STEPs Math Quiz 3

Thursday, August 7, 2025 **Version 1**

Name: ______________________ National ID: ______________________

Select your section instructor: ☐ Majid ☐ Asaad

1. (6 points) A company produces sofas. The profit, in riyals, from selling x units is modeled by:

$$P(x) = -x^3 + 6x^2 + 15x + 20$$

Find the maximum profit and determine the value of x at which it occurs.

2. (15 points) Find the derivative of the following functions (find $f'(x)$ or $\frac{dy}{dx}$ for each relation):

(a) (1 point) $f(x) = 2x^5 - 3x^4 + 7x^2 + 1$

(b) (3 points) $f(x) = \frac{x^2+1}{x^2-1}$

(c) (4 points) $f(x) = (x+2)^2(x-5)^3$

(d) (1 point) $f(x) = 3\sqrt{x} + 2e^x - 7\ln(x)$

(e) (3 points) $f(x) = \ln(\sqrt{4x^2+7})$

(f) (3 points) $f(x) = 3e^{x^2+2x+\sqrt{x}}$

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1. (6 points) A company produces sofas. The profit, in riyals, from selling x units is modeled by:

$$P(x) = -x^3 + 6x^2 + 15x + 20$$

Find the maximum profit and determine the value of x at which it occurs.

Solution:

$$P'(x) = -3x^2 + 12x + 15 = 0$$

$$\implies x^2 - 4x - 5 = 0 \implies (x-5)(x+1) = 0$$

$$\implies x = 5 \text{ or } x = -1$$

x can't be negative, so $x = 5$ is the only valid possible value.

$$\text{The maximum profit is } P(5) = -5^3 + 6 \cdot 5^2 + 15 \cdot 5 + 20 = 120$$

Verification: $P(4) = -4^3 + 6 \cdot 4^2 + 15 \cdot 4 + 20 = 112 < 120 = P(5)$

2. (15 points) Find the derivative of the following functions (find $f'(x)$ or $\frac{dy}{dx}$ for each relation):

(a) (1 point) $f(x) = 2x^5 - 3x^4 + 7x^2 + 1$

Solution:

$$f'(x) = 10x^4 - 12x^3 + 14x$$

(b) (3 points) $f(x) = \frac{x^2+1}{x^2-1}$

Solution:

$$f'(x) = \frac{2x(x^2-1) - (x^2+1)(2x)}{(x^2-1)^2}$$

$$= \frac{-4x}{(x^2-1)^2}$$

(c) (4 points) $f(x) = (x+2)^2(x-5)^3$

Solution:

$$f'(x) = 2(x+2)(x-5)^3 + 3(x+2)^2(x-5)^2$$

$$= (x+2)(x-5)^2(5x-4)$$

(d) (1 point) $f(x) = 3\sqrt{x} + 2e^x - 7\ln(x)$

Solution:

$$f'(x) = \frac{3}{2\sqrt{x}} + 2e^x - \frac{7}{x}$$

(e) (3 points) $f(x) = \ln(\sqrt{4x^2+7})$

Solution:

$$f'(x) = \frac{1}{\sqrt{4x^2+7}} \cdot \frac{8x}{2\sqrt{4x^2+7}} = \frac{4x}{4x^2+7}$$

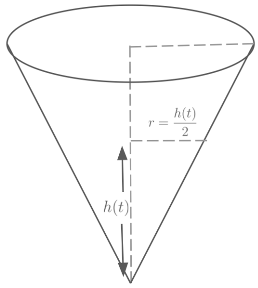
(f) (3 points) $f(x) = 3e^{x^2+2x+\sqrt{x}}$

Solution:

$$f'(x) = 3e^{x^2+2x+\sqrt{x}} \cdot (2x + 2 + \frac{1}{2\sqrt{x}})$$

3. (9 points) A conical container is being filled with water. The container has a height of 8 meters and a base radius of 4 meters. At time t, the water is $h(t)$ meters deep, and the radius is $r(t) = \frac{h(t)}{2}$. The height grows over time according to $h(t) = 2t^{\frac{1}{2}} + 3t^{\frac{1}{3}}$.

