STEPs Program Math Class Syllabus

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Course Description

This intensive summer session provides a comprehensive review of fundamental mathematical concepts leading into and including introductory calculus. Topics include probability and statistics, functions and their graphs, limits and continuity, and differential calculus, with additional review of integral calculus and number sense as time permits. Topics of the last week can be adjusted based on pace of the class.

All exercises, labs, and other supporting materials will be available on the class website `https://steps-math.github.io`.

Course Objectives

By the end of this course, students should be able to:

- Compute elementary probabilities and interpret statistical measures.
- Analyze polynomial, exponential, and logarithmic functions, including transformations.
- Identify discontinuities and evaluate limits from graphs and formulas.
- Understand basic notion of derivatives as slope of tangent line, rate of change, and limit.
- Differentiate elementary functions using standard rules and apply derivatives to optimization problems.

Meeting Times

- **Dates:** June 29, 2025 – August 14, 2025
- **Days:** Saturday through Thursday (no class on Fridays)
- **Time:** 1 hour per day during 8:00 AM – 11:30 AM (for a total of 40 hours)
- **Location:** Rooms TBD

Class Structure

1. **Regular Sessions:** The first part of each class will be focused on theoretical concepts and examples (30–40 minutes), and the second part will be focused on problem solving and applications (20–30 minutes).
2. **Lab Sessions:** The full class will be dedicated to problem solving and applications through guided exercises using graphing (Desmos, GeoGebra, etc) and other supporting tools.

3. **Review Quizzes:** After each major topic, a quiz session will be held. The first 20-25 minutes will be dedicated to reviewing key concepts and solving sample problems as a group, followed by a 25-30 minute open-notes quiz to gauge students' understanding of the material.

Topics

Week	Main Topic	Core Topics	Key Concepts and Skills
1	**Probability & Statistics**	• Sample space & events • Discrete & continuous distributions • Conditional probability • Descriptive statistics	• Counting rules, Venn diagrams • Conditional probability & independence • Mean, median, variance, standard deviation • Posterior probabilities (Bayes' theorem)
2–3	**Functions & Graphs**	• Polynomial, exponential, & logarithmic functions • Domain, range, and function notation • Transformations: shifts, stretches • Implicit vs. explicit forms	• Graphing and sketching • Log and exponential identities • Combining functions
3–4	**Limits & Continuity**	• Introduction to limits • Limit laws and techniques • One-sided & infinite limits • Continuity & Intermediate Value Theorem	• Removable and essential discontinuities • Infinite behaviour of polynomials • Squeeze theorem • ε–δ concept (graphical introduction)

Week	Main Topic	Core Topics	Key Concepts and Skills
5–7	**Differential Calculus**	• Derivative definitions • Rules: power, sum, product, quotient, chain • Applications: optimization, related rates	• Derivative as slope, rate of change, and limit • Higher-order derivatives • Critical points, max/min problems (closed intervals, first/second derivative tests) • Related rates modeling
If time permits	**Integral Calculus**	• Riemann sums and approximation • Definite & indefinite integrals • Fundamental Theorem of Calculus	—
If needed	**Number Sense & Operations**	• Calculating numbers faster • LCM & GCD • Order of operations (PEMDAS) and calculation strategies	—

Detailed Weekly Schedule and Learning Material

Week 1 (June 29–July 4):

- **Day 1 (Sunday, June 29):** ***Orientation (no class)***
- **Day 2 (Monday, June 30):** Course overview. Probability introduction (meaning of probability), events, sample space, and basic counting rules.
- **Day 3 (Tuesday, July 1):** Dependent vs independent events and their formal definition. Independence as conditional probability. Venn diagrams.
- **Day 4 (Wednesday, July 2):** Descriptive statistics: mean, median, variance, and standard deviation. Applications in real life.
- **Day 5 (Thursday, July 3):** **Lab1:** Toy probability experiment / expectation lab.
- **(Friday, July 4):** ***No class***

Week 2 (July 5–11):

- **Day 6 (Saturday, July 5):** Formal definition of a function, simple examples of functions (linear, quadratic), how to draw them and how to figure out their domain and range.
- **Day 7 (Sunday, July 6):** Polynomial functions, their graphs, their domain and range, and their infinity behavior.

