

# CAUSALITY & CORRELATION

How are continuous variables related  
and are these relations causal?

→ IHenrik von Wehrden

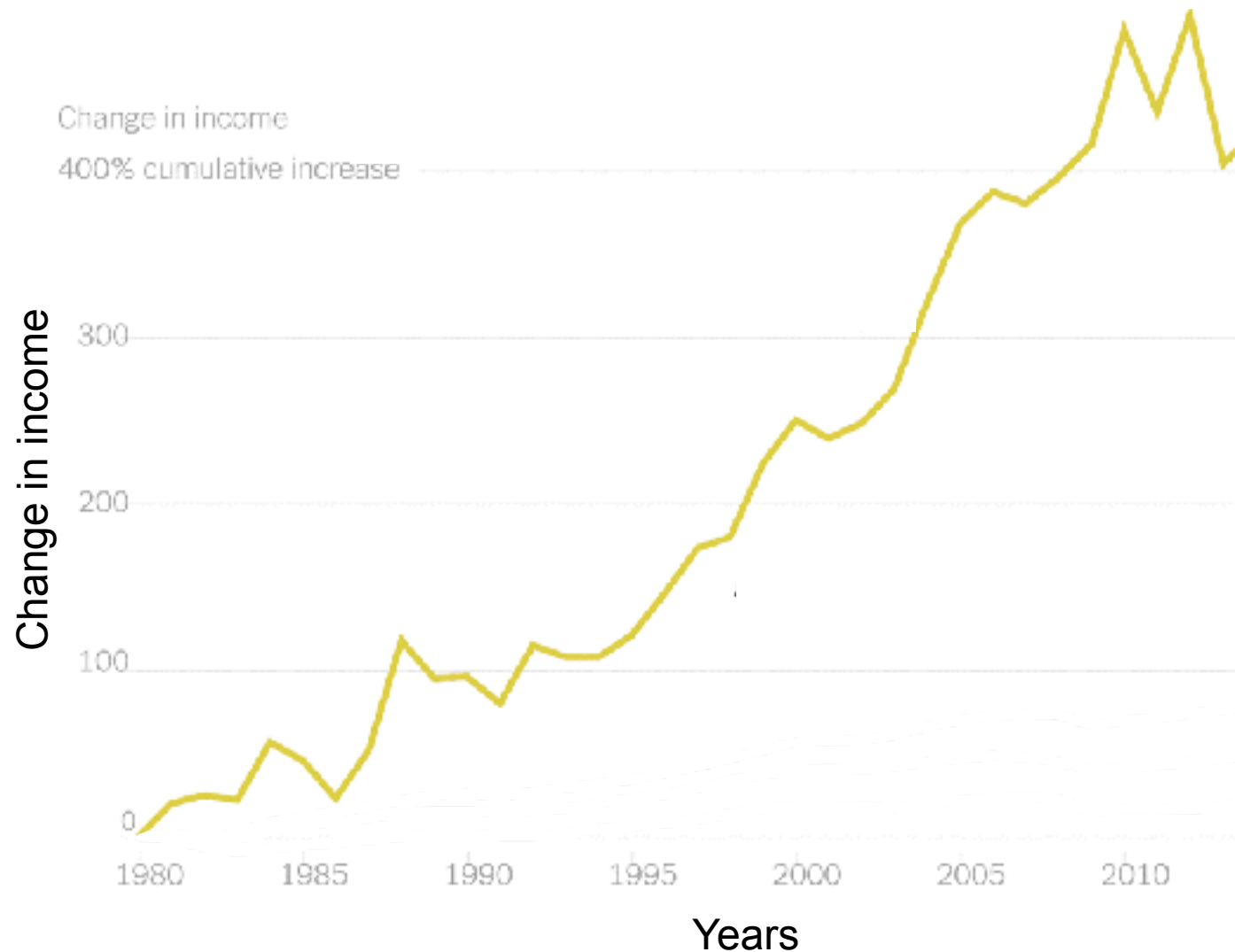
# CAUSALITY OR CORRELATION

Are relations really causal?  
And does this matter?

→ Henrik von Wehrden

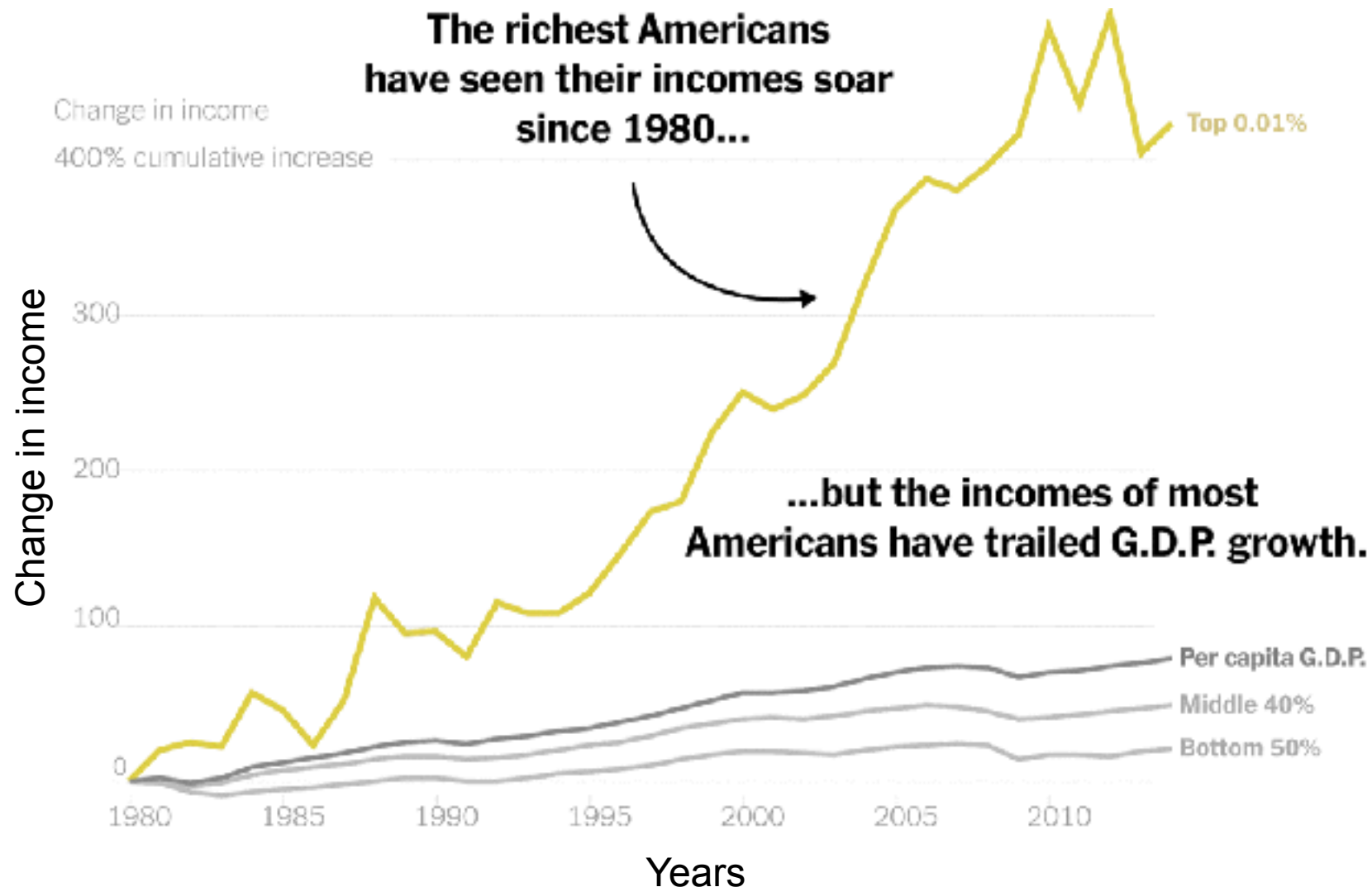
# LET US START SIMPLE: STATISTICAL CORRELATIONS

How are two continuous variables in relation with each other?



# LET US START SIMPLE: STATISTICAL CORRELATIONS

How are two continuous variables in relation with each other?