(a) (2 points) The volume V of a cone is given by the expression $V = \frac{\pi}{3}r^2h$. Express the volume V of the water in the cone as a function of h only.



Solution:

$$V = \frac{\pi}{3}\left(\frac{h}{2}\right)^2 h = \frac{\pi}{12}h^3$$

(b) (5 points) Find $\frac{dV}{dt}$ in terms of t.

Solution:

$$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt} = \frac{\pi}{12} \cdot 3h^2 \cdot \frac{dh}{dt} = \frac{\pi}{4}h^2 \cdot \frac{dh}{dt}$$

$$= \frac{\pi}{4}\left(2t^{\frac{1}{2}} + 3t^{\frac{1}{3}}\right)^2 \cdot \left(2 \cdot \frac{1}{2}t^{-\frac{1}{2}} + 3 \cdot \frac{1}{3}t^{-\frac{2}{3}}\right)$$

$$= \frac{\pi}{4}\left(2t^{\frac{1}{2}} + 3t^{\frac{1}{3}}\right)^2 \cdot \left(t^{-\frac{1}{2}} + t^{-\frac{2}{3}}\right)$$

(c) (2 points) How fast is the volume of water changing at $t = 1$ minute?

Solution:

$$\frac{dV}{dt} = \frac{\pi}{4}\left(2t^{\frac{1}{2}} + 3t^{\frac{1}{3}}\right)^2 \cdot \left(t^{-\frac{1}{2}} + t^{-\frac{2}{3}}\right)$$

$$= \frac{\pi}{4}\left(2 \cdot 1^{\frac{1}{2}} + 3 \cdot 1^{\frac{1}{3}}\right)^2 \cdot \left(1^{-\frac{1}{2}} + 1^{-\frac{2}{3}}\right)$$

$$= \frac{\pi}{4}(2+3)^2 \cdot (1+1)$$

$$= \frac{\pi}{4} \cdot 5^2 \cdot 2 = \frac{25\pi}{2}$$

Sudoku Bonus (1pt)
(each row, column, and 2x3 box must contain 1-6)

3	2	1	4	5	6
4	5	6	3	1	2
6	4	3	5	2	1
5	1	2	6	4	3
1	3	5	2	6	4
2	6	4	1	3	5

4. (3 points) Determine the values of m, n such that the following function is continuous for all real numbers:

$$f(x) = \begin{cases} -x^2 + nx + m, & x < 2 \\ 2x - m, & 2 \le x \le 4 \\ -x - n, & x > 4 \end{cases}$$

Solution:

$$f(x) \text{ is continuous at } x = 2 \implies -2^2 + 2n + m = 2 \cdot 2 - m \implies 2n + 2m = 8 \implies n + m = 4$$

$$f(x) \text{ is continuous at } x = 4 \implies 2 \cdot 4 - m = -4 - n \implies 8 - m = -4 - n \implies m = 12 + n$$

$$n + m = 4 \implies n + 12 + n = 4 \implies 2n = -8 \implies n = -4$$

$$m = 12 + n = 12 - 4 = 8$$

STEPs Math Quiz 3

Thursday, August 7, 2025 **Version 2**

Name: ____________________ National ID: ____________________

Select your section instructor: ☐ Majid ☐ Asaad

1. (6 points) A company produces bicycles. The profit, in riyals, from selling x units is modeled by:

$$P(x) = -x^3 + \frac{9}{2}x^2 + 12x + 12$$

Find the maximum profit and determine the value of x at which it occurs.

