

Financial Time Series Analysis of Stock Price of Traveler's Insurance

Tess Steplyk

Executive Summary

This report provides an analysis and evaluation of the current and forecasted stock price of Traveler's insurance. Methods of analysis include ARMA models, GARCH models, and lag testing. Other calculations include model fitting, forecasting, and risk and volatility estimations. The calculations can be found in the appendices and in the report. Results of data analyzed show that it would be in the best interest in a long position to sell the stock or option.

The report finds the prospects of the company in its current position are very positive, especially when comparing to the company in 2008 and 2009. The company has continued since 2009 to almost have a consistent increase in its stock price, and it is still growing. Look at figure 1 to see a preliminary idea of Traveler's stock growth.

Also, the report investigates the fact that the analysis conducted has limitations. Some of the limitations include: simply looking at only the 2008 stock market crash, not the smaller in comparison drops in stock price from 2009 to 2016. For example, in figure 1 for the closing prices of Traveler's, it is easily seen that in late 2015 dropped from about \$110 to about \$90 within a short period.

Introduction

Stock price forecasting is useful in solving real problems. I am attempting to find the trading strategies for Traveler's stock and the volatility for the stock, as well. Traveler's Insurance was founded over 163 years ago and represents a publicly traded American insurance company. It is the second largest writer of commercial property casualty insurance and the third largest writer of personal insurance in the United States. As a property and casualty insurance company, Traveler's focuses on writing policies that cover vehicles, real estate, and other properties, as well as liabilities for accidents and negligence. As of 2013, Traveler's wrote \$20.6 billion in net premiums, meaning the amount of money property and causality policies expect to receive over the time of the contract.

By focusing on a property and casualty insurance company, I can show the before and after effects of the stock market crash. It was particularly important for me to use a

property and casualty insurance company because the crash specifically affected homes and businesses, which are who P&C companies write policies for. Analyzing the trading strategies and volatility will determine whether it is beneficial to buy or sell Traveler's stock, even when including the stock market crash.

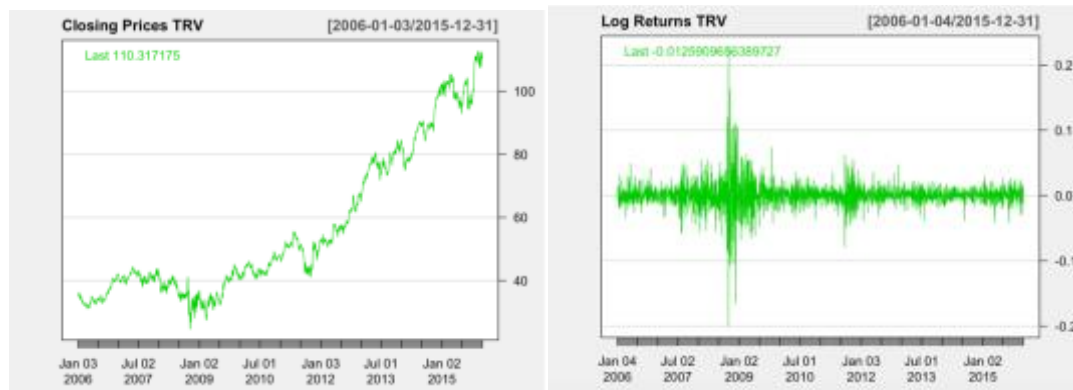


Figure 1: closing price and log returns on Traveler's stock

Data Description

The dates of focus will then extend from January 1st, 2006 (2006-01-01) to January 1st, 2016 (2016-01-01), for looking particularly at the before and after effects of the stock market crash. I have extended my data to spread over ten years to get the best look at the true effects of before, during, and after the stock market crash. The stock data will be taken from Yahoo! Finance (www.finance.yahoo.com) with the Quantmod package in R using 2517 observations.

The stock price provides us seven variables: date, day's open stock price, high stock price, low stock price, close stock price, trading volume, and adjusted close stock price. I will, however, mainly be focusing on just date, day's open stock price, close stock price, and adjusted close stock price particularly. I will be using log returns throughout the report as log returns allow us to assume lognormality, and assuming the distribution is normal is much easier to analyze and the residual easier as well. Also, we are looking for deviations from the random walk, so autoregressive coefficient with a significance suggests there is a real effect, and potentially one that can be traded.

Preliminary Analysis

Performing the Ljung-Box test first (Appendix 2), the outcome was a very small p-value of 0.00006. This is significant and allows us to conclude there are serial

correlations in the return (r_t) series. To find these serial correlations for the log return of Traveler's stock, we will need to show and build linear and volatility models.

	TRV
nobs	2516
Minimum	-0.200671
Maximum	0.227578
1. Quartile	-0.006934
3. Quartile	0.007829
Mean	0.000457
Median	0.000657
SE Mean	0.000373
Variance	0.00035
Stdev	0.018697
Skewness	0.327397
Kurtosis	23.810386

Table 1: Summary Statistics for log return

Given the information from table 1, I can test the hypothesis of symmetry and normal kurtosis to examine the normality of the data. Traveler's stock rejects the hypothesis, and can conclude that the log returns are asymmetric and have heavy tails. Therefore, I will need to consider non-normal distributions when fitting models. With these things in mind, I attempted to fit a model to each stock individually.

