

# STEPS Math Quiz 3 Review Solutions

Wednesday, August 6, 2025

1. Differentiate each of the following functions (find  $f'(x)$  or  $\frac{dy}{dx}$  for each part)

(a)  $f(x) = 3x^5 - 4x^3 + 2x^2 + 7x - 2$

**Solution:**  $f'(x) = 15x^4 - 12x^2 + 4x + 7$

(b)  $f(x) = \frac{x^3 - 3x^2}{x}$

**Solution:**  $f'(x) = 2x - 3$  (first simplify the function, then differentiate)

(c)  $f(x) = \sqrt{x} + \frac{1}{x}$

**Solution:**  $f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{x^2}$

(d)  $f(x) = \frac{2x-1}{x^2+1}$

**Solution:**  $f'(x) = \frac{-2x^2+2x+2}{(x^2+1)^2}$  (use the quotient rule)

(e)  $f(x) = \frac{\ln(x)}{x^2}$

**Solution:**  $f'(x) = \frac{1-2\ln(x)}{x^3}$  (use the quotient rule)

(f)  $f(x) = e^{-7x}$

**Solution:**  $f'(x) = -7e^{-7x}$  (use the chain rule)

(g)  $f(x) = x^2 \cdot e^x$

**Solution:**  $f'(x) = 2xe^x + x^2e^x = e^x(2x + x^2)$  (use the product rule)

(h)  $f(x) = (2x^2 + 3)^6$

**Solution:**  $f'(x) = 24x(2x^2 + 3)^5$  (use the chain rule)

(i)  $f(x) = \ln(3x^2 + 6x + 10)$

**Solution:**  $f'(x) = \frac{6x+6}{3x^2+6x+10}$  (use the chain rule)

(j)  $f(x) = \sqrt{5x + \frac{1}{x}}$

**Solution:**  $f'(x) = \frac{5 - \frac{1}{x^2}}{2\sqrt{5x + \frac{1}{x}}}$  (use the chain rule)

(k)  $f(x) = e^{3x^2+2x}$

**Solution:**  $f'(x) = (6x + 2)e^{3x^2+2x}$  (use the chain rule)

(l)  $f(x) = (x^2 - 3)(x + 4)^3$

**Solution:**  $f'(x) = (x+4)^2(5x^2+8x-9)$  (use the product rule and chain rule)

(m)  $f(x) = 2^{4x^2-2}$

**Solution:**  $f'(x) = 8x \cdot 2^{4x^2-2} \cdot \ln(2)$  (use the chain rule)

(n)  $f(x) = \frac{1}{\sqrt{4x^2+2}}$

**Solution:**  $f'(x) = \frac{-4x}{(4x^2+2)^{3/2}}$  (use the chain rule and power rule)

(o)  $x^2y + y^2 = 7$

**Solution:**  $\frac{d}{dx}[x^2y + y^2] = \frac{d}{dx}[7] \implies 2xy + x^2\frac{dy}{dx} + 2y\frac{dy}{dx} = 0 \implies \frac{dy}{dx} = \frac{-2xy}{x^2+2y}$

(p)  $y^2 \ln(x) = x^2$

**Solution:**  $\frac{d}{dx}[y^2 \ln(x)] = \frac{d}{dx}[x^2] \implies 2y\frac{dy}{dx} \cdot \ln(x) + y^2 \cdot \frac{1}{x} = 2x \implies \frac{dy}{dx} = \frac{2x - \frac{y^2}{x}}{2y \ln(x)} = \frac{2x^2 - y^2}{2xy \ln(x)}$   
(use chain rule and implicit differentiation)

2. A balloon's radius is growing over time according to  $r(t) = 3t^2 + 1$ .

- (a) Find the rate of change of the radius at time  $t$ .
- (b) At what time is the radius increasing at a rate of 30 cm/s?
- (c) If the volume of the balloon is  $V = \frac{4}{3}\pi r^3$ , express  $\frac{dV}{dt}$  using the chain rule.

**Solution:**

(a)  $r'(t) = 6t$

(b)  $6t = 30 \implies t = 5$

(c)  $\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt} = 4\pi r^2 \cdot r'(t) = 4\pi r^2 \cdot 6t = 24\pi r^2 t$

3. For a certain rectangle the length of one side is always three times the length of the other side.

- (a) If the shorter side is decreasing at a rate of 2 inches/minute at what rate is the longer side decreasing?
- (b) At what rate is the enclosed area decreasing when the shorter side is 6 inches long and is decreasing at a rate of 2 inches/minute?