Solution:

$$P'(x) = -3x^2 + 9x + 12 = 0$$

$$\implies x^2 - 3x - 4 = 0 \implies (x-4)(x+1) = 0$$

$$\implies x = 4 \text{ or } x = -1$$

x can't be negative, so $x = 4$ is the only valid possible value.

$$\text{The maximum profit is } P(4) = -4^3 + \frac{9}{2} \cdot 4^2 + 12 \cdot 4 + 12 = 68$$

Verification: $P(3) = -3^3 + \frac{9}{2} \cdot 3^2 + 12 \cdot 3 + 12 = 61.5 < 68 = P(4)$

2. (15 points) Find the derivative of the following functions (find $f'(x)$ or $\frac{dy}{dx}$ for each relation):

(a) (1 point) $f(x) = 3x^5 - 2x^4 + 6x^2 + 5$

Solution:

$$f'(x) = 15x^4 - 8x^3 + 12x$$

(b) (3 points) $f(x) = \frac{2x^2+1}{x^2-2}$

Solution:

$$f'(x) = \frac{(2x^2+1)'(x^2-2) - (2x^2+1)(x^2-2)'}{(x^2-2)^2}$$

$$= \frac{4x(x^2-2) - (2x^2+1)(2x)}{(x^2-2)^2}$$

$$= \frac{4x^3 - 8x - 4x^3 - 2x}{(x^2-2)^2}$$

$$= \frac{-10x}{(x^2-2)^2}$$

(c) (4 points) $f(x) = (x+3)^2(x-4)^3$

Solution:

$$f'(x) = 2(x+3)(x-4)^3 + 3(x+3)^2(x-4)^2$$

$$= (x+3)(x-4)^2(5x+1)$$

(d) (1 point) $f(x) = 4\sqrt{x} + 3e^x - 5\ln(x)$

Solution:

$$f'(x) = \frac{2}{\sqrt{x}} + 3e^x - \frac{5}{x}$$

(e) (3 points) $f(x) = \ln(\sqrt{3x^2+4})$

Solution:

$$f'(x) = \frac{1}{\sqrt{3x^2+4}} \cdot \frac{6x}{2\sqrt{3x^2+4}} = \frac{3x}{3x^2+4}$$

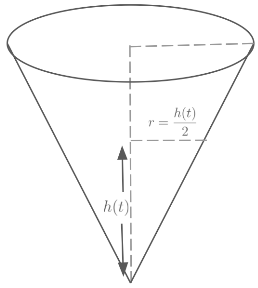
(f) (3 points) $f(x) = 3e^{x^3+3x+e^x}$

Solution:

$$f'(x) = 3e^{x^3+3x+e^x} \cdot (3x^2 + 3 + e^x)$$

3. (9 points) A conical container is being filled with water. The container has a height of 8 meters and a base radius of 4 meters. At time t, the water is $h(t)$ meters deep, and the radius is $r(t) = \frac{h(t)}{2}$. The height grows over time according to $h(t) = 2t^{\frac{1}{2}} + 4t^{\frac{1}{4}}$.

(a) (2 points) The volume V of a cone is given by the expression $V = \frac{\pi}{3}r^2 h$. Express the volume V of the water in the cone as a function of h only.



Solution:

$$V = \frac{\pi}{3}\left(\frac{h}{2}\right)^2 h = \frac{\pi}{12}h^3$$

(b) (5 points) Find $\frac{dV}{dt}$ in terms of t.

Solution:

$$\frac{dV}{dt} = \frac{dV}{dh}\cdot\frac{dh}{dt} = \frac{\pi}{12}\cdot 3h^2\cdot\frac{dh}{dt} = \frac{\pi}{4}h^2\cdot\frac{dh}{dt}$$

$$= \frac{\pi}{4}\left(2t^{\frac{1}{2}} + 4t^{\frac{1}{4}}\right)^2\cdot\left(2\cdot\frac{1}{2}t^{-\frac{1}{2}} + 4\cdot\frac{1}{4}t^{-\frac{3}{4}}\right)$$

$$= \frac{\pi}{4}\left(2t^{\frac{1}{2}} + 4t^{\frac{1}{4}}\right)^2\cdot\left(t^{-\frac{1}{2}} + t^{-\frac{3}{4}}\right)$$

(c) (2 points) How fast is the volume of water changing at $t = 1$ minute?