Beginning with the assumption of Traveler's being a simple linear time series, we will use the basic model (Appendix 1) and try to fit μ_t using ACF, PACF, EACF, and AIC/BIC to find the correct number of lags. After lags are found, I will be able to fit the data on AR, MA, and ARMA models.

After analyzing ARMA models, I will need to analyze ARCH models with the Ljung-Box test. Then, use the F-statistic to also check the time series for my ARCH model. Assuming it is significant, I can then fit the asset volatility (α_t) on various GARCH volatility models. Particularly focusing on GARCH models with normal and student-t distribution, iGARCH model, EGARCH model, and GARCH-M model. Then effectively using the residual, QQ-normal, and model fit tests to compare how well the various models fit to the true time series being tested.



	(1,1)	(1,2)	(2,1)	(2,2)	i(1,1)	t(1,2)	m(1,2)	i(1,1)	t(2,2)	m(2,2)
Sign Bias	0.713413	0.73997	0.713407	0.79861	0.79049	0.20062	0.73297	0.79049	0.338537	0.73283
Neg Sign Bias	0.007838***	0.02567**	0.007836***	0.01156**	0.01192**	0.01767**	0.02280**	0.01192**	0.008299***	0.02255**
Pos Sign Bias	0.582362	0.45116	0.582383	0.44606	0.4396	0.97782	0.45104	0.4396	0.796472	0.45216
Joint Effect	0.027558**	0.06927 *	0.027551**	0.02646**	0.02749**	0.12546	0.06222*	0.02749**	0.065651*	0.06168*
Likelihood	7403	7405	7403	7405	7392	7420	7405	7392	7423	7405

Table 2:P-values and Likelihood for bias tests of GARCH models

Proposed Model and Fitted Results

Model Fitting:

To try to identify the ARMA effect of the Traveler's model, I have tested and fit the models suggested with the EACF plot. Focusing on MA(2), AR(2), and ARMA(2,2) as well as adding in a white noise ARMA(0,0) to fit alongside the other three and to see if the simple model could be used. After performing the Ljung-Box test and finding the AIC for all four models, I compiled a table of the data.

All achieved very similarly small AIC, however the white noise and MA(2) model failed the Ljung-Box test with extremely small p-values. Then, both the ARMA(2,2) and the AR(2) models ended up passing the Ljung-Box test. Looking at the table, we can see both fit the returns very well, but because AR(2) has the smallest AIC, I have chosen the AR(2) model when going forward with the Traveler's stocks data.

	mu	ma1	ma2	omega	alpha1	beta1	beta2	eta11	skew	shape
coefficients	0.000477	-0.105814	-0.005911	0.000252	0.106705	0.647938	0.25465	0.526839	0.956255	5.549828
std error	0.000195	0.01968	0.019197	0.000076	0.019852	0.07244	0.068228	0.114073	0.02643	0.593082

Table 3: Coefficients and SE of tGARCH Model

I will discuss proposed linear models, and first plotting log returns, along with their ACF and PACF. As shown, both plots show significant correlations out to high lags, so I will need to examine the EACF plot of the Traveler's data (table 4). The EACF plot shows no clear good model therefore we will have to use AR(2) and ARMA(0,2) as they may work, but will require testing since it is unclear.

For the model, I have removed insignificant coefficients that are significant at level of 0.95 and refit the model, which means I will remove any parameter of each model with t- ratio less than 1.96 in absolute value, but will keep the intercept to see the trends in the time series.

```
## AR/MA
##    0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x x o o x o x x o o x o o o
## 1 x x o o x o o o o o x o o o
## 2 o x o o o o o x o o x x o o
## 3 x o x o x o o o o x x o o o
## 4 x x x o o o o o o x o x o
## 5 x x x x x o o o o o x o x o
## 6 x x x o o o o o o x x o o
## 7 x x x o x o o o o o x o o o
```

Table 4: EACF table for log returns

The GARCH effects by fitting the GARCH(1,1), GARCH(1,2), GARCH(2,1), and GARCH(2,2) models are shown. The maximum likelihood model (MLE) out of these four are the GARCH(1,2) and GARCH(2,2) models are exactly equal, so I also fit a tGARCH(1,2),

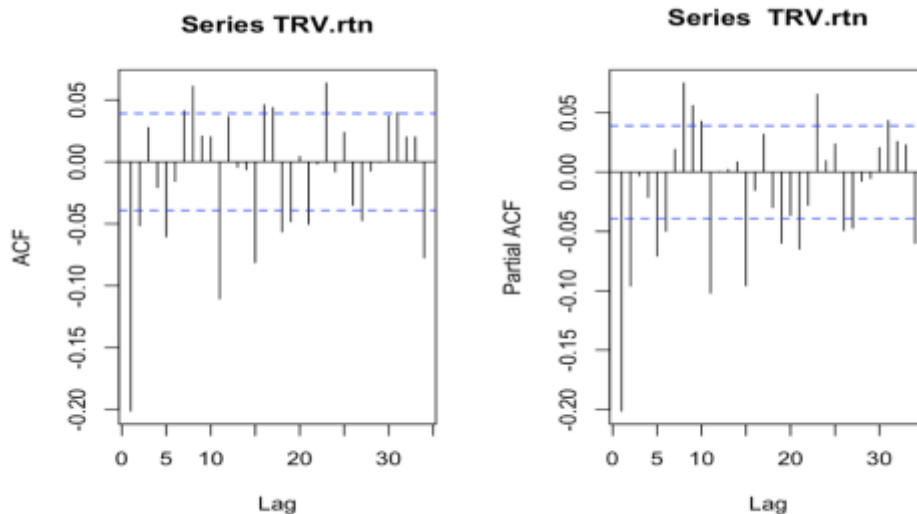
GARCH-m(1,2), and iGARCH(1,1) model. The likelihoods for each of these models is given. Using these likelihoods, the MA(1) x tGARCH(1,2) model is the best fit for the data as it has the highest likelihood and passes all goodness of fit tests. Then also fit a tGARCH(2,2), GARCH-m(2,2), and using iGARCH(1,1) model again. The likelihoods of these models are shown in the same table. Using these, the MA(1) x tGARCH(2,2) model is the best fit for the GARCH(2,2) model because it has the highest likelihood and passes all goodness of fit tests.

