

# Linear Algebra (0031)

## Project 2

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**Solution.**

$$\text{Let } A = \begin{bmatrix} 3 & 5 \\ 5 & 8 \end{bmatrix}$$

$$\begin{aligned} T_0 &= U_0^T A U_0 \\ &= A \left( \cdot : U_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \end{aligned}$$

Given that  $T_{k-1} = U_k R_k$ ,  $T_k = R_k U_k$

·  $k = 1$

$$\begin{aligned} T_0 &= U_1 R_1 \text{ (QR Factorization)} \\ &= \begin{bmatrix} 0.514 & 0.857 \\ 0.857 & -0.514 \end{bmatrix} \begin{bmatrix} 5.831 & 9.432 \\ 0 & 0.171 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} T_1 &= R_1 U_1 \\ &= \begin{bmatrix} 5.831 & 9.432 \\ 0 & 0.171 \end{bmatrix} \begin{bmatrix} 0.514 & 0.857 \\ 0.857 & -0.514 \end{bmatrix} \\ &= \begin{bmatrix} 11.080 & 0.149 \\ 0.147 & -0.088 \end{bmatrix} \end{aligned}$$

·  $k = 2$

$$\begin{aligned} T_1 &= U_2 R_2 (\text{QR Factorization}) \\ &= \begin{bmatrix} 1 & 0.013 \\ 0.013 & -1 \end{bmatrix} \begin{bmatrix} 11.081 & 0.148 \\ 0 & 0.090 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} T_1 &= R_2 U_2 \\ &= \begin{bmatrix} 11.081 & 0.148 \\ 0 & 0.090 \end{bmatrix} \begin{bmatrix} 1 & 0.013 \\ 0.013 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 11.083 & -0.004 \\ 0.001 & -0.090 \end{bmatrix} \end{aligned}$$

·  $k = 3$

$$\begin{aligned} T_2 &= U_3 R_3 (\text{QR Factorization}) \\ &= \begin{bmatrix} 1 & 0.013 \\ 0.013 & -1 \end{bmatrix} \begin{bmatrix} 11.081 & 0.148 \\ 0 & 0.090 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} T_3 &= R_3 U_3 \\ &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 11.083 & -0.004 \\ 0 & 0.090 \end{bmatrix} \\ &= \begin{bmatrix} 11.083 & -0.004 \\ 0 & -0.090 \end{bmatrix} \end{aligned}$$

· **Calculate  $Q_3$**

$$\begin{aligned} Q_3 &= U_0 U_1 U_2 U_3 \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.514 & 0.857 \\ 0.857 & -0.514 \end{bmatrix} \begin{bmatrix} 1 & 0.013 \\ 0.013 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0.013 \\ 0.013 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 0.525 & -0.850 \\ 0.850 & 0.525 \end{bmatrix} \begin{bmatrix} 1 & 0.013 \\ 0.013 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 0.514 & 0.857 \\ 0.857 & -0.514 \end{bmatrix} \end{aligned}$$

· **Calculate reconstructed A**

$$\begin{aligned}
 A &= Q_3 T_3 Q_3^T \\
 &= \begin{bmatrix} 0.514 & 0.857 \\ 0.857 & -0.514 \end{bmatrix} \begin{bmatrix} 11.083 & -0.004 \\ 0 & -0.090 \end{bmatrix} \begin{bmatrix} 0.514 & 0.857 \\ 0.857 & -0.514 \end{bmatrix} \\
 &= \begin{bmatrix} 5.697 & -0.08 \\ 9.498 & 0.043 \end{bmatrix} \begin{bmatrix} 0.514 & 0.857 \\ 0.857 & -0.514 \end{bmatrix} \\
 &= \begin{bmatrix} 2.860 & 4.923 \\ 4.919 & 8.118 \end{bmatrix}
 \end{aligned}$$

**Console Output After Running The Program**

```

1 Terminated after 3 iterations
2 Tk
3 2 x 2
4      11.090      0.000
5      0.000     -0.090
6
7 Qk=U0*...*Uk
8 2 x 2
9      0.526     -0.851
10     0.851      0.526
11
12 Reconstructed A=Qk * Tk * Qk^T
13 2 x 2
14      3.000      5.000
15      5.000      8.000
16

```

**Result.**

$$\begin{aligned}
 T_3 &= \begin{bmatrix} 11.083 & -0.004 \\ 0 & -0.090 \end{bmatrix} \approx \begin{bmatrix} 11.090 & 0.000 \\ 0.000 & -0.090 \end{bmatrix} \\
 Q_3 &= \begin{bmatrix} 0.514 & 0.857 \\ 0.857 & -0.514 \end{bmatrix} \approx \begin{bmatrix} 0.526 & -0.851 \\ 0.851 & 0.526 \end{bmatrix} \\
 A &= \begin{bmatrix} 2.860 & 4.923 \\ 4.919 & 8.118 \end{bmatrix} \approx \begin{bmatrix} 3.000 & 5.000 \\ 5.000 & 8.000 \end{bmatrix}
 \end{aligned}$$