**Solution:**

(a) Let the longer side be  $x$  and the shorter side be  $y$ . Then  $x = 3y$ . Then,  $\frac{dx}{dt} = 3 \cdot \frac{dy}{dt} = 3 \cdot (-2) = -6$  inches/minute.

(b)  $y = 6 \implies x = 18$  and  $A = x \cdot y = 18 \cdot 6 = 108$  square inches. Then,  $\frac{dA}{dt} = \frac{dA}{dy} \cdot \frac{dy}{dt} = x \cdot \frac{dy}{dt} = 18 \cdot (-2) = -36$  square inches/minute.

4. A tank of water in the shape of a cone is being filled with water at a rate of 12 m<sup>3</sup>/sec. The base radius of the tank is 26 meters and the height of the tank is 8 meters. At what rate is the depth of the water in the tank changing when the radius of the top of the water is 10 meters?

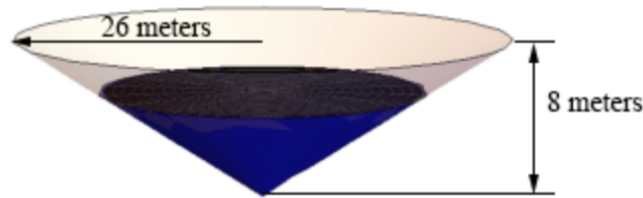


Figure 1: Water tank exercise

**Solution:**

- (a) Let the radius of the water be  $r$  and the height of the water be  $h$ . Then,  $\frac{r}{h} = \frac{26}{8} \implies r = \frac{13}{4}h$ .  
 Also,  $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \left(\frac{13}{4}h\right)^2 h = \frac{169}{48}\pi h^3$ . Then,  $\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt} = 3 \cdot \frac{169}{48}\pi h^2 \cdot \frac{dh}{dt} = 12$  (given by the problem to be  $12 \text{ m}^3/\text{sec}$ ).  
 Solving for  $\frac{dh}{dt}$ , we get  $\frac{dh}{dt} = \frac{12}{\frac{169}{16}\pi h^2} = \frac{192}{169\pi h^2}$ .  
 When  $r = 10$ ,  $h = \frac{40}{13}$  and  $\frac{dh}{dt} = \frac{192}{169\pi \left(\frac{40}{13}\right)^2} = \frac{192}{169\pi \cdot \frac{1600}{169}} = \frac{192}{1600\pi} = \frac{3}{25\pi} \approx 0.038 \text{ m/sec}$ .

5. Find two positive numbers whose product is 810 and for which the sum of one and 10 times the other is a minimum.

**Solution:**

- (a) Let the two numbers be  $x$  and  $y$ . Then,  $xy = 810$  and  $x + 10y$  is minimized.  
 We can express  $y$  in terms of  $x$  as  $y = \frac{810}{x}$ . Then,  $x + 10y = x + 10 \cdot \frac{810}{x} = x + \frac{8100}{x}$ .  
 To find the minimum, we take the derivative of  $x + \frac{8100}{x}$  and set it to 0.  
 $\frac{d}{dx}\left[x + \frac{8100}{x}\right] = 1 - \frac{8100}{x^2} = 0 \implies x^2 = 8100 \implies x = 90$ .  
 When  $x = 90$ ,  $y = \frac{810}{90} = 9$ .  
 The minimum sum is  $90 + 10 \cdot 9 = 180$ .

6. We have  $48 \text{ m}^2$  of material to build a box with a square base and no top. Determine the dimensions of the box that will maximize the enclosed volume.

- Solution:** Let the side length of the square base be  $x$  and the height of the box be  $y$ . Then,  $x^2 + 4xy = 48$  and  $V = x^2 y$  is maximized.  
 We can express  $y$  in terms of  $x$  as  $y = \frac{48 - x^2}{4x}$ . Then,  $V = x^2 \cdot \frac{48 - x^2}{4x} = \frac{48x - x^3}{4}$ .  
 To find the maximum, we take the derivative of  $\frac{48x - x^3}{4}$  and set it to 0.  
 $\frac{d}{dx}\left[\frac{48x - x^3}{4}\right] = 12 - \frac{3x^2}{4} = 0 \implies 48 - 3x^2 = 0 \implies x^2 = 16 \implies x = 4$ .  
 When  $x = 4$ ,  $y = \frac{48 - 16}{16} = 2$ .  
 The maximum volume is  $4^2 \cdot 2 = 32 \text{ m}^3$ .