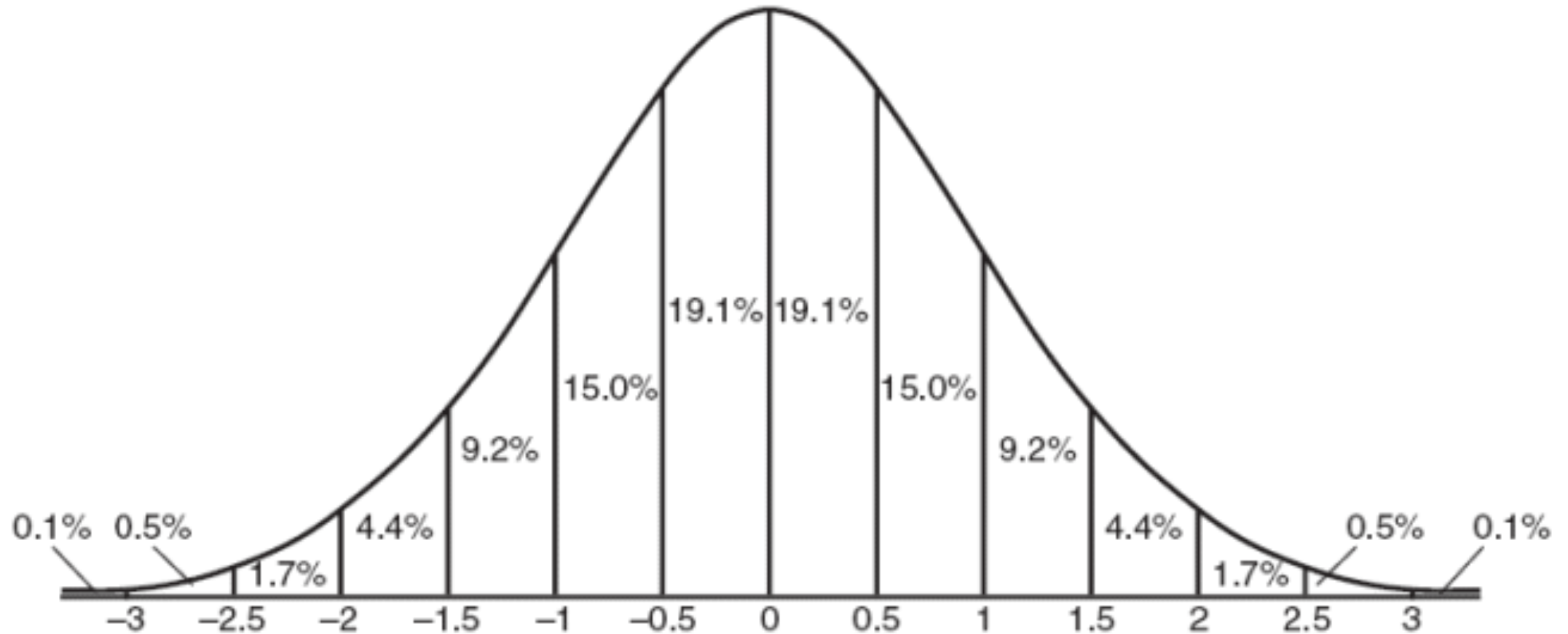


# HISTORY OF CORRELATIONS

- Early focus by Pearson widely theoretical
- Fisher unraveled linear relations
- Explanatory measures have no real threshold
- Statistical power is highly relevant
- Shift from hard to soft science



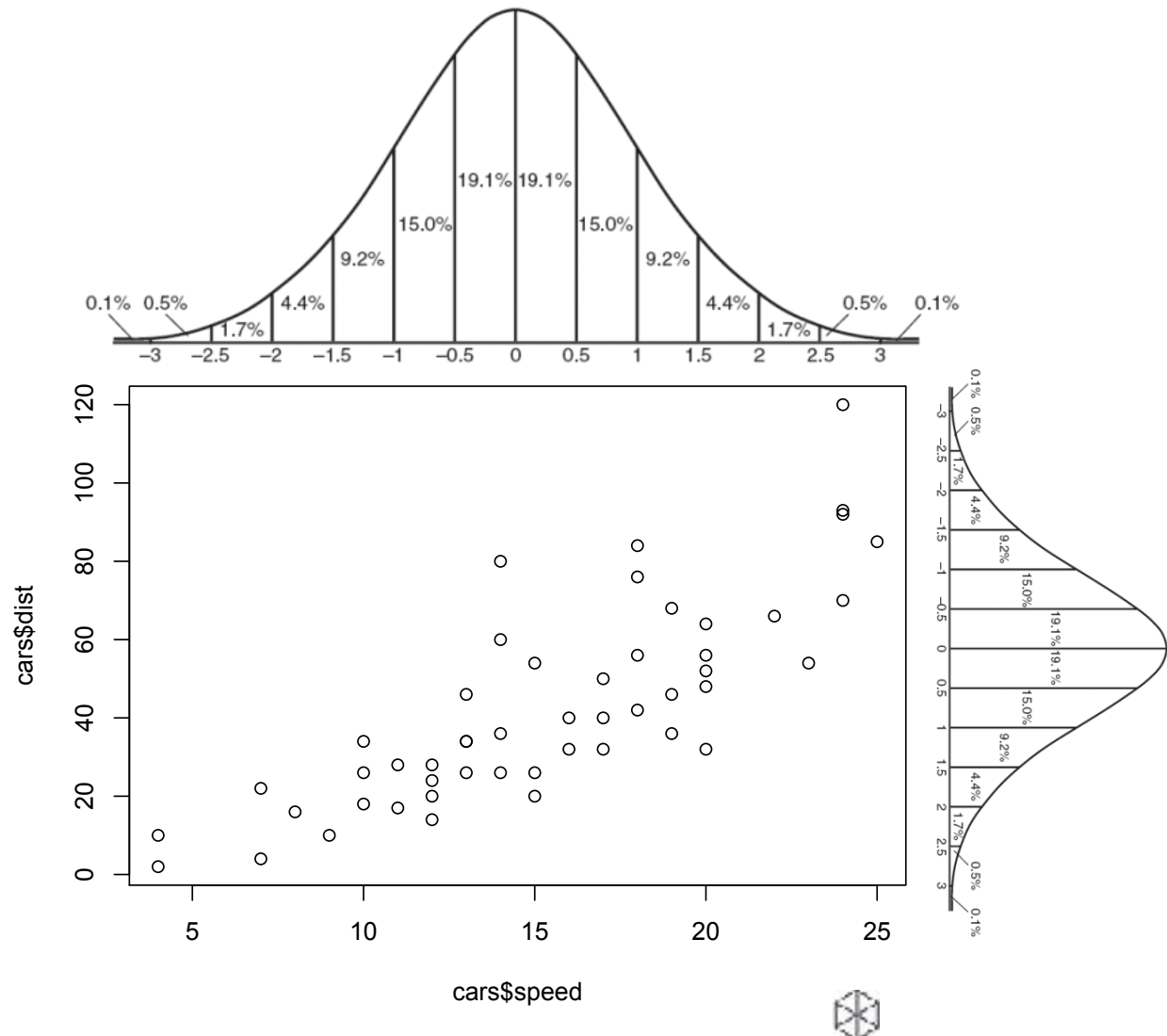
# THE NORMAL DISTRIBUTION



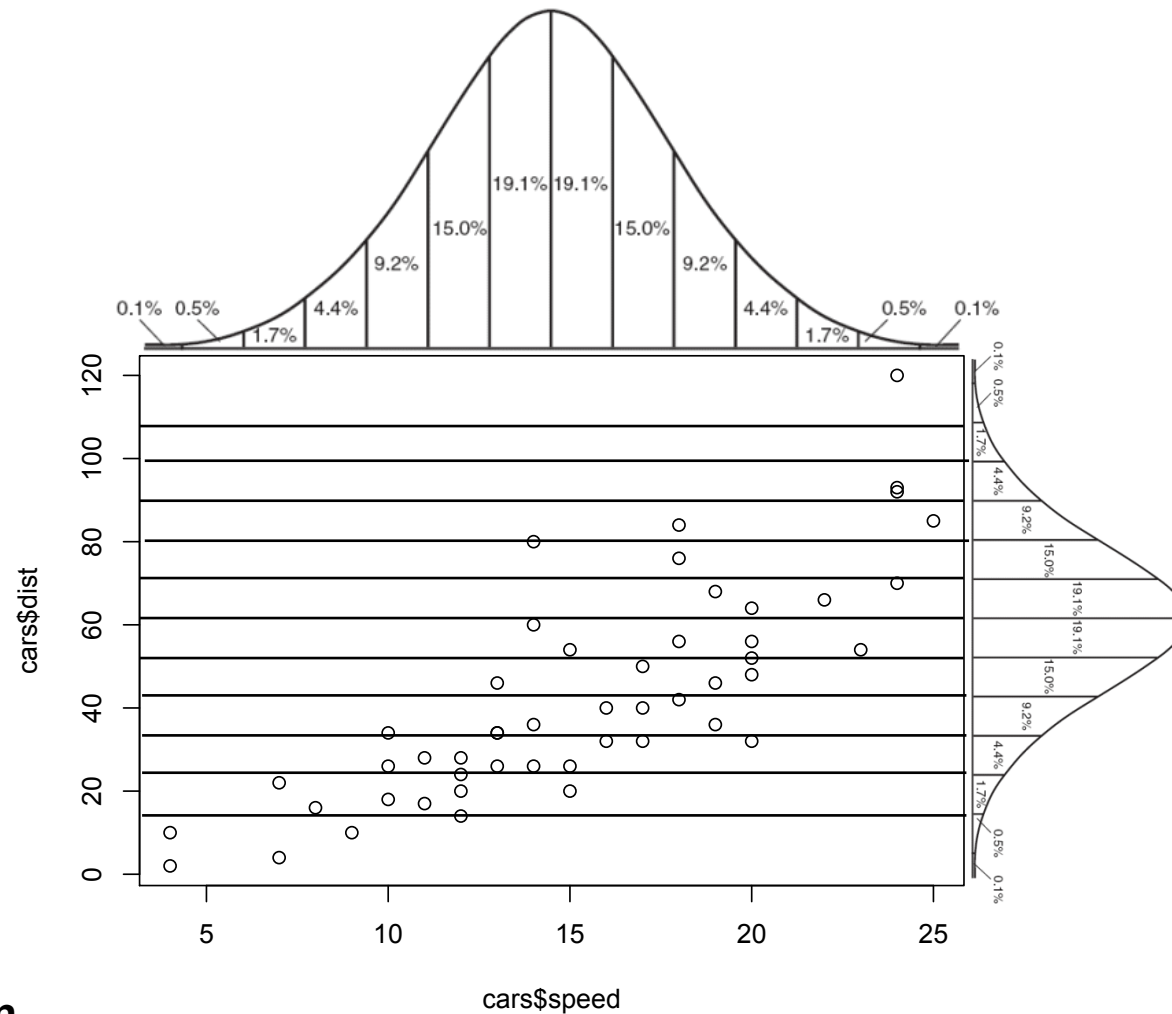
# LET US START SIMPLE: STATISTICAL CORRELATIONS

$$r_{xy} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}}$$

- $r_{xy}$  – the correlation coefficient of the linear relationship between the variables  $x$  and  $y$
- $x_i$  – the values of the  $x$ -variable in a sample
- $\bar{x}$  – the mean of the values of the  $x$ -variable
- $y_i$  – the values of the  $y$ -variable in a sample
- $\bar{y}$  – the mean of the values of the  $y$ -variable