- **Day 8 (Monday, July 7):** Root and rational functions, their graphs, their domain and range, points of discontinuity, and their infinity behavior.
- **Day 9 (Tuesday, July 8):** Exponential and log functions, their graphs, their domain and range, points of discontinuity, and their infinity behavior.
- **Day 10 (Wednesday, July 9):** Operations on functions. Transformations: shifting, squeezing, reflecting, and composing functions.
- **Day 11 (Thursday, July 10): Lab2:** Function plotting lab (Desmos/Geogebra).
- **(Friday, July 11): *No class***

Week 3 (July 12–18):

- **Day 12 (Saturday, July 12):** Operations on functions (continued). Inverse functions and implicit vs. explicit functions.
- **Day 13 (Sunday, July 13): *Review Quiz Session 1* (Probability and Functions)**
- **Day 14 (Monday, July 14):** Introduction of discontinuities, discontinuities in graphs. Desmos exercises.
- **Day 15 (Tuesday, July 15):** Formal definition of discontinuities, removable vs essential discontinuities, and various functions behaviors
- **Day 16 (Wednesday, July 16):** Introduction to limits, limits on graphs, and limits of infinity.
- **Day 17 (Thursday, July 17):** Introductory limits problems and exercises.
- **(Friday, July 18): *No class***

Week 4 (July 19–25):

- **Day 18 (Saturday, July 19):** Formal definition of limits, laws of limits (addition/multiplication), and examples on previous functions (could also add step function).
- **Day 19 (Sunday, July 20):** Laws of limits continued. More examples and problems.
- **Day 20 (Monday, July 21):** Existence of limits, one sided limits, and infinity limits.
- **Day 21 (Tuesday, July 22):** Squeeze Theorem. Intermediate Value Theorem. Examples and Applications.
- **Day 22 (Wednesday, July 23): Lab3:** Limits lab. Case study of $\sin(\frac{1}{x})$, and guided exploration of $\epsilon - \delta$ definition of limits.
- **Day 23 (Thursday, July 24): *Review Quiz Session 2.* (Limits and Continuity)**
- **(Friday, July 25): *No class***

Week 5 (July 26–August 1):

- **Day 24 (Saturday, July 26):** Derivatives Introduction 1: Derivatives as rate of change, connections to real-life examples.
- **Day 25 (Sunday, July 27):** Derivatives introduction 2: Derivatives as slopes of functions, connections to real-life examples.

- **Day 26 (Monday, July 28):** Derivatives Introduction 3: Formal algebraic definition of derivatives. Calculate derivatives of simple functions (linear/quadratic).
- **Day 27 (Tuesday, July 29): Lab4:** Derivatives Lab 1. Guided examples of various important derivatives. (e.g. $x^n, \frac{1}{x^n}, e^x, \ln(x)$)
- **Day 28 (Wednesday, July 30):** Derivatives rules 1: addition, multiplication, constant multiple, power rule, and many examples.
- **Day 29 (Thursday, July 31):** Derivatives rules 2: division, show it as another form of multiplication and many examples involving polynomials, rational, and root functions.
- **(Friday, August 1):** ***No class***

Week 6 (August 2–8):

- **Day 30 (Saturday, August 2):** Derivatives rules 3: chain rule, examples and problems.
- **Day 31 (Sunday, August 3):** ***Review Quiz Session 3*** **(Derivatives)**
- **Day 32 (Monday, August 4):** Derivatives in real life. Derivatives role in optimization problems. Fence perimeter/area and factory production problems.
- **Day 33 (Tuesday, August 5): Lab5:** Derivatives Lab 2. Guided example of an elaborate optimization problem.
- **Day 34 (Wednesday, August 6):** Derivatives and graphs of functions. Derivatives use to understand behavior of functions. First and second derivative tests.
- **Day 35 (Thursday, August 7):** Trigonometric functions derivatives. Geometrical proof of $\frac{d}{dx}\sin(x)$. problems and examples.
- **(Friday, August 8):** ***No class***

Week 7 (August 9–14):

- **Day 36 (Saturday, August 9):** L'hopital's rule, Mean Value Theorem, and applications.
- **Day 37 (Sunday, August 10):** Implicit differentiation, differentiability vs. continuity.
- **Day 38 (Monday, August 11): Lab6:** Derivates Lab 3. Related-rates problem.
- **Day 39 (Tuesday, August 12):** Inverse of derivatives. Brief introduction to integrals.
- **Day 40 (Wednesday, August 13):** Review of graph sketching.
- **Day 41 (Thursday, August 14):** Review of calculus concepts.