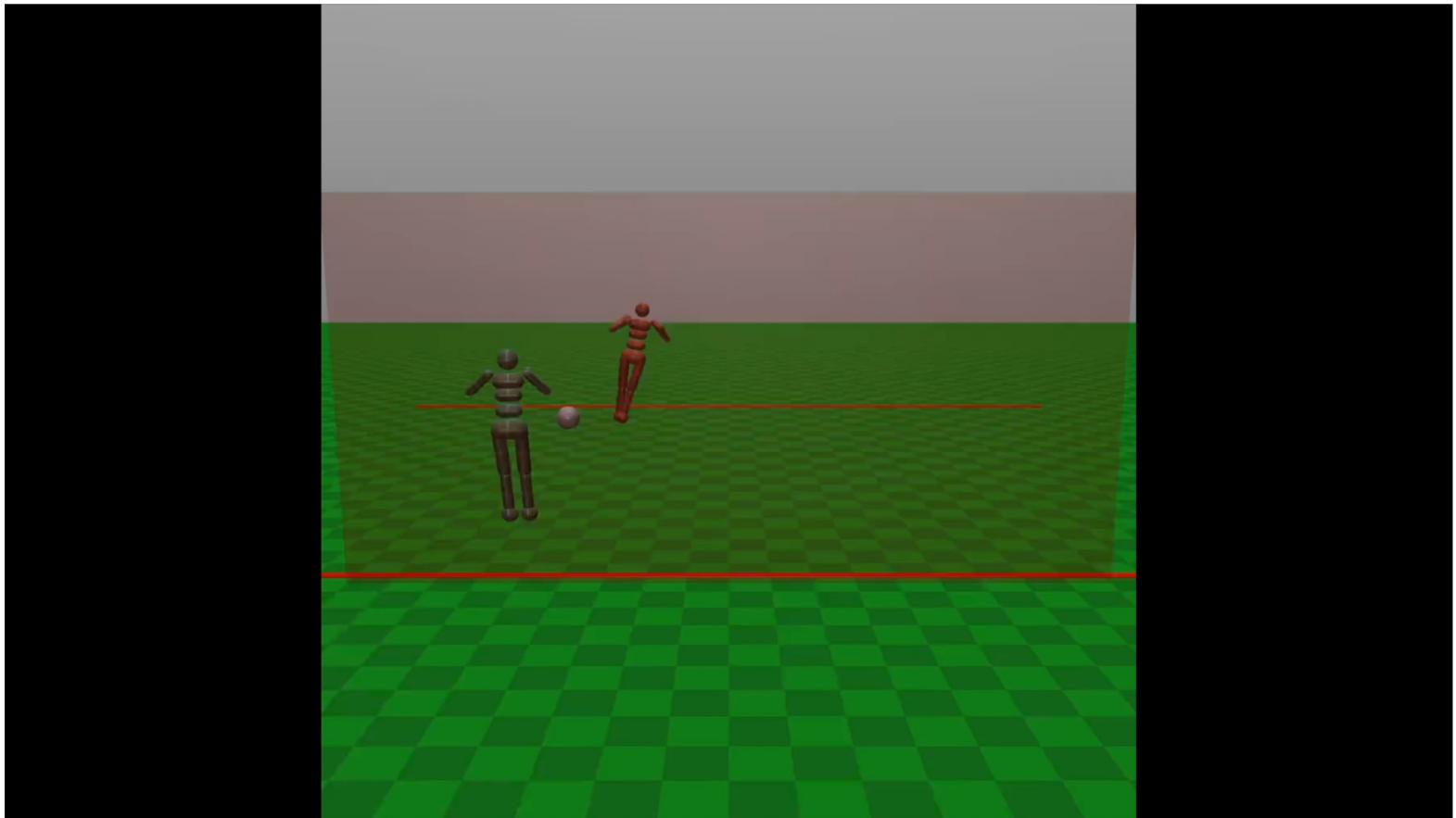


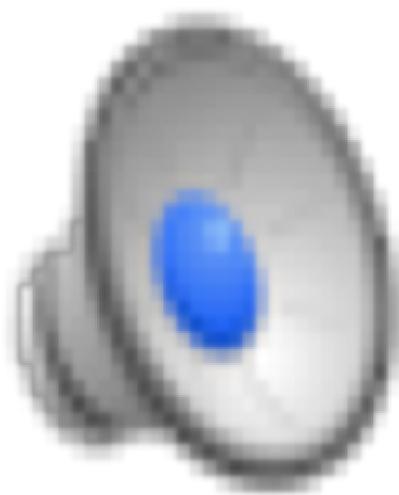
[Video: HELICOPTER]