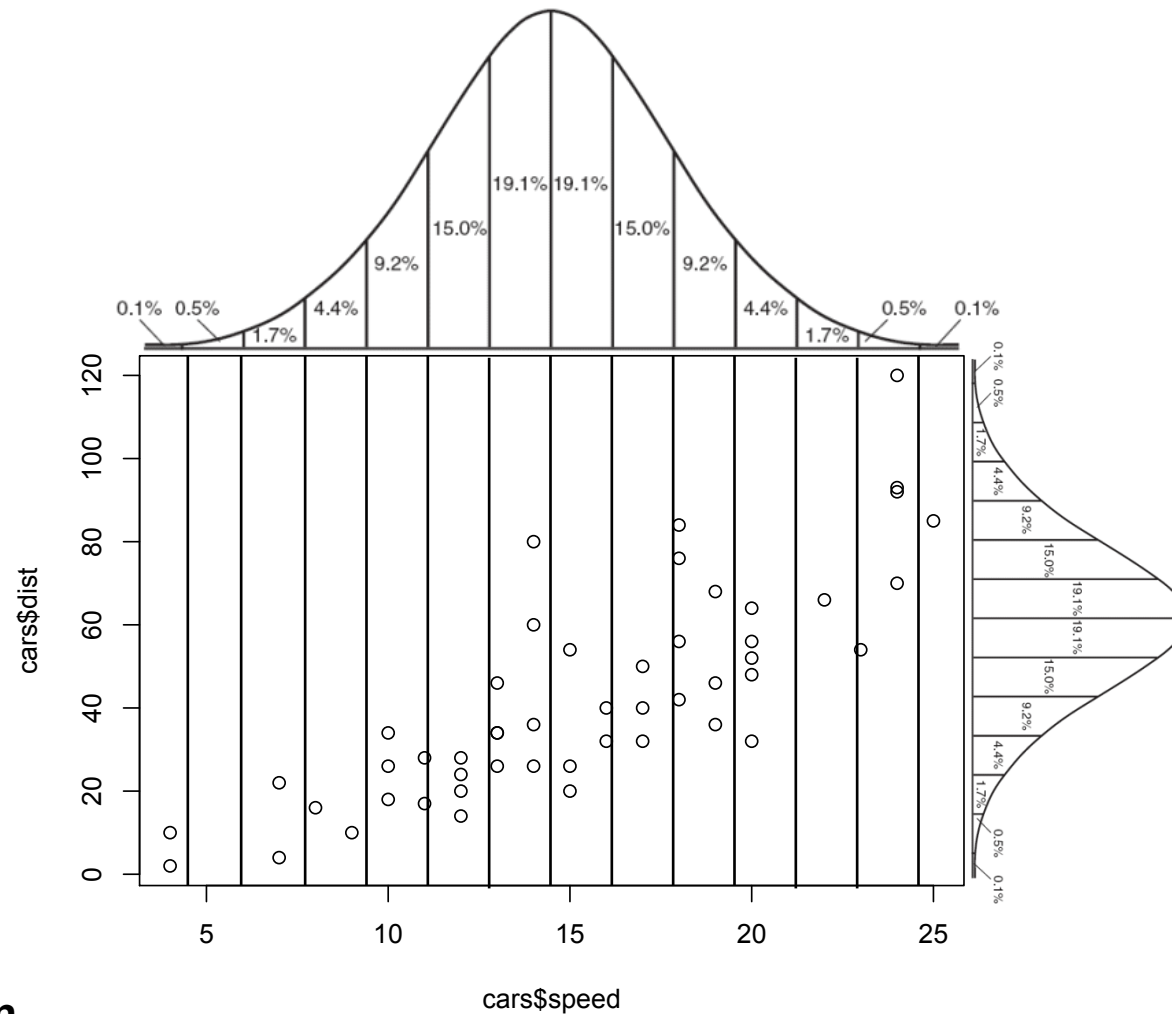


# LET US START SIMPLE: STATISTICAL CORRELATIONS





# LET US START SIMPLE: STATISTICAL CORRELATIONS



# PROBABILITY - STATISTICS

## The lady tasting tea



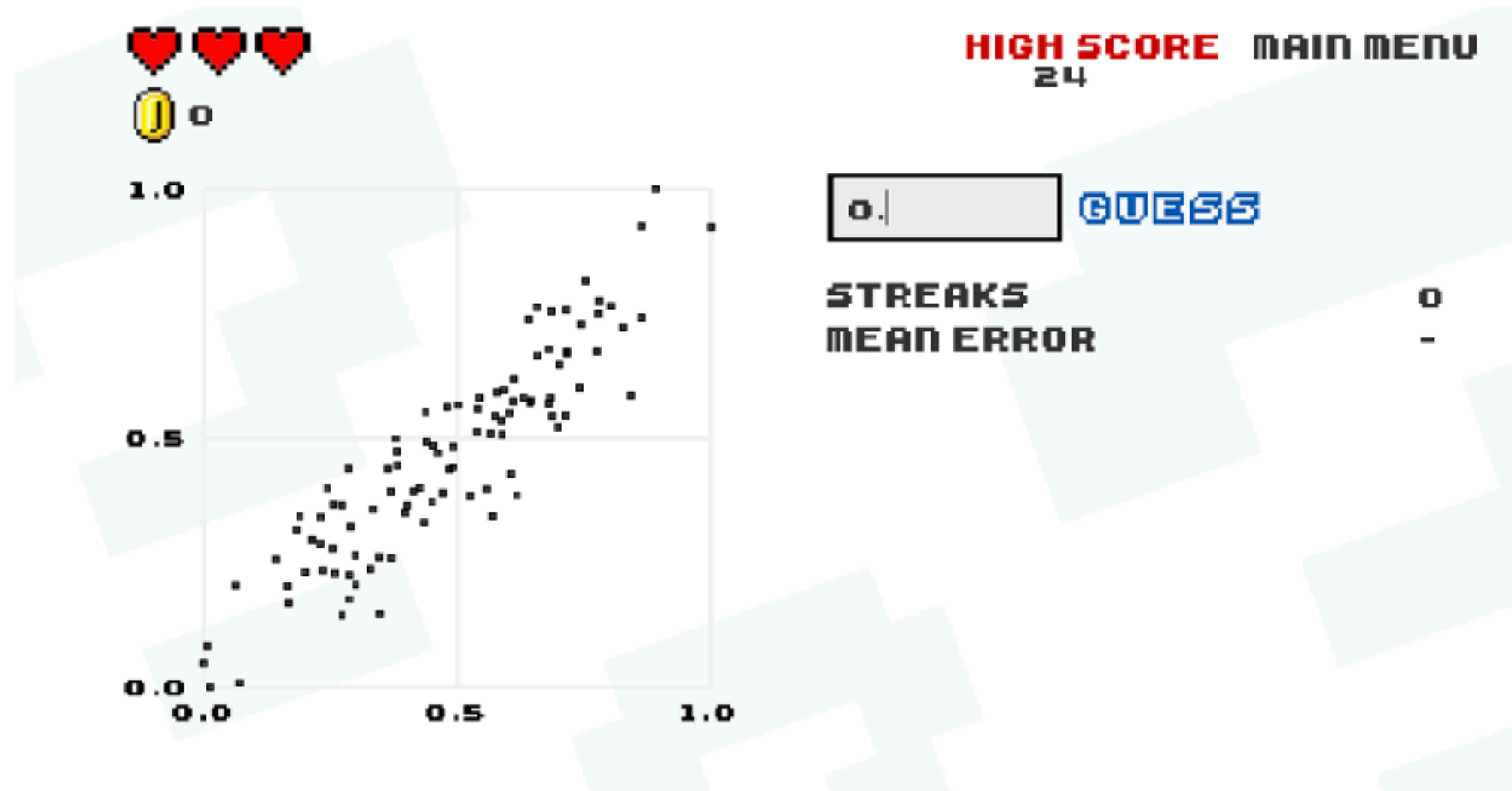
# What is the probability?



