Last Name (PRINT): _____ First Name: _____

Student Number: _____ UTORid: _____

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UNIVERSITY OF TORONTO MISSISSAUGA
DECEMBER 2022 FINAL EXAMINATION

CSC311H5F
Introduction to Machine Learning

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Duration - 3 Hours

Examination Aids: One page double sided letter

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*Please note, once this exam has begun, you **CANNOT** re-write it.*

- Please fill out the information at the top of this cover page.
- **DO NOT** write anything in the QR code area.
- This examination has **8** questions. There are a total of **14 pages**, **DOUBLE-SIDED**.
- **DO NOT** open or turn over the exam paper until the exam has started.
- Answer questions clearly and completely; illegible answers will not be marked.
- Show all your work and provide justifications to your answers unless explicitly asked not to do so.
- Remember that you must earn a grade of at least **40%** on this exam to pass this course.

Question	Out of
Q1	7
Q2	16
Q3	10
Q4	5
Q5	6
Q6	6
Q7	12
Q8	8
Total	70

Use this page for rough work. If you want work on this page to be marked, please indicate this clearly *at the location of the original question.*

1. **[7 marks]** For each of the following quantities, circle whether **increasing** the quantity would **increase the bias** of our model , **increase the variance** of our model, or **neither**. No explanation is required.

   (a) The maximum depth of a decision tree classifier.

   Circle one: Increase the Bias or **Increase the Variance** or **Neither**

   (b) The number of PCA features to use as input to a decision tree classifier.

   Circle one: Increase the Bias or **Increase the Variance** or **Neither**

   (c) The number of individual decision trees in a random forest classifier.

   Circle one: Increase the Bias or **Increase the Variance** or **Neither**

   (d) The weight of the regularizer λ in the regularized cost function $\mathcal{E}_{\text{reg}}(\mathbf{w}) = \mathcal{E}(\mathbf{w}) + \frac{\lambda}{2}\sum_j w_j^2$.

   Circle one: Increase the Bias or **Increase the Variance** or **Neither**

   (e) The value of k in a k-Nearest Neighbour model.

   Circle one: Increase the Bias or **Increase the Variance** or **Neither**

   (f) The number of data points in our training set.

   Circle one: Increase the Bias or **Increase the Variance** or **Neither**

   (g) The number of input features to use in a Naive Bayes classifier.

   Circle one: Increase the Bias or **Increase the Variance** or **Neither**

2. **[16 marks]** For each of the following questions, circle either **True** or **False**, and briefly justify your choice.

(a) **[2 marks]** In a Naive Bayes model, we make the conditional independence assumption that

$$p(\mathbf{x}|c) = p(x_1, x_2, \ldots, x_D|c)$$

Circle one: True or **False**

Explanation:

(b) **[2 marks]** A decision tree divides the input space into overlapping regions, one for each leaf of the tree.
Circle one: True or **False**

Explanation:

(c) **[2 marks]** In the "E" step of the Expectation-Maximization algorithm to fit a Gaussian Mixture Model, we compute the expected value of the mean and covariance matrix of each Gaussian.
Circle one: True or **False**

Explanation:

(d) **[2 marks]** An ensemble of models will typically achieve a higher accuracy compared to a single model.
Circle one: True or **False**

Explanation:

(e) **[2 marks]** PCA features are uncorrelated.
Circle one: True or **False**

Explanation:

(f) **[2 marks]** The entropy of an unfair coin is lower than the entropy of a fair coin.
Circle one: True or **False**

Explanation:

(g) **[2 marks]** The more data we have, the more important the prior is for MAP inference.
Circle one: True or **False**

Explanation:

(h) **[2 marks]** PCA directions of a given data set correspond to the eigenvectors of the empirical covariance matrix.
Circle one: True or **False**

Explanation:

3. [**10 marks**] Answer each question below.