Solution:

$$\frac{dV}{dt} = \frac{\pi}{4}\left(2t^{\frac{1}{2}} + 4t^{\frac{1}{4}}\right)^2\cdot\left(t^{-\frac{1}{2}} + t^{-\frac{3}{4}}\right)$$

$$= \frac{\pi}{4}\left(2\cdot 1^{\frac{1}{2}} + 4\cdot 1^{\frac{1}{4}}\right)^2\cdot\left(1^{-\frac{1}{2}} + 1^{-\frac{3}{4}}\right)$$

$$= \frac{\pi}{4}(2+4)^2\cdot(1+1)$$

$$= \frac{\pi}{4}\cdot 6^2\cdot 2 = 18\pi$$

Sudoku Bonus (1pt)
(each row, column, and 2x3 box must contain 1-6)

2	4	1	5	3	6
3	5	6	1	4	2
1	3	5	6	2	4
6	2	4	3	1	5
5	1	2	4	6	3
4	6	3	2	5	1

4. (3 points) Determine the values of m, n such that the following function is continuous for all real numbers:

$$f(x) = \begin{cases} -x^2 + nx + m, & x < 2 \\ 2x - m, & 2 \le x \le 4 \\ -x - n, & x > 4 \end{cases}$$

Solution:

$$f(x) \text{ is continuous at } x = 2 \implies -2^2 + 2n + m = 2\cdot 2 - m \implies 2n + 2m = 8 \implies n + m = 4$$

$$f(x) \text{ is continuous at } x = 4 \implies 2\cdot 4 - m = -4 - n \implies 8 - m = -4 - n \implies m = 12 + n$$

$$n + m = 4 \implies n + 12 + n = 4 \implies 2n = -8 \implies n = -4$$

$$m = 12 + n = 12 - 4 = 8$$

STEPs Math Quiz 3

Thursday, August 7, 2025 **Version 3**

Name: ______________________ National ID: ______________________

Select your section instructor: ☐ Majid ☐ Asaad

1. (6 points) A company produces trucks. The profit, in riyals, from selling x units is modeled by:

$$P(x) = -x^3 + 3x^2 + 9x + 17$$

Find the maximum profit and determine the value of x at which it occurs.

Solution:

$$P'(x) = -3x^2 + 6x + 9 = 0$$
$$\implies x^2 - 2x - 3 = 0 \implies (x-3)(x+1) = 0$$
$$\implies x = 3 \text{ or } x = -1$$

x can't be negative, so $x = 3$ is the only valid possible value.

$$\text{The maximum profit is } P(3) = -3^3 + 3 \cdot 3^2 + 9 \cdot 3 + 17 = 44$$

Verification: $P(2) = -2^3 + 3 \cdot 2^2 + 9 \cdot 2 + 17 = 39 < 44 = P(3)$

2. (15 points) Find the derivative of the following functions (find $f'(x)$ or $\frac{dy}{dx}$ for each relation):

(a) (1 point) $f(x) = 2x^6 - 5x^3 + x^2 + 15$

Solution:

$$f'(x) = 12x^5 - 15x^2 + 2x$$

(b) (3 points) $f(x) = \frac{x^2+3}{2x^2-1}$

Solution:

$$f'(x) = \frac{(x^2+3)'(2x^2-1) - (x^2+3)(2x^2-1)'}{(2x^2-1)^2}$$
$$= \frac{2x(2x^2-1) - (x^2+3)(4x)}{(2x^2-1)^2}$$
$$= \frac{4x^3 - 2x - 4x^3 - 12x}{(2x^2-1)^2}$$
$$= \frac{-14x}{(2x^2-1)^2}$$

(c) (4 points) $f(x) = (x+1)^2(x-6)^3$

Solution:

$$f'(x) = 2(x+1)(x-6)^3 + 3(x+1)^2(x-6)^2$$
$$= (x+1)(x-6)^2(5x-9)$$

(d) (1 point) $f(x) = 6\sqrt{x} + \frac{1}{2}e^x - 3\ln(x)$

Solution:

$$f'(x) = \frac{3}{\sqrt{x}} + \frac{1}{2}e^x - \frac{3}{x}$$

(e) (3 points) $f(x) = \ln(\sqrt{9x^2+7})$

Solution:

$$f'(x) = \frac{1}{\sqrt{9x^2+7}} \cdot \frac{18x}{2\sqrt{9x^2+7}} = \frac{9x}{9x^2+7}$$