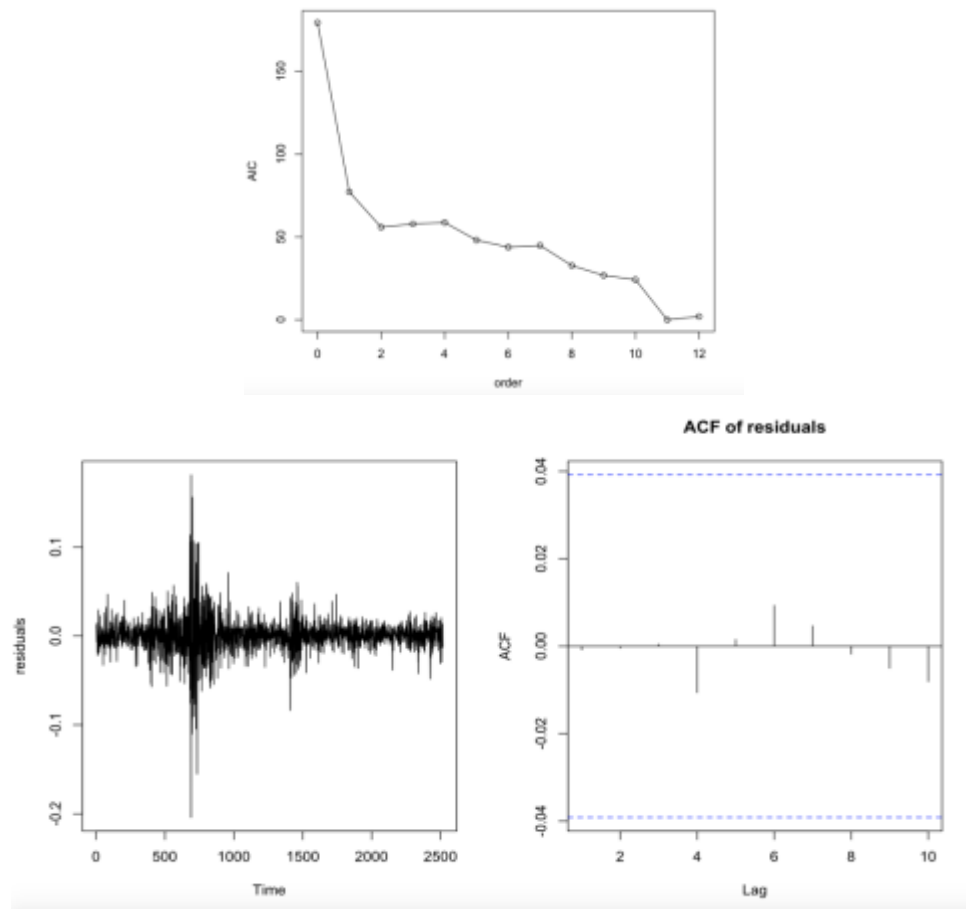
Therefore, for both models the corresponding tGARCH model is the best fit, and since it fits our data so well, then last test I will use is to check the distribution of the model's residuals. Q-Q plot of residuals is shown, and because of the fact both residuals closely follow a straight line in the plot, I will determine that the residuals are distributed as expected, and I also have more evidence that the model fits the model returns correctly. The specification is also outlined.

Lag:

The ACF graph of the log return of Traveler's stock, Figure 3, shows it is stationary, therefore a "not going anywhere" series³. A stationary series means the series is an exponentially-decaying pattern. I plotted both the ACF and PACF, and both show similar actions. Since that suggests a ARMA model, I have shown in Table 1 the EACF.

The EACF table shows an ARMA(0,2) because the simplified table shows a triangular pattern of O with its upper left vertex at the order $(p,q) = (0,2)$. A couple exceptions of X being at $q = 4, 6$, and 7 . Since the EACF table does not show these specific values of sample ACF corresponding to those X, we will assume they are only slightly greater than $2/\sqrt{2517} = 0.04$. So, that if 1% critical value is used, those X would become O in the simplified EACF table. Consequently, the EACF suggests that the log returns of Traveler's stock is an ARMA(0,2). This claim also agrees with our Sample ACF of TRV in Figure 3.





They show significant correlations for multiple lags out to lag 34. There is a lag 7, 8, 16, 17, and 23 for ACF and lag 8, 9, 10, and 23 PACF is above the upper confidence level. Then ACF and PACF share lag 11, 15, and 34 below the lower confidence level. We care much more about the effect of smaller time lapse, we will pick lag equal to 2 instead of 11.