# HOW MUCH DOES A MODEL EXPLAIN?



# HOW MUCH DOES A MODEL EXPLAIN?



# STATISTICAL CORRELATIONS: ONE VARIABLE RELATES ON ONE VARIABLE.

```
> cor.test(cars$dist, cars$speed)
```

Pearson's product-moment correlation

data: cars\$dist and cars\$speed  
t = 9.464, df = 48, p-value = 1.49e-12 **P-VALUE: 0.0000000000000149**  
alternative hypothesis: true correlation is not equal  
0

95 percent confidence interval:

0.6816422 0.8862036

sample estimates:

cor **CORRELATION COEFFICIENT**  
0.8068949 **RANGES FROM -1 TO 1**



# STATISTICAL CORRELATIONS: ONE VARIABLE RELATES ON ONE VARIABLE.

## Correlation

Correlation coefficient ranges from -1 to 1

Inspect the relation for flaws

Correlations can be inductive and deductive

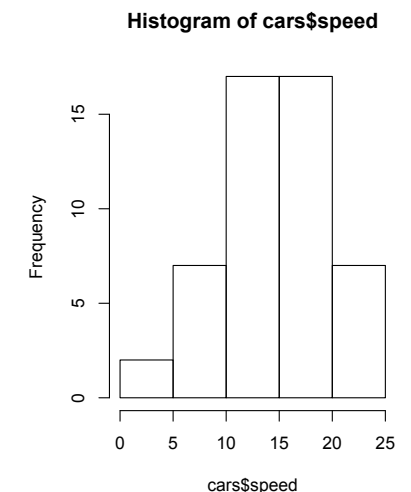
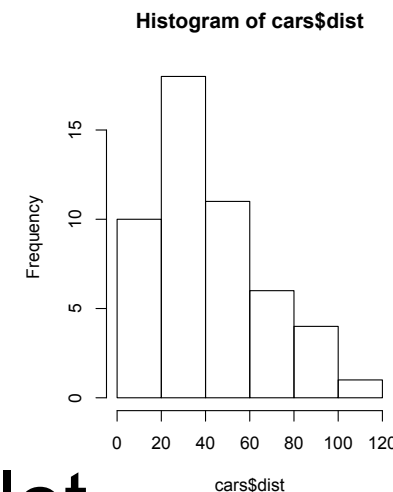
Check for data density

Data gaps

Changes in the relation

Diagnostics:

x-y-plot, Normal distribution?, Scatterplot



# STATISTICAL CORRELATIONS: ONE VARIABLE RELATES ON ONE VARIABLE.

| Data                    |              |
|-------------------------|--------------|
| Normal distribution     | Pearson's R  |
| Non normal distribution | Spearman's R |





# STATISTICAL CORRELATIONS: ONE VARIABLE RELATES ON ONE VARIABLE.

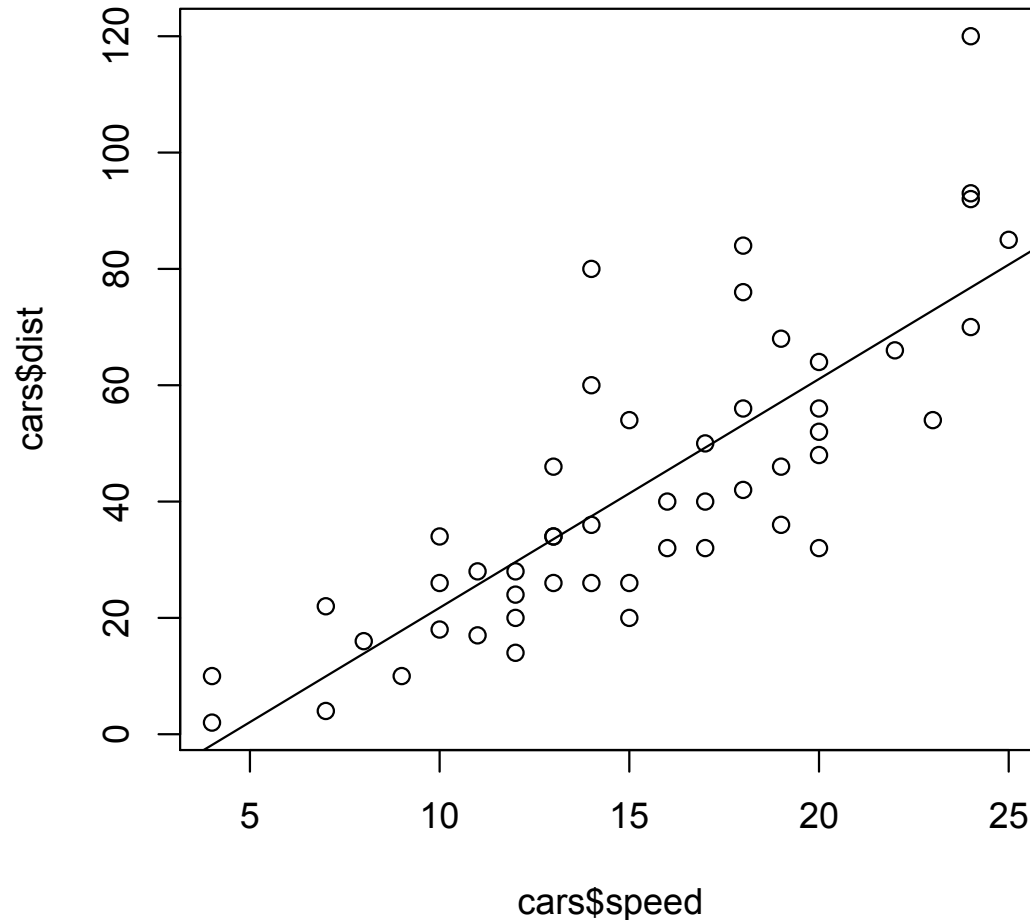
## Pearson correlations

linear relation between two variables, normal distribution?;  $-1 \leq R \leq 1$ .

- `cor(x,y)` → simple values
- `cor(data)` → simple values, correlation matrix
- `cor.test(x,y)` → extensive result with many details



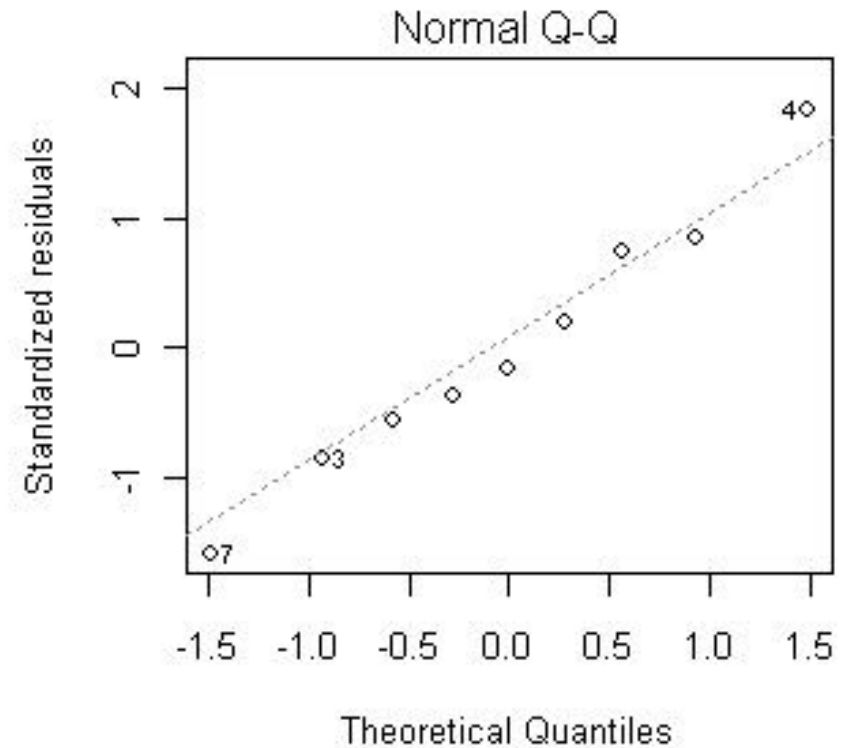
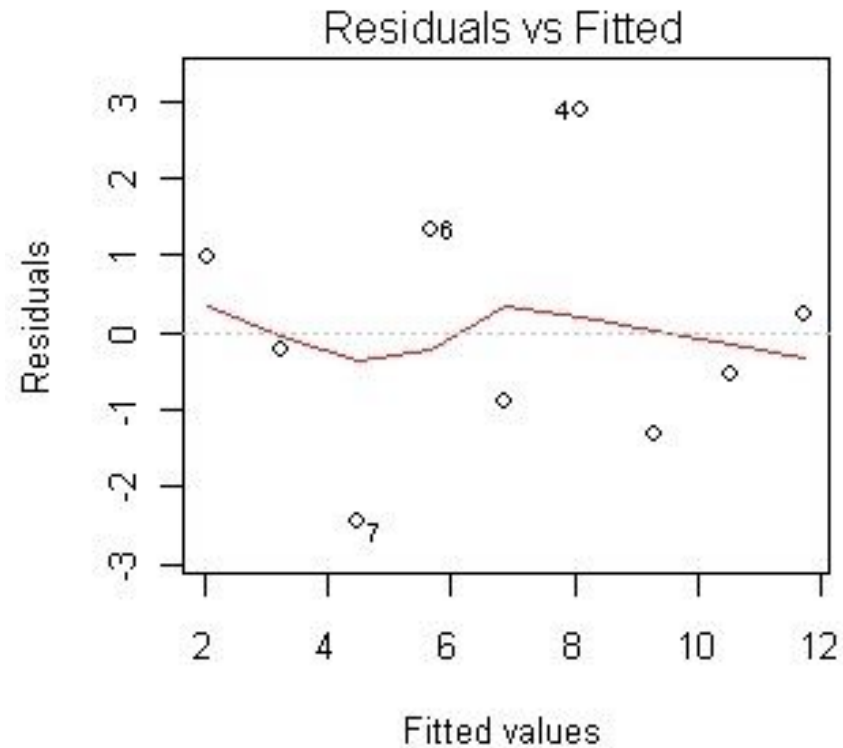
# AND NOW THE REGRESSION: ONE VARIABLE IS EXPLAINED BY ANOTHER VARIABLE, AND HENCE DEPENDS ON IT



Breaking distance depends on the speed!