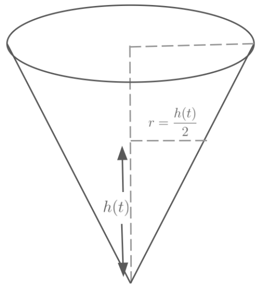
(f) (3 points) $f(x) = 2e^{2x^3+x+\ln x}$

Solution:

$$f'(x) = 2e^{2x^3+x+\ln x} \cdot (6x^2 + 1 + \frac{1}{x})$$

3. (9 points) A conical container is being filled with water. The container has a height of 8 meters and a base radius of 4 meters. At time t, the water is $h(t)$ meters deep, and the radius is $r(t) = \frac{h(t)}{2}$. The height grows over time according to $h(t) = 3t^{\frac{1}{3}} + 4t^{\frac{1}{4}}$.

(a) (2 points) The volume V of a cone is given by the expression $V = \frac{\pi}{3}r^2 h$. Express the volume V of the water in the cone as a function of h only.



Solution:

$$V = \frac{\pi}{3}\left(\frac{h}{2}\right)^2 h = \frac{\pi}{12}h^3$$

(b) (5 points) Find $\frac{dV}{dt}$ in terms of t.

Solution:

$$\frac{dV}{dt} = \frac{dV}{dh}\cdot\frac{dh}{dt} = \frac{\pi}{12}\cdot 3h^2\cdot\frac{dh}{dt} = \frac{\pi}{4}h^2\cdot\frac{dh}{dt}$$
$$= \frac{\pi}{4}\left(3t^{\frac{1}{3}} + 4t^{\frac{1}{4}}\right)^2\cdot\left(3\cdot\frac{1}{3}t^{-\frac{2}{3}} + 4\cdot\frac{1}{4}t^{-\frac{3}{4}}\right)$$
$$= \frac{\pi}{4}\left(3t^{\frac{1}{3}} + 4t^{\frac{1}{4}}\right)^2\cdot\left(t^{-\frac{2}{3}} + t^{-\frac{3}{4}}\right)$$

(c) (2 points) How fast is the volume of water changing at $t = 1$ minute?

Solution:

$$\frac{dV}{dt} = \frac{\pi}{4}\left(3t^{\frac{1}{3}} + 4t^{\frac{1}{4}}\right)^2\cdot\left(t^{-\frac{2}{3}} + t^{-\frac{3}{4}}\right)$$
$$= \frac{\pi}{4}\left(3\cdot 1^{\frac{1}{3}} + 4\cdot 1^{\frac{1}{4}}\right)^2\cdot\left(1^{-\frac{2}{3}} + 1^{-\frac{3}{4}}\right)$$
$$= \frac{\pi}{4}(3+4)^2\cdot(1+1)$$
$$= \frac{\pi}{4}\cdot 7^2\cdot 2 = \frac{49\pi}{2}$$

Sudoku Bonus (1pt)
(each row, column, and 2x3 box must contain 1-6)

5	3	2	4	6	1
1	4	6	5	2	3
4	6	3	2	1	5
2	5	1	3	4	6
6	2	5	1	3	4
3	1	4	6	5	2

4. (3 points) Determine the values of m, n such that the following function is continuous for all real numbers:

$$f(x) = \begin{cases} -x^2 + nx + m, & x < 2 \\ 2x - m, & 2 \le x \le 4 \\ -x - n, & x > 4 \end{cases}$$

Solution:

$$f(x) \text{ is continuous at } x = 2 \implies -2^2 + 2n + m = 2\cdot 2 - m \implies 2n + 2m = 8 \implies n + m = 4$$
$$f(x) \text{ is continuous at } x = 4 \implies 2\cdot 4 - m = -4 - n \implies 8 - m = -4 - n \implies m = 12 + n$$
$$n + m = 4 \implies n + 12 + n = 4 \implies 2n = -8 \implies n = -4$$
$$m = 12 + n = 12 - 4 = 8$$