The Mondrian Problem

The Mondrian problem is a beautiful open problem in combinatorics and number theory, named after the painter Piet Mondrian, known for his grid-based artwork.

Problem Setup

Given an $N \times N$ square:
1. Tile it completely with finitely many rectangles.
2. Each rectangle must have integer side lengths.
3. Rectangles must be pairwise non-congruent (i.e., no two rectangles can have the same dimensions up to rotation).
4. No overlap, no gaps, no overhangs: the rectangles must exactly fill the $N \times N$ square.

Definition of M(N)

For a valid tiling of an $N \times N$ square:
• Let A_max = the maximum area among the rectangles.
• Let A_min = the minimum area among the rectangles.

Define the defect of the tiling as: defect = A_max – A_min.

Then M(N) is defined as the minimal possible defect over all valid tilings of the $N \times N$ square:

M(N) = min_over_valid_tilings (A_max) – A_min).

Notes

• The rectangles do not need to have equal areas. In fact, the problem is interesting precisely because of the tension between non-congruence and integer side constraints.
• The search for tilings that minimize the defect is computationally challenging as N grows.

Open Problem

The central open question of the Mondrian problem is:

Does there exist any N such that M(N) = 0?

That is, is it possible to tile an $N \times N$ square with pairwise non-congruent, integer-sided rectangles, with no overlap, no gaps, no overhangs, such that all rectangles have exactly the same area?

Despite extensive computational searches, no such N has been found to date. It remains an open problem whether such an N exists.

Auxiliary notes and codebase goals

- Solving the Mondrian problem for largeish N is in itself a hard challenge without several optimisations. In this code base, we aim to use R + Rcpp (and as needed a jump to C) to provide fast, accurate solutions for the problem that given an N, output M(N) and also output all configurations that have this M(N) value