Using the maximum likelihood estimator to identify an AR model for the r_t series (Appendix 2), we identify the AIC as a function of lags being included in the model to be 11. Figure 5 shows the plotted AR(11) model fitted, and confirms it to be 11 as the plot shows this is where the AIC minimizes, at lag 11. This AIC outcome, in turn, agrees with the ACF and PACF result. As both the ACF and PACF have very large lag at 11 below lower confidence interval line.

Next, I want to confirm my results above for the ACF and PACF plot and want to test for the presence of a GARCH effect in the returns. Calculating the Ljung-Box test statistic for the returns and their square will help with this. As seen in table 4 we reject the null hypothesis for both the returns and the squared return, concluding that an ARMA effect and GARCH effect exist in these returns. To explore these effects further,

I first explore the ARMA effect, then the residual GARCH effect.

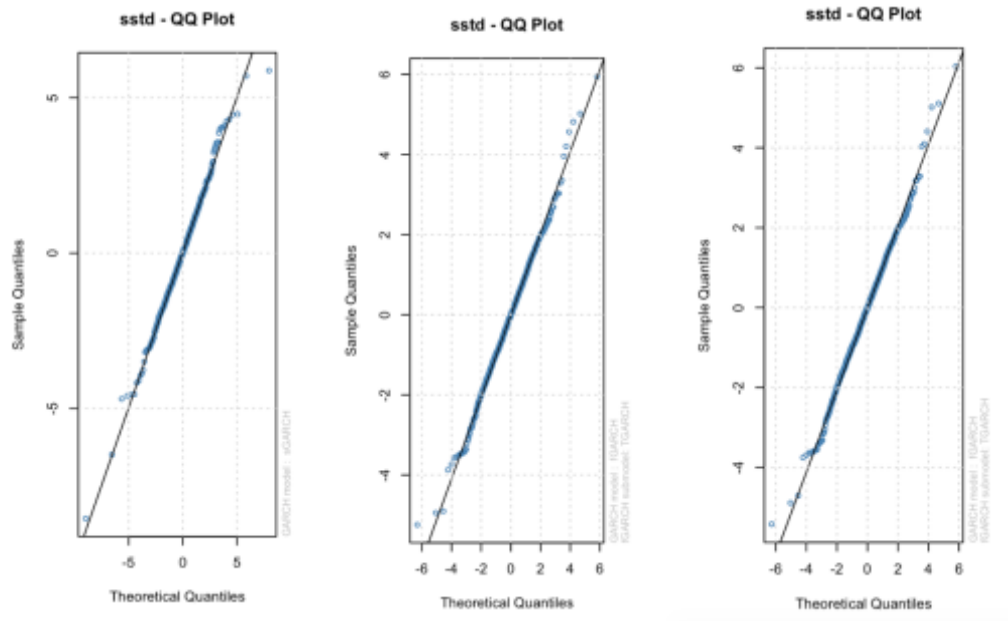


Figure 2: (from L to R) student t-distribution QQ plot for GARCH(1,1), tGARCH(1,2), and tGARCH(2,2)

AIC and BIC

It appears the AIC helps determine the tGARCH(2,2) is still the best model to use considering it has the smallest AIC. tGARCH(2,2) also holds the smallest BIC. tGARCH(2,2) will be the best representation.

Model	AIC	BIC
GARCH(2,0)	-5.73604	-5.724454
tGARCH(1,2)	-5.890476	-5.867303
tGARCH(2,2)	-5.890793	-5.862985

Table 5: AIC and BIC of three chosen models for closer look

VaR

To to check the VaR of the model is analyzing the plots of VaR forecast, which are shown by figure 3. The solid line means the lower bound of the return under specific model, which implies the protection of the losses. For all plots for these three candidate

models, we have a red dot below the lower bound, which mean an extreme loss happened at that time. However, notice that for the second and third models, the red spots are nearer to the solid lines. Thus, I would say the TGARCH (1,1) and EGARCH (1,1) models are better protecting loss than GARCH (1,1) model is.