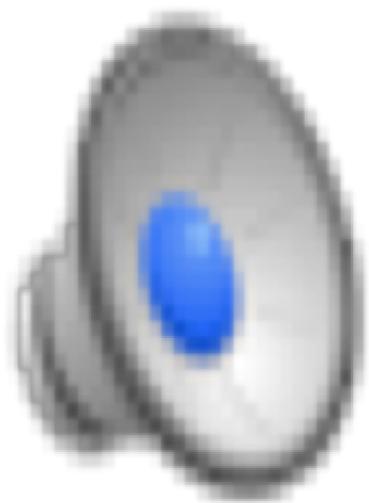
Iteration 0

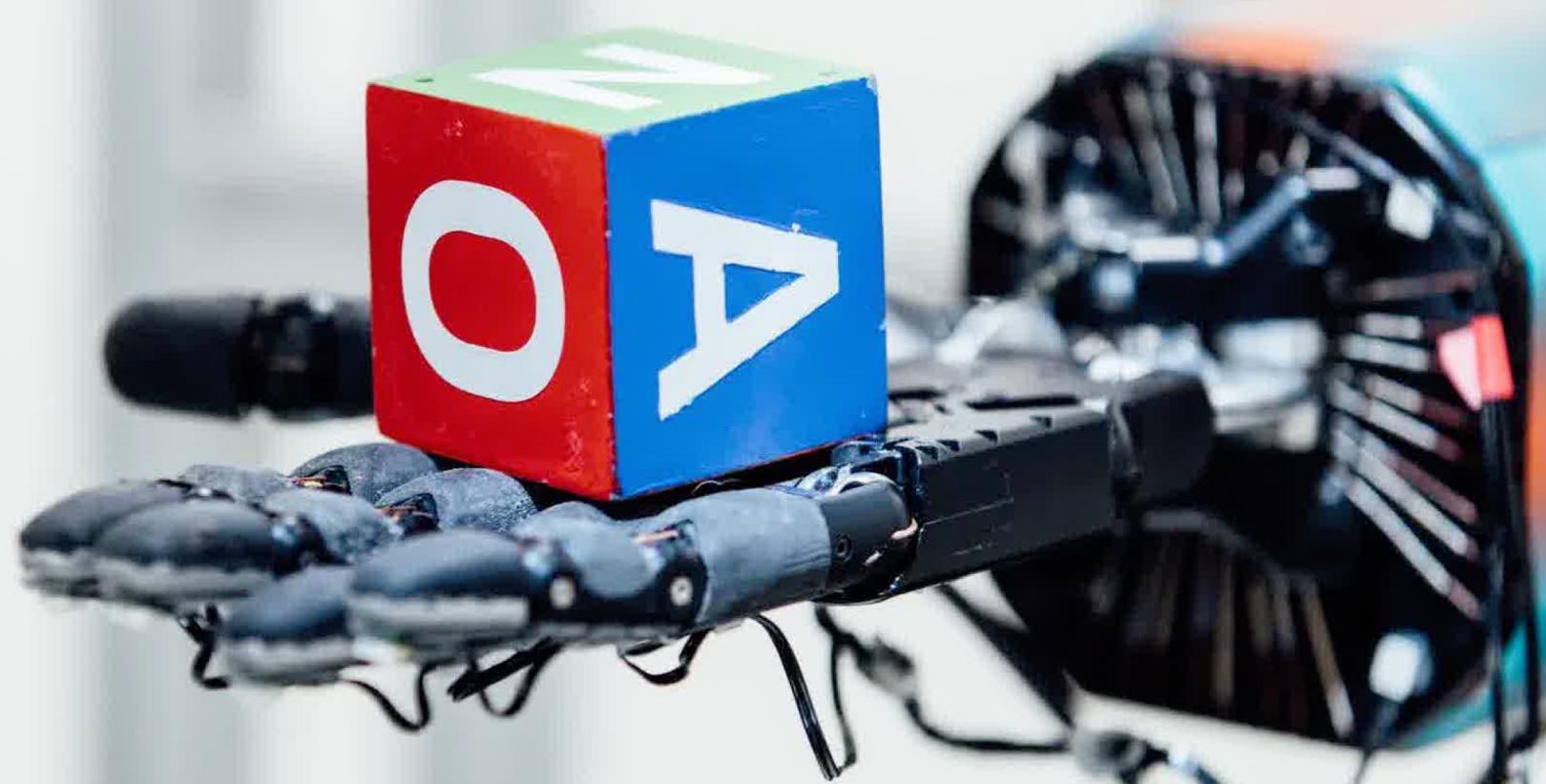


[Video: GAE]









Pieter Abbeel -- UC Berkeley | Gradescope |
Covariant.AI



Conclusion

- We're done with Part I: Search and Planning!
- We've seen how AI methods can solve problems in:
 - Search
 - Constraint Satisfaction Problems
 - Games
 - Markov Decision Problems
 - Reinforcement Learning
- Next up: Part II: Uncertainty and Learning!