# Model inspection



# Assumptions: of a linear regression

- independent and dependent Variables are continuous
- dependent variables and residuals are normally distributed (Transformations, GLM)
- Homogeneity variances (Transformations, GLM)



# REGRESSION ON A SHOESTRING

## Linear regression

- Relation between an independent variable **x** and a dependend variable **y**

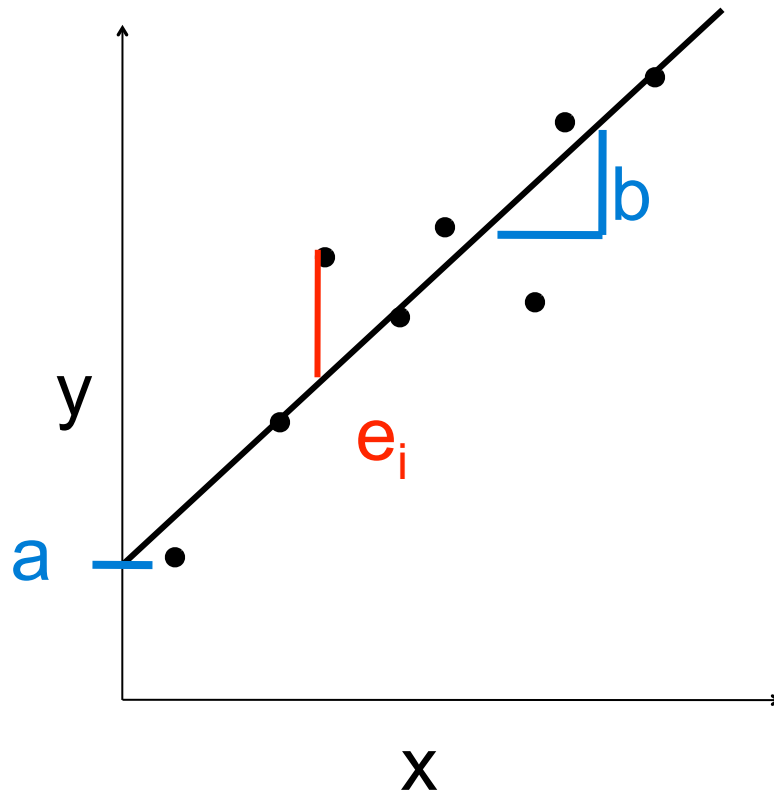
$$y = a + bx$$

**a** Intercept with the y-axis

**b** Estimate of the regression



# REGRESSION AS A FIGURE AND A FORMULA

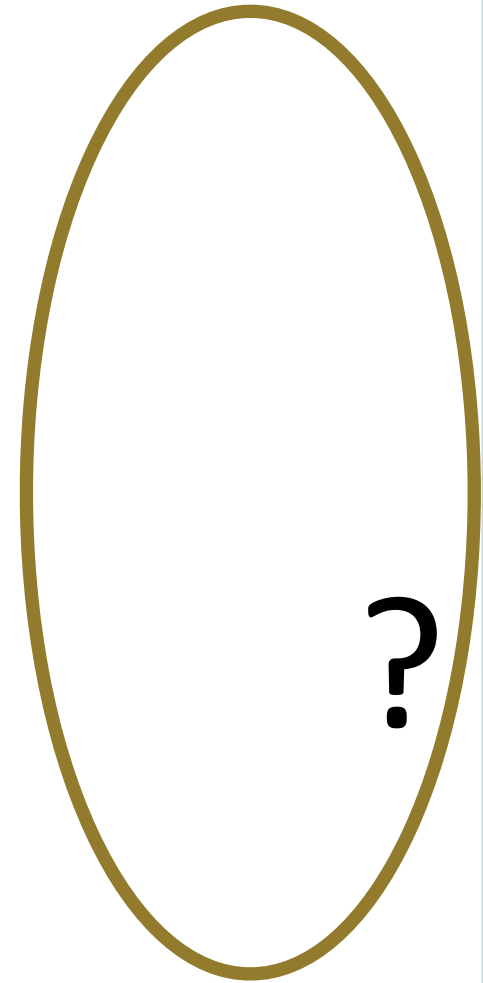
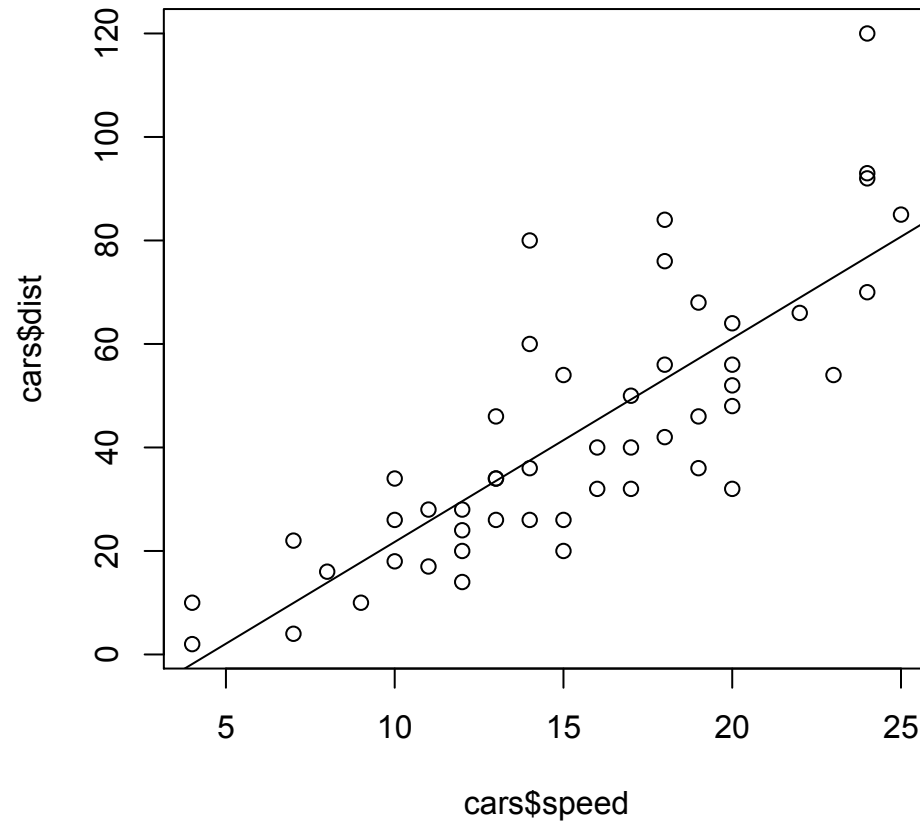


$$y = a + bx$$

Residual  $e_i$



# PREDICTIONS: SOMETIMES OUTSIDE OF THE DATA



# UNCERTAINTY - HOW CERTAIN ARE WE?

There are measures of uncertainty!  
Context matters!  
What could be unknown?  
No model is perfect!





# R EXAMPLE OF A LINEAR REGRESSION

```
> summary(model)
```

Call:

```
lm(formula = growth ~ tannin)
```

Residuals:

| Min     | 1Q      | Median  | 3Q     | Max    |
|---------|---------|---------|--------|--------|
| -2.4556 | -0.8889 | -0.2389 | 0.9778 | 2.8944 |

Coefficients:

|             | Estimate | Std. Error | t value | Pr(> t )     |
|-------------|----------|------------|---------|--------------|
| (Intercept) | 11.7556  | 1.0408     | 11.295  | 9.54e-06 *** |
| tannin      | -1.2167  | 0.2186     | -5.565  | 0.000846 *** |

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.693 on 7 degrees of freedom