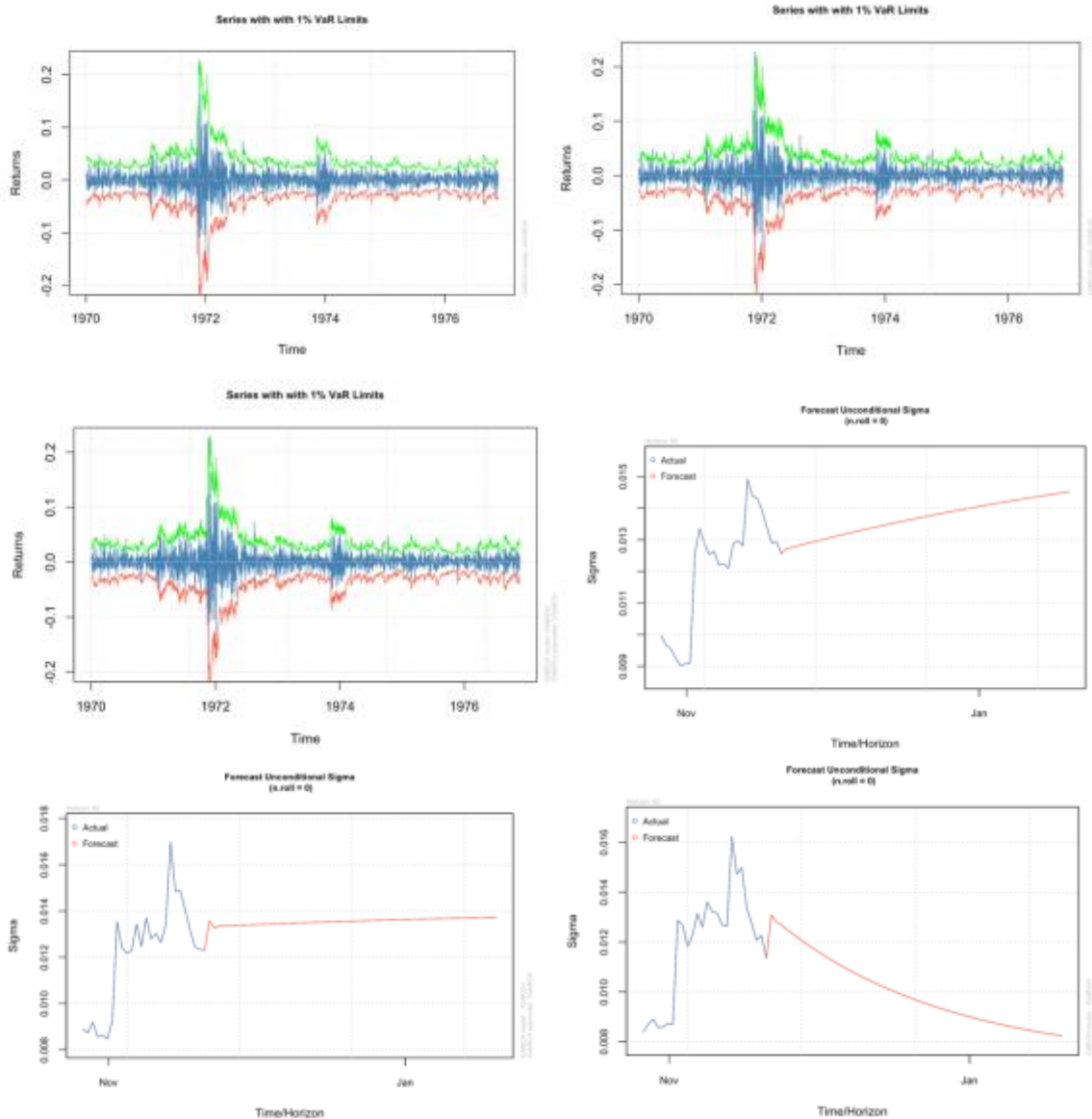


Figure 3: Top three represent the series with 1% VaR limits for GARCH(1,1), tGARCH(1,2), and tGARCH(2,2). Bottom three are VaR forecast for the unconditional sigma for same three GARCH

Taking a look at the forecast for unconditional sigma for tGARCH(2,2), which is the sixth graph in figure 2, we see a sharp downward slope for the forecast. This negative slope red line means the lower bound of the return under specific model, which implies the protection of the losses. As we can see, the forecasted red line for GARCH(1,1) and tGARCH(1,2) do not slope downward at all. Thus, tGARCH(2,2)

proves to be the best at protecting losses and the best fitted model using VaR.

Forecasts

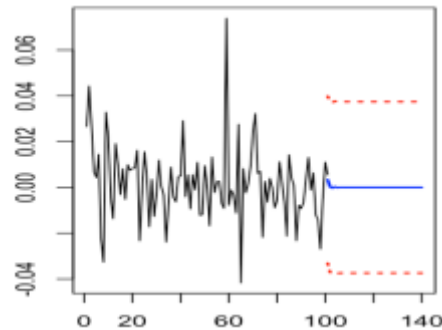
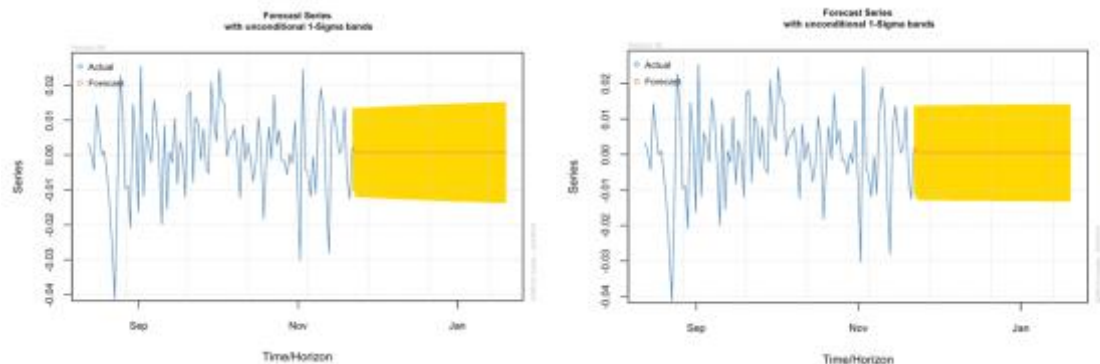


Figure 4: 60 step ahead forecast with upper and lower confidence bounds

Traveler's stock returns have been fitted and now we will need to forecast. I will next look at the fitted values of the model and then make predictions about future event forecasts. Above in figure 3, we predict using a ARMA model (see Appendix 2.6), but watch to fit the tGARCH model since we are using it as a better fit. Using a fitted volatility plot for the tGARCH model, we see a large spike around 2009, and many small spikes elsewhere. This matches the behavior seen in the closing price plot of Traveler's and of the stock market crash, as 2009 shows a period of very high variance. Sixty day forecasts for these returns predict small negative returns and an increasing volatility (figure 4).



