Why Genius Square Works – A Mathematical Perspective

This writeup captures the reasoning around The Genius Square game, starting from why the puzzle has 7 blockers on a 6×6 board, through combinatorial counts of blocker placements, symmetries, and a general formula for blocker counts given k-omino sets.

The Pieces and the 7 Blockers

A Genius Square board is 6×6 = 36 squares. The game comes with 9 polyomino pieces:
- 5 tetrominoes (4 cells each) → 20 cells
- 2 triominoes (3 cells each) → 6 cells
- 1 domino (2 cells) → 2 cells
- 1 monomino (1 cell) → 1 cell

Total occupied = 29 cells. The remaining 36 – 29 = 7 squares are reserved for blockers. This is the origin of the 7-blocker rule.

Counting Blocker Configurations

Since there are 36 cells and we must choose 7 for blockers, the number of possible blocker configurations is:

C(36, 7) = 8,347,680.

Symmetries and Rotations

The 6×6 board has 4 rotational symmetries (0°, 90°, 180°, 270°). With 7 blockers, no configuration can be invariant under a nontrivial rotation:
- 90° or 270° rotation would force blockers in 4-cycles → requires multiples of 4 blockers.
- 180° rotation would force blockers in 2-cycles → requires an even number of blockers.
Since 7 is odd, no invariants exist. Hence, each blocker pattern belongs to an orbit of size 4.

Therefore, the number of distinct blocker layouts up to rotation is:
C(36, 7) / 4 = 2,086,920.

Reflections

Including reflections (the full dihedral group D_4 with 8 symmetries) is more subtle. Vertical and horizontal reflections require even blocker counts (so none with 7). Diagonal reflections do allow some invariant patterns because the diagonal has 6 fixed cells. By Burnside's Lemma, the total number of distinct blocker patterns under full D_4 is 1,044,690.

Generalizing the Blocker Formula

We can flip the perspective. Suppose we decide to use all unique polyominoes from size 1 up to size k, one of each. Let p(n) denote the number of distinct n-ominoes. The total area covered is:

$A = \Sigma [n \times p(n)]$ for n=1..k.

We then pick the smallest square N×N such that $N^2 \geq A$. The number of blockers is:

$B = N^2 - A$.

Examples

- k = 4 (polyominoes up to tetrominoes): A = 29, $N^2 = 36$, B = 7 → Genius Square.
- k = 5 (pentominoes, 12 of them): A = 89, $N^2 = 100$, B = 11.
- k = 6 (hexominoes, 35 of them): A = 299, $N^2 = 324$, B = 25.

Thus, Genius Square is a neat instantiation of this formula for k = 4.