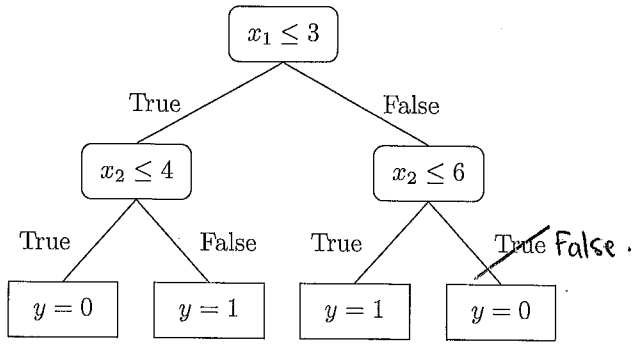
(a) [**4 marks**] Explain why, when training a decision tree classifier, we do *not* choose splits that ~~minimize~~ maximize the classification accuracy. Provide an example to illustrate your point.

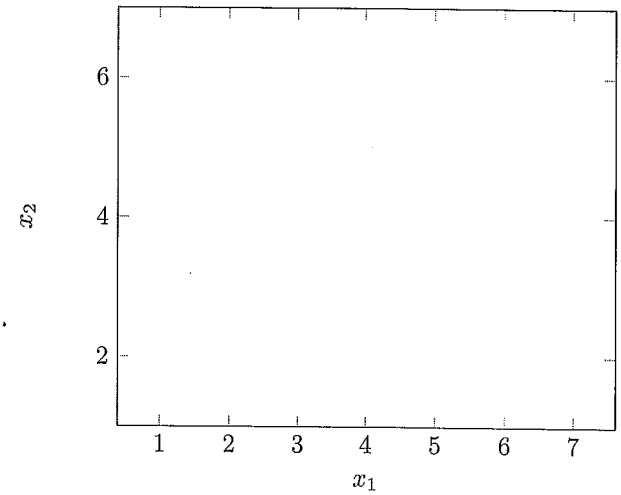
(b) [**4 marks**] A random forest classifier consists of many individual trees. However, each tree is trained differently. What are two differences in how each tree is trained? Why are these differences important?

(c) [**2 marks**] In Stochastic Gradient Descent, what happens if the batch size is too large? Explain briefly.

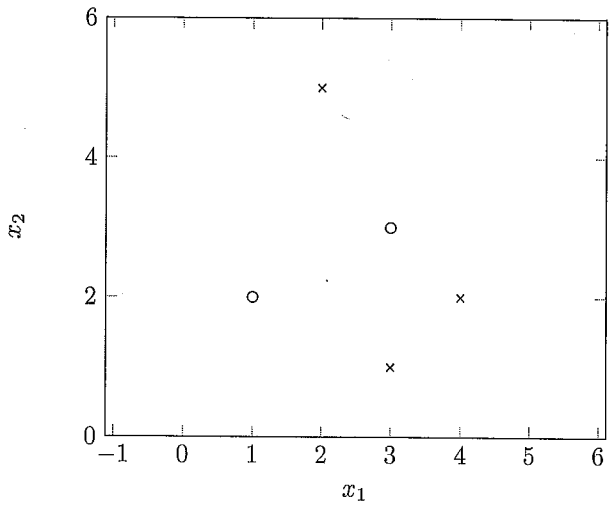
4. [**5 marks**] Draw the decision boundaries for the following models.

   (a) [**3 marks**] Draw the decision boundary for the following decision tree.

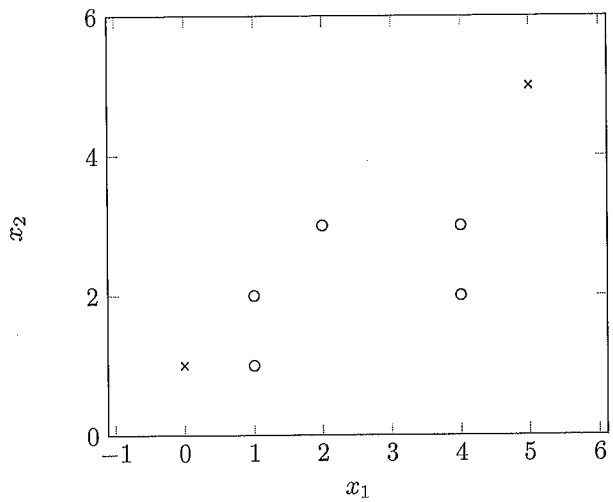




   (b) [**2 marks**] Draw the decision boundary for the 1-nearest neighbour model trained on the following data set.