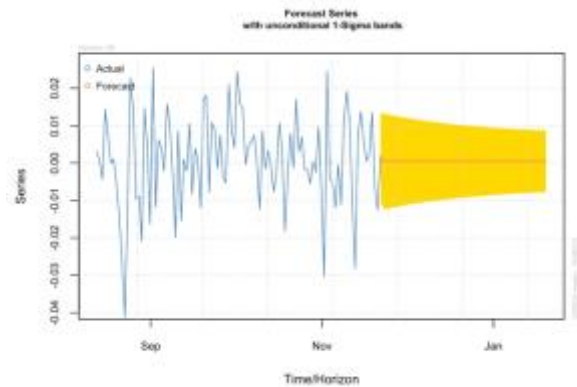


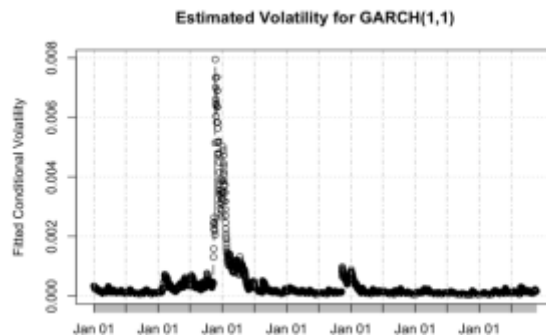
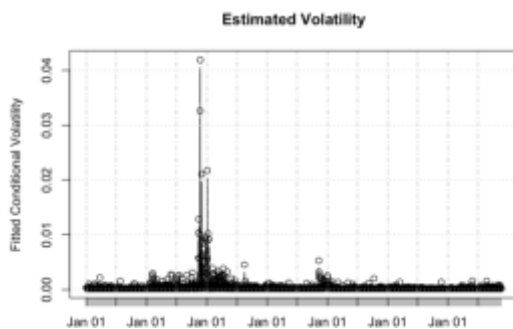
Figure 5: the three GARCH 60 day forecast series, rolling forecast with moving window

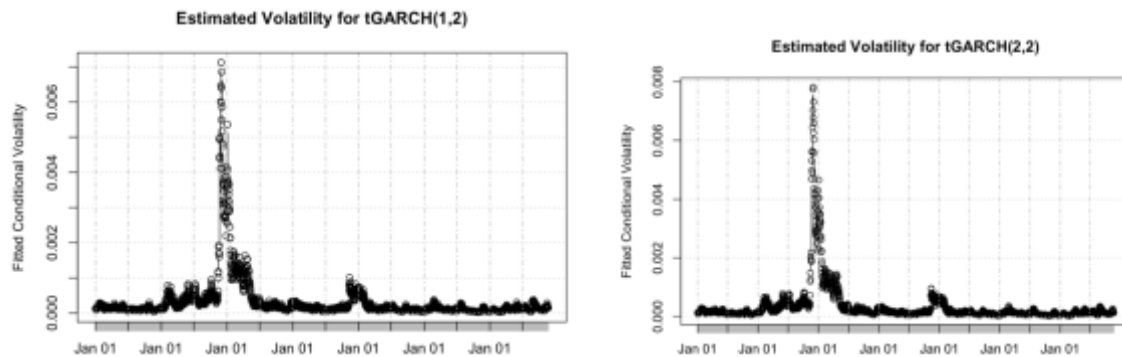
Model Diagnosis

Volatilities

From the figures below, I have included more than just the tGARCH(2,2) model in order to compare the estimated volatilities of the models that also fit the data quite well. Now looking just at the estimated volatilities for tGARCH (2,2) model, we can see that the volatility of Traveler's stock price is quite smooth and stagnant from 2006 to year 2016. There, of course, is a very extreme increase in volatility (can easily be seen in all four models) at the end of 2008 and beginning of 2009.

This extreme increase in volatility is consistent to what we intuitively believe would occur, due to the stock market crash and everything seen and discussed thus far. Therefore, it would be better for investors to avoid such huge fluctuations like that of 2008 and 2009. And to determine this we should buy the stocks or options of Traveler's. Because, even though there is an extreme volatility changing happened, we know it is due to the financial crisis. More testing could be done to see if this will this happen in the future, but reading my models and intuition, we will go with buying as the stocks volatility after 2008 has decreased significantly.





Discussion

Using ARMA and GARCH models I could fit a model to the log return of the Traveler's stock. Both models fit well and pass both the sign-bias tests and the Pearson goodness of fit tests. The models also have quite straight QQ plots, suggesting a normal distribution exists in the model residuals. All together these facts suggest the models fit the data quite well, and that so far I can go along with the conclusions made by the models.

Throughout the report, I have decided to keep the GARCH(1,1) and tGARCH(1,2) for a few visual models including the QQ-plots, AIC and BIC table, VaR limits, forecast, and forecast series although I am testing the tGARCH(2,2) model. This is particularly for comparison, and to visually show how tGARCH(2,2) is in fact the best model.

This report did an analysis of the stock price of Traveler's insurance with 2517 observations Jan. 1st 2006 to Jan. 1st 2016. By fitting linear models, many GARCH (GARCH, iGARCH, eGARCH, tGARCH, and GARCH-m) volatility models with normal and mainly student-t distribution, comparing information criterion (AIC/BIC), VaR and forecast results, and conclude that tGARCH(2,2) is the best fitted model of Traveler's stock price.

