QR Decomposition

The **QR** decomposition decomposes a matrix as

$$\boldsymbol{A} = \boldsymbol{Q} \cdot \boldsymbol{R}$$

where $\boldsymbol{R}$ is upper triangular, while $\boldsymbol{Q}$ is orthogonal, that is,

$$\boldsymbol{Q}^T \cdot \boldsymbol{Q} = \mathbf{1}$$

where $\boldsymbol{Q}^T$ is the transpose matrix of $\boldsymbol{Q}$. Although the decomposition exists for a general rectangular matrix, we shall restrict our treatment to the case when all the matrices are square, with dimensions *NXN*.

Like the other matrix factorizations (LU, SVD, Cholesky), QR decomposition can be used to solve systems of linear equations. To solve

$$\boldsymbol{A} \cdot x = \boldsymbol{b}$$

first form $\boldsymbol{Q}^T \cdot \boldsymbol{b}$ and then solve

$$\boldsymbol{R} \cdot x = \boldsymbol{Q}^T \cdot \boldsymbol{b}$$

by backsubstitution. Since *QR* decomposition involves about twice as many operations as *LU* decomposition, it is not used for typical systems of linear equations. However, we will meet special cases where QR is the method of choice.

The standard algorithm for the QR decomposition involves successive Householder transformations.