5. **[6 marks]** In this question, we would like to use k-Means clustering to group the following 5 data points (labeled "o") into two clusters. We initialize the cluster centers to $(0, 1)$ and $(5, 5)$ (labeled "x").



Perform one iteration of k-means clustering to update the location of the cluster centers. Describe each step and show your work.

6. **[6 marks]** Consider neural network models A and B below, both of which can be trained to solve binary classification problems. The difference between the two models is that Model A models the target as a scalar $t \in \{0, 1\}$ while Model B models the target as a one-hot vector $\mathbf{t} = \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$ with $t_i \in \{0, 1\}$ with either $t_1 = 1$ or $t_2 = 1$ but not both. Model A therefore uses the sigmoid activation function to compute $y = \sigma(z)$, while Model B uses the softmax activation function to compute $\mathbf{y} = \text{softmax}(\mathbf{z})$.

Model A	Model B
$\mathbf{h} = \text{ReLU}(\mathbf{W}^{(1)}\mathbf{x} + \mathbf{b}^{(1)}$	$\mathbf{h} = \text{ReLU}(\mathbf{W}^{(1)}\mathbf{x} + \mathbf{b}^{(1)}$
$z = \mathbf{W}^{(2)}\mathbf{h} + b^{(2)}$	$\mathbf{z} = \mathbf{W}^{(2)}\mathbf{h} + \mathbf{b}^{(2)}$
$y = \sigma(z)$	$\mathbf{y} = \text{softmax}(\mathbf{z})$

Both models uses the same input features $\mathbf{x} \in \mathbb{R}^D$, and both models have the same hidden activation size $\mathbf{h} \in \mathbb{R}^M$.

(a) **[2 marks]** Compute the number of parameters in Model A.

(b) **[2 marks]** Compute the number of parameters in Model B.

(c) **[2 marks]** Which model is more likely to overfit? Explain.

7. **[12 marks]** In this question, we will fit the parameters to a Naive Bayes model that classifies whether an email is spam ($c = 1$) or not-spam ($c = 0$).

   (a) **[2 marks]** Write down the data matrix $\mathbf{X}$ containing the bag-of-words features for the following emails, the first two of which are labeled spam ($c = 1$) and the last email is not spam ($c = 0$). Assume that the vocabluary consists of only the words appearing in these emails.

   - buy now
   - buy it now
   - it is now

   (b) **[2 marks]** What parameters do we need to fit in order to fit a Naive Bayes model to this data? Clearly define each parameter.

(c) **[4 marks]** Use MLE to fit the parameters from part (b). Do not derive the maximum likelihood estimate formula for the parameters, but make it clear what formula you are using.

(d) **[4 marks]** Using those parameters, produce a discrete estimate for whether a new email consisting of the words "buy it" is spam or not spam. Show all your work. You do not need to multiply together the individual probabilities.

8. **[8 marks]** Suppose we would like to model the distribution of data $\mathcal{D} = \{x^{(1)}, x^{(2)}, \ldots, x^{(N)}\}$ using a univariate Gaussian distribution:

$$\mathcal{N}(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

(a) **[2 marks]** Show that the log likelihood of the data can be written

$$\ell(\mu, \sigma^2) = -\frac{N}{2}\log(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{N}(x^{(i)} - \mu)^2$$

(b) **[2 marks]** Show that the maximum likelihood estimate of μ is

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x^{(i)}$$

(c) [**4 marks**] Show that the maximum likelihood estimate of σ^2 is

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x^{(i)} - \mu)^2$$

Use this page for rough work. If you want work on this page to be marked, please indicate this clearly *at the location of the original question.*