I would recommend the investors to buy the stock of JPM, because under my best fitted-model I predicted that the return of this stock will increase in the future and it won't have many extreme changes. Since the mean of the stock prices is positive and the volatilities are quite smooth, I would like to conclude by saying the Traveler's stock behaved well and smooth after the 2008 stock market crash despite mentioning at the beginning about the small falls in stock price, the analysis shows they were not significant.

Additional References

- (1) <http://www.investopedia.com/articles/active-trading/111314/top-10-insurance-companies-metrics.asp>
- (2) <https://coolstatsblog.com/2013/08/07/how-to-use-the-autocorrelation-function-acf/>
- (3) *Analysis Financial Time Series*, Tsay

Appendix 1

- basic model:

$$r_t = \mu_t + a_t, \text{ where } a_t = \sigma_t \varepsilon_t$$

- MA(2)

$$\text{Model Representation: } r_t = \Phi_0 + a_t + \theta_1 a_{t-1} + \theta_2 a_{t-2}$$

- AR(2)

$$\text{Model Representation: } r_t = \Phi_0 + \Phi_1 r_{t-1} + \Phi_2 r_{t-2} + a_t$$

- ARMA(2,2)

$$\text{Model Representation: } r_t = \Phi_0 + \Phi_1 r_{t-1} + \Phi_2 r_{t-2} + a_t + \theta_1 a_{t-1} + \theta_2 a_{t-2}$$

- Call profit/payoff

$$Payoff_{call} = \max[0, S_t - K]$$

$$Profit_{call} = \max[0, S_t - K] - FV(Premium)$$

$$Profit_{call} = \begin{cases} C_0, & K > S_t \\ K - S_t - C_0, & \text{otherwise} \end{cases}$$

Appendix 2

Additional R Code

2. acf values for 35 lags

#estimated acf values for all 35 lags, serial correlations do not appear to be significant for the first 10 lags

```
acf(TRV.rt.n)$acf
##      [,1]
## [1,] -0.201422803
## [2,] -0.051592498
## [3,]  0.027661952
## [4,] -0.020802815
## [5,] -0.060622735
```

```
## [6,] -0.015839695
## [7,] 0.041255347
## [8,] 0.061032443
## [9,] 0.020830276
## [10,] 0.020232676
## [11,] -0.110825605
## [12,] 0.036476413
## [13,] -0.004454993
## [14,] -0.006390649
## [15,] -0.081437966
## [16,] 0.046132298
## [17,] 0.043977293
## [18,] -0.056615497
## [19,] -0.048407461
## [20,] 0.004455702
## [21,] -0.050475906
## [22,] -0.001695683
## [23,] 0.063638542
## [24,] -0.008223648
## [25,] 0.023810587
## [26,] -0.035182060
## [27,] -0.047521979
## [28,] -0.007416420
## [29,] -0.000483221
## [30,] 0.037093552
## [31,] 0.039859298
## [32,] 0.020149712
## [33,] 0.020162285
## [34,] -0.077607970
```

2 Box test - always small p-value

```
Box.test(TRV.rtn, lag=11) # Box-Pierce test: reject null, v small p-val
```

```
##
```

```
## Box-Pierce test
```

```
##
```

```
## data: TRV.rtn
```

```
## X-squared = 168.34, df = 11, p-value < 2.2e-16
```

```
Box.test(TRV.rtn, lag=11, type="Ljung") # Ljung-Box test: reject null, v small p-val
```

```
##
```

```
## Box-Ljung test
```

```
##
```

```
## data: TRV.rtn
```

```
## X-squared = 168.73, df = 11, p-value < 2.2e-16
```

```
Box.test(TRV.rtn, lag=34) # Box-Pierce test: reject null, v small p-val
```

```
##
```

```
## Box-Pierce test
```

```
##
```

```
## data: TRV.rtn
```

```
## X-squared = 264.56, df = 34, p-value < 2.2e-16
```

```
Box.test(TRV.rtn, lag=34, type="Ljung") # Ljung-Box test: reject null, v small p-val
```

```
##
```

```
## Box-Ljung test
```

```
##
```

```
## data: TRV.rtn
```

```
## X-squared = 265.9, df = 34, p-value < 2.2e-16
```

3. eacf numbers

```

print(mseacf[0:7, 0:13], digits=1)
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
## [1,] -0.20 -0.052 0.028 -0.021 -0.06 -0.016 0.041 0.061 2e-02 0.020
## [2,] -0.39 -0.149 -0.009 -0.010 -0.05 -0.026 0.017 0.038 -2e-02 0.005
## [3,] -0.03 -0.188 0.012 -0.026 -0.01 0.008 -0.002 0.050 4e-03 0.018
## [4,] -0.15 -0.009 -0.176 -0.008 -0.05 0.008 0.001 0.003 -6e-03 0.041
## [5,] -0.28 0.050 -0.149 0.033 -0.04 0.013 0.003 0.007 3e-04 0.024
## [6,] -0.47 0.332 0.049 -0.051 0.06 -0.004 -0.005 0.006 -4e-04 0.006
## [7,] 0.34 0.313 0.495 0.006 0.03 -0.017 -0.003 0.031 5e-03 0.007
##      [,11] [,12] [,13]
## [1,] -0.11 0.036 -0.004
## [2,] -0.11 0.024 -0.016
## [3,] -0.11 -0.045 -0.002
## [4,] -0.11 0.007 -0.010
## [5,] -0.09 -0.028 -0.045
## [6,] -0.09 0.038 -0.047
## [7,] -0.09 -0.048 -0.023