Multiple R-Squared: 0.8157, Adjusted R-squared: 0.7893

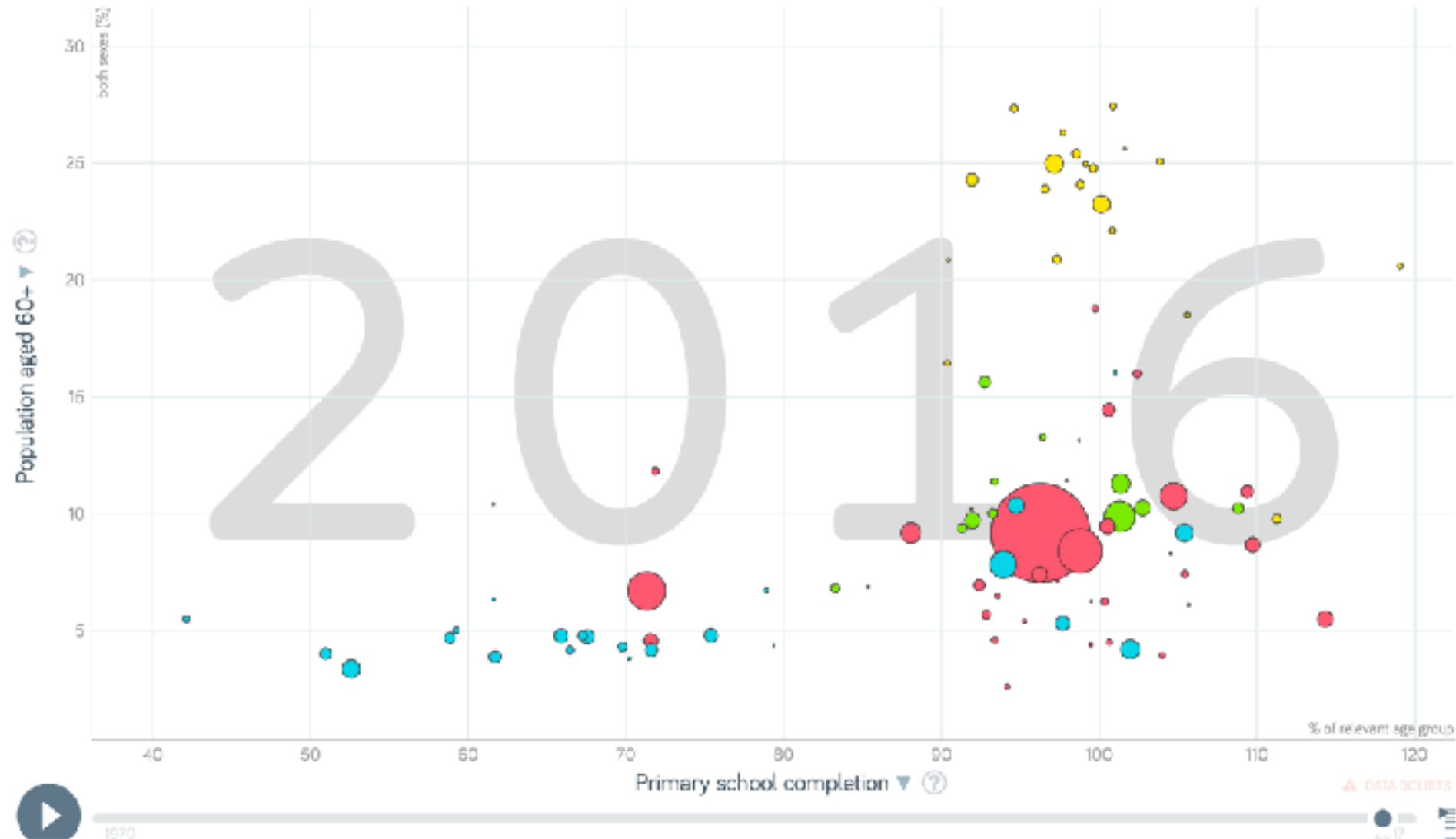
F-statistic: 30.97 on 1 and 7 DF, p-value: 0.000846



