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ML Zoomcamp 2023 – Introduction to Machine Learning – Part 11

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Overview:

Linear algebra refresher – Part 3/3

This is the third and last part of the refresher and covers special matrix types and its implementation in Python.

Special matrix types

Identity matrix

An identity matrix is a special type of square matrix in linear algebra. It is denoted as `I` and has ones along its main diagonal (from the top-left to the bottom-right) and zeros in all other positions.

The identity matrix is typically represented as follows:

I = [[1, 0, 0, …, 0],
[0, 1, 0, …, 0],
[0, 0, 1, …, 0],

...
[0, 0, 0, ..., 1]]

Here, the matrix `I` is of size `n x n`, where `n` represents the number of rows (or columns) in the matrix.

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Identity matrix of size 4 x 4

The identity matrix is unique because it behaves like the number one in matrix operations. When the identity matrix is multiplied with another matrix `U`, the result is `U` itself. Mathematically, this can be expressed as:

I * U = U

Similarly, when matrix `U` is multiplied by the identity matrix, the result is also `U`:

U * I = U

The identity matrix has various applications in linear algebra and matrix operations. It serves as the neutral element for matrix