```

4. GARCH fit and Ljung-Box residuals example

```

ret urn=as.vect or(dff(log(TRV$TRV. Adjust ed))[-1,])
spec1=ugarchspec(
  variance.model =list(model = "sGARCH", garchOrder = c(1, 0)),
  mean.model =list(ar ma Order = c(0, 2), i nd ude.mean=FALSE),
  fixed.pars=list(ma1=0),
  distribution.model = "sst d") # specify model structure
m1=ugarchfit(ret urn, spec=spec1) #fit ARMA(0, 2)-ARCH(1) model show(m1)
m1 # show model fit
##
## *-----*
## *          GARCH Model Fit          *
## *-----*
##
## Conditional Variance Dynamics
## -----
## GARCH Model : sGARCH(1, 0)
## Mean Model : ARFIMA(0, 0, 2)
## Distribution: sst d
##
## Optimal Parameters
## -----
##      Estimate Std. Error t value Pr(>|t|)
## ma1      0.000000      NA      NA      NA
## ma2     -0.036049     0.019461  -1.8523 0.063979
## omega    0.000157     0.000016  9.5411 0.000000
## alpha1   0.812269     0.111159   7.3072 0.000000
## skew     0.943342     0.020032  47.0928 0.000000
## shape    3.155313     0.218190  14.4613 0.000000
##
## Robust Standard Errors:
##      Estimate Std. Error t value Pr(>|t|)
## ma1      0.000000      NA      NA      NA
## ma2     -0.036049     0.028226  -1.2771 0.20156
## omega    0.000157     0.000017  9.4107 0.00000
## alpha1   0.812269     0.113046   7.1853 0.00000
## skew     0.943342     0.019814  47.6091 0.00000
## shape    3.155313     0.223255  14.1332 0.00000
##
## LogLikelihood : 7220.939
##

```

```

## Information Criteria
## -----
##
## Akai ke      -5.7360
## Bayes       -5.7245
## Shi bat a    -5.7360
## Hannan- Qui nn -5.7318
##
## Wei ghted Ljung-Box Test on Standardized Residuals
## -----
##          statistic  p-value
## Lag[ 1]          15.96 6.459e-05
## Lag[ 2*(p+q)+(p+q)-1][ 5] 21.01 0.000e+00
## Lag[ 4*(p+q)+(p+q)-1][ 9] 28.52 7.248e-13
## d.o.f=2
## H0 : No serial correlation
##
## Wei ghted Ljung-Box Test on Standardized Squared Residuals
## -----
##          statistic  p-value
## Lag[ 1]          1.026 0.311162
## Lag[ 2*(p+q)+(p+q)-1][ 2] 7.676 0.007883
## Lag[ 4*(p+q)+(p+q)-1][ 5] 86.469 0.000000
## d.o.f=1
##
## Wei ghted ARCH LM Tests
## -----
##          Statistic Shape Scale P-Value
## ARCH Lag[ 2]    13.28 0.500 2.000 0.0002683
## ARCH Lag[ 4]    97.32 1.397 1.611 0.0000000
## ARCH Lag[ 6]   154.64 2.222 1.500 0.0000000
##
## Nybl om stability test
## -----
## Joint Statistic 8.9086
## Individual Statistics:
## ma2 0.1156
## omega 6.7631
## alpha1 3.7039
## skew 0.5075
## shape 4.6144
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic 1.28 1.47 1.88
## Individual Statistic 0.35 0.47 0.75
##
## Sgn Bias Test
## -----
##          t-value  prob sig
## Sgn Bias 0.6580 0.5106
## Negative Sgn Bias 0.2051 0.8375
## Positive Sgn Bias 0.5442 0.5863
## Joint Effect 1.3337 0.7211
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## -----
## group statistic p-value(g-1)
## 1 20 10.76 0.9317

```



```
## 2 30 18.62 0.9307
## 3 40 27.31 0.9204
## 4 50 40.40 0.8043
##
##
## Elapsed time : 1.054744
```

5.

```
vcov(m4) # variance-covariance matrix of estimated coefficients
##
## ar1 ar2 ar3 ar4 intercept
## ar1 3.975e-04 8.796e-05 3.847e-05 1.276e-06 -1.953e-09
## ar2 8.796e-05 4.167e-04 9.632e-05 3.845e-05 -1.626e-09
## ar3 3.847e-05 9.632e-05 4.166e-04 8.792e-05 4.053e-10
## ar4 1.276e-06 3.845e-05 8.792e-05 3.970e-04 8.637e-10
## intercept -1.953e-09 -1.626e-09 4.053e-10 8.637e-10 7.288e-08
```

6.

```
pred <- predict(m5, n.ahead=40) # 40 step ahead forecast
plot(as.vector(TRV.rtn)[900:1000], xlim=c(1, 140), type="l", ylab="", xlab="") # plot last 100 observations
lines(101:140, pred$pred, col="blue", lwd=2) # add predicted values
lines(101:140, pred$pred+2*pred$se, col="red", lty=3, lwd=2) # add upper confidence bound for
lines(101:140, pred$pred-2*pred$se, col="red", lty=3, lwd=2) # add lower confidence bound for predicted value
```