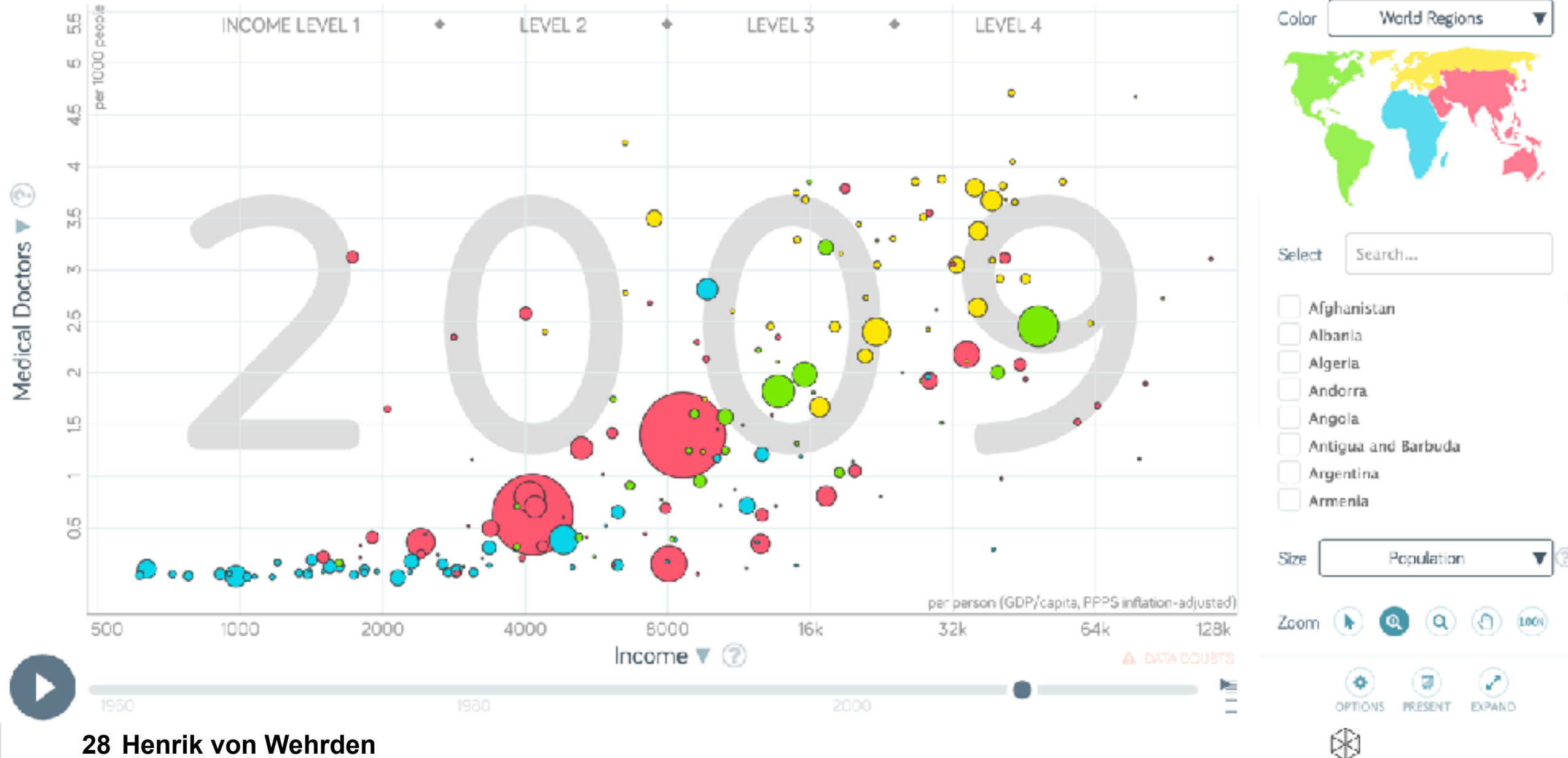
# LEARN TO READ CORRELATION PLOTS



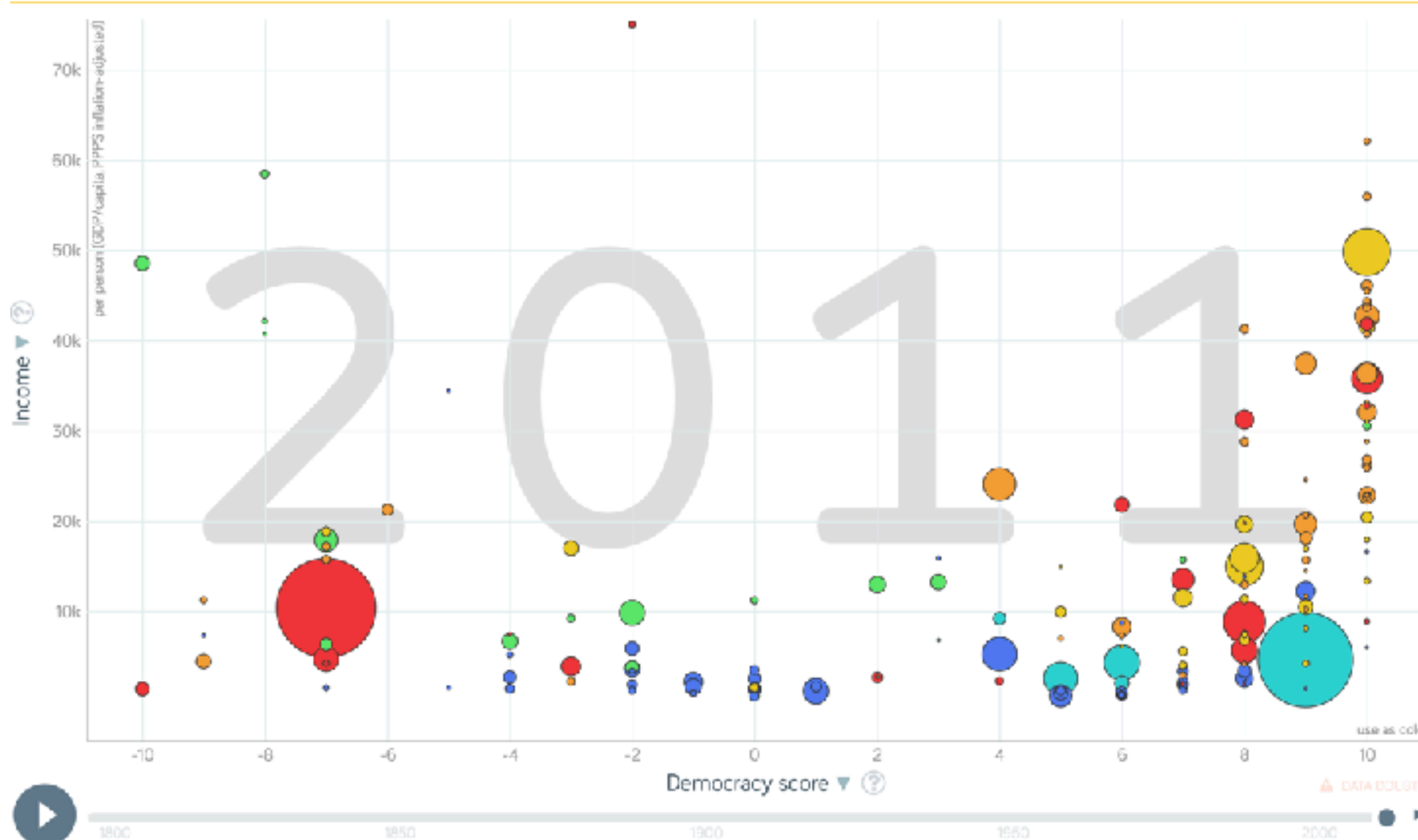
# LEARN TO READ CORRELATION PLOTS



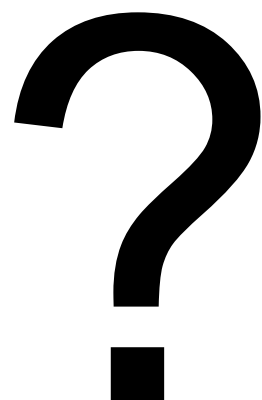
# LEARN TO READ CORRELATION PLOTS



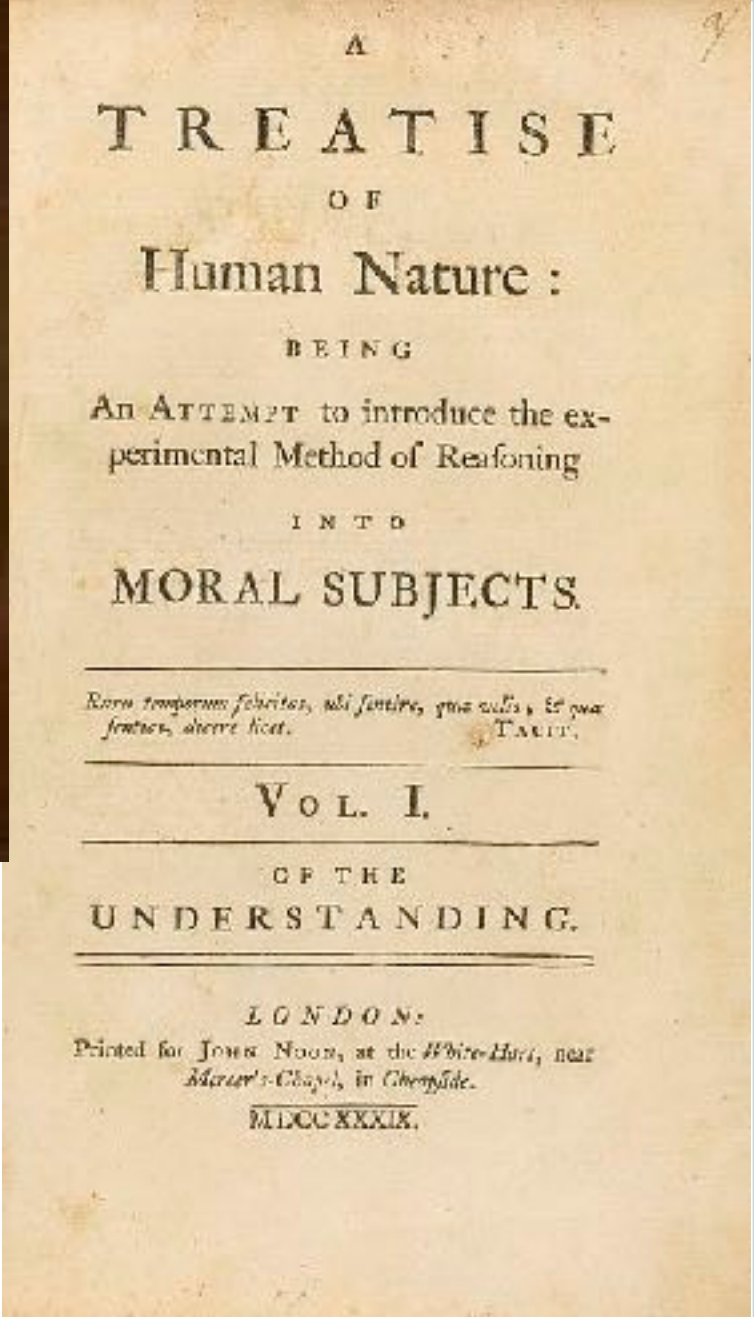
# LET US START SIMPLE: STATISTICAL CORRELATIONS



# CAUSALITY



Hume



Knowledge and probability

|                    | Immediate  | Inferential             |
|--------------------|------------|-------------------------|
| Relations of ideas | intuition  | demonstrative reasoning |
| Matters of fact    | perception | probable reasoning      |

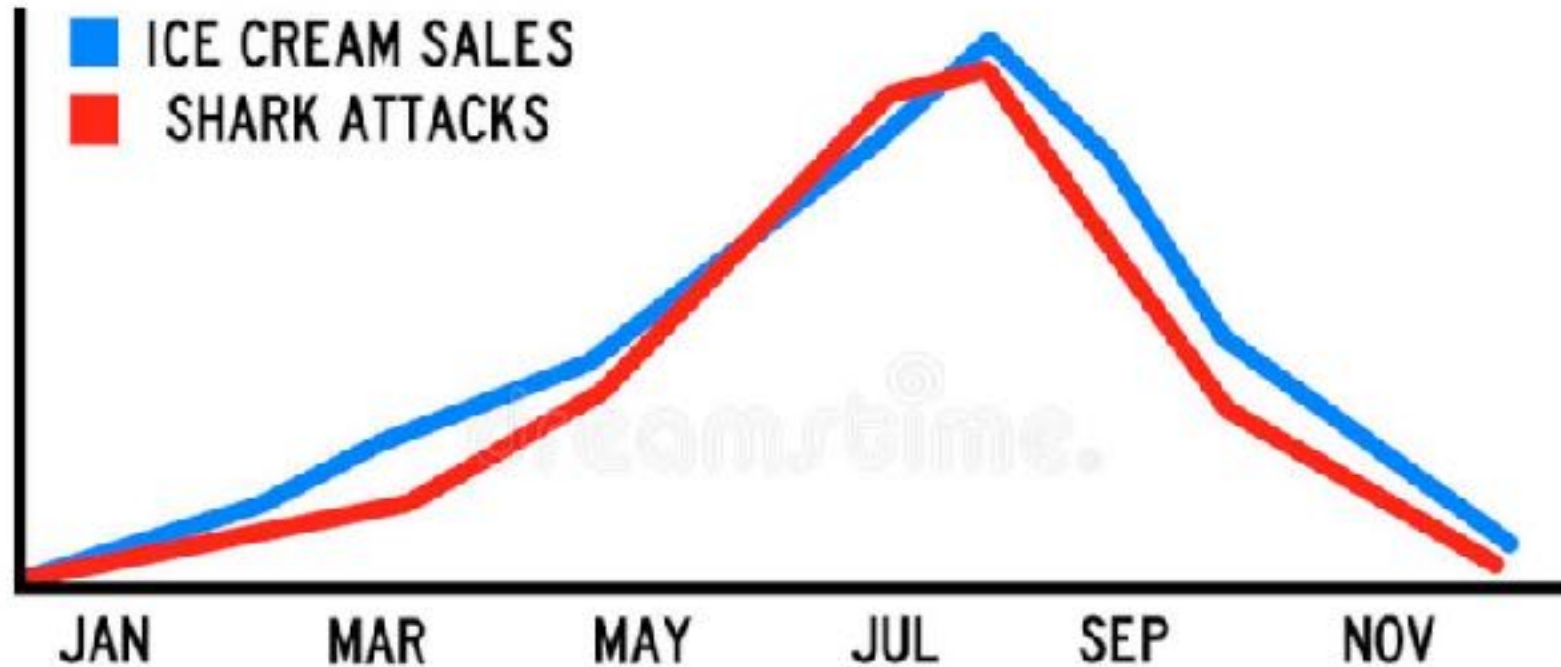


# A GLIMPSE AT HUME'S CAUSALITY CRITERIA

1. The same cause produces the same effect.
2. If several objects create the same effect, then there must be a uniting criterion among them causing the effect.
3. If two objects have a different effect, there must be a reason that explains the difference.



# DO YOU WANT TO PREDICT OR TO EXPLAIN?



Both ice cream sales and shark attacks increase when the weather is hot and sunny, but they are not caused by each other (they are caused by good weather, with lots of people at the beach, both eating ice cream and having a swim in the sea)





# SOME GENERAL LAWS THAT MAY BE VALUABLE

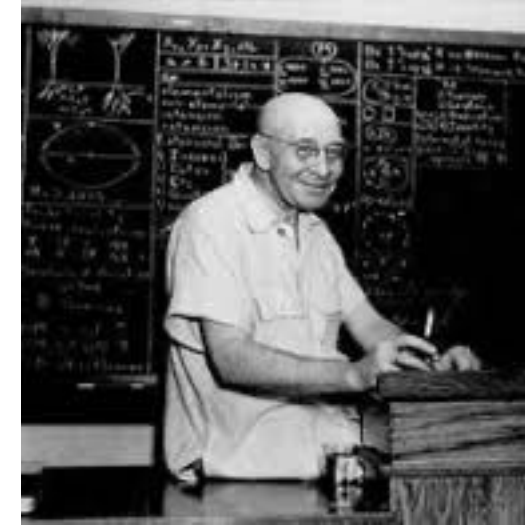



All models are  
wrong  
Some models  
are useful.

33 Henrik von Wehrden



Everything needs  
to be as simple as  
possible, and as  
complex as  
necessary.



A map is not the  
territory it represents,  
but, if correct, it has  
a similar structure to  
the territory, which  
accounts for its  
usefulness. 

# WHAT SHOULD MODELS BE?



Georg Box



William of Ockham



Alfred Korzybski

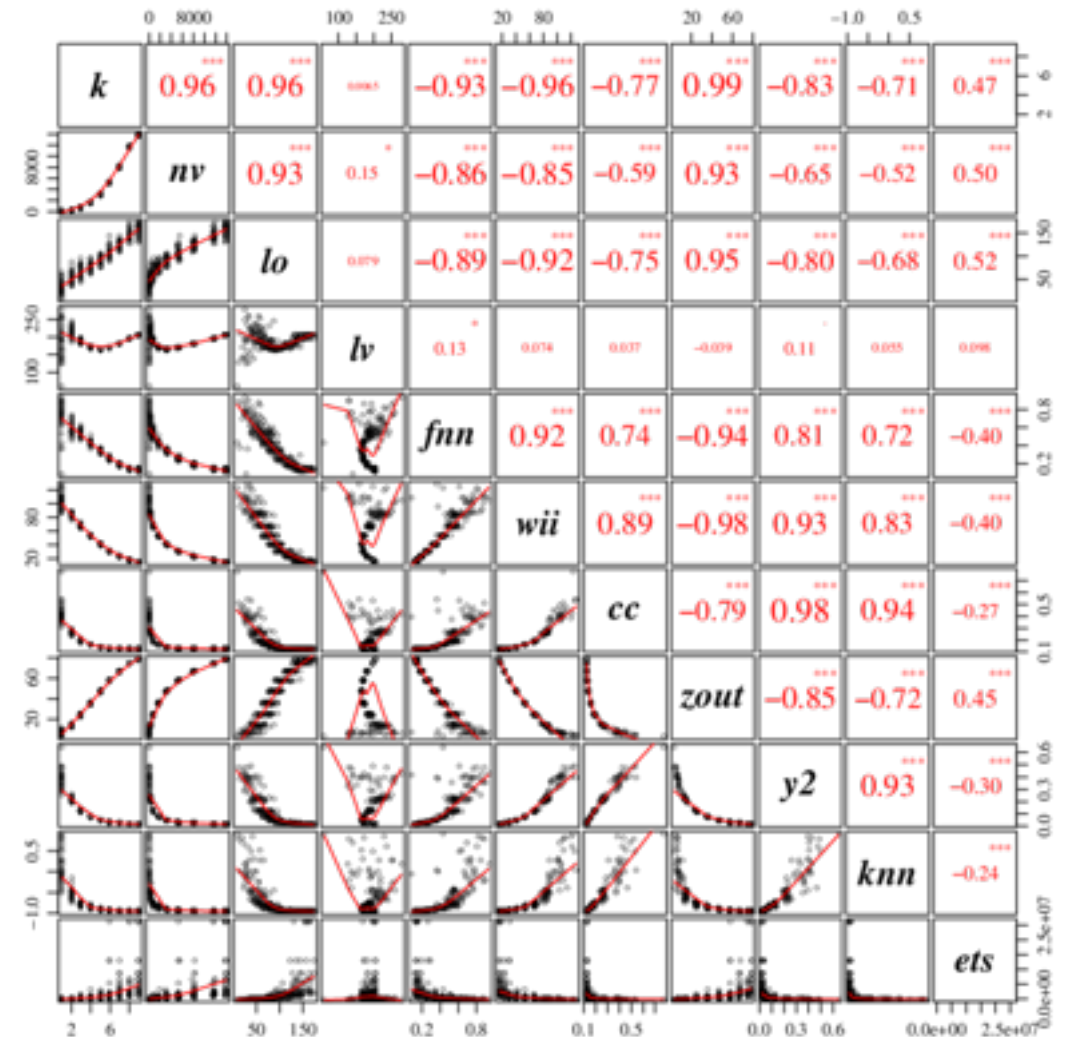
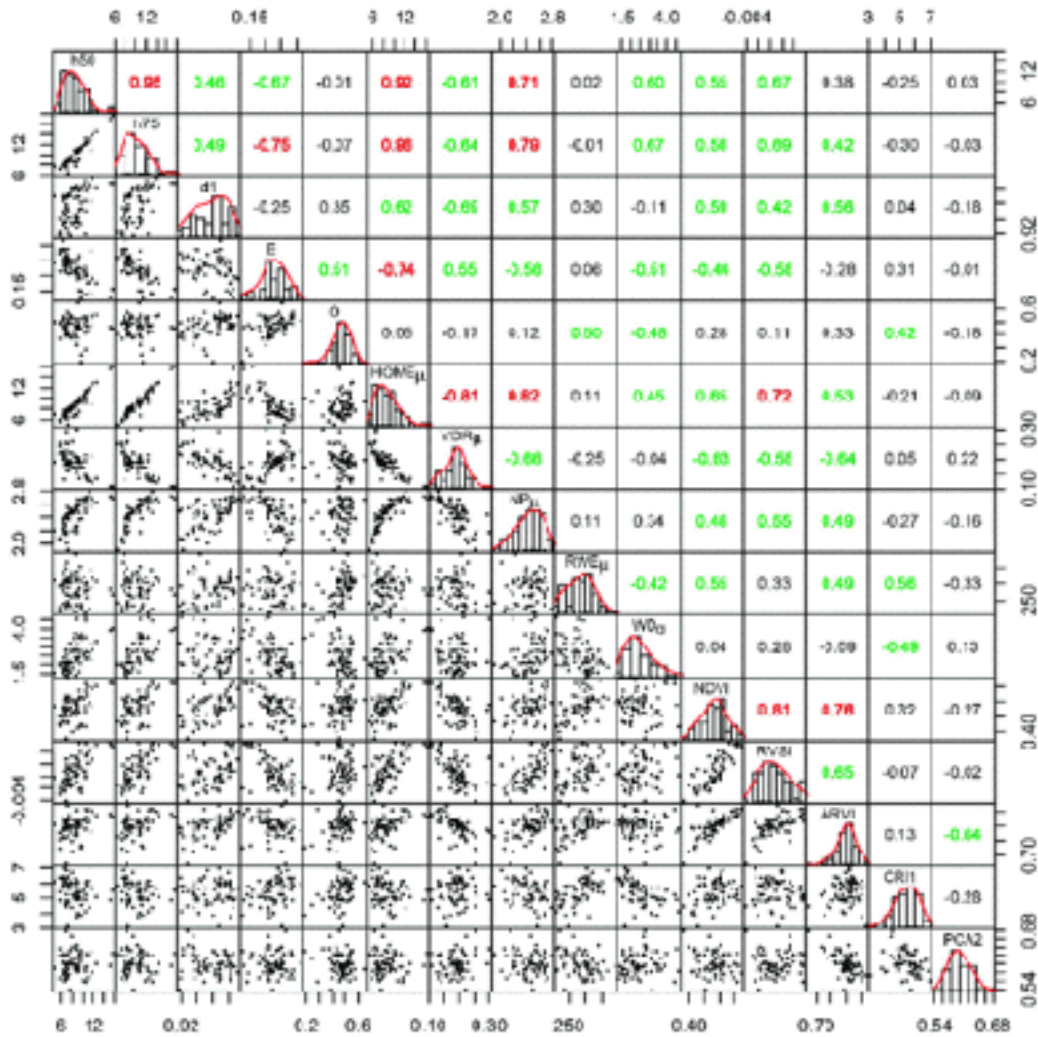
Approximations?

Parsimonious!

Generalisation?



# AFTER TEN THOUSANDS OF PLOTS AND MODELS, INTUITION



# CAUSALITY OR CORRELATION? ONE SUMMARY.

- In a century of numbers, correlations mattered
- Correlation can predict, and may even help to explain
- Correlation models are generalisation, approximations, and (ideally) parsimonious
- Causality can be altogether different thing
- Suggestions of causality are rooted in logic
- Whether you may ultimately understand relations between to continuous variables is a matter of theory



# CONTACT

Henrik von Wehrden | Faculty of Sustainability  
Universitätsallee 1 | 21335 Lüneburg  
Fon 04131.677-1671 | [henrik.von\\_wehrden@leuphana.de](mailto:henrik.von_wehrden@leuphana.de)  
<https://henrikvonwehrden.web.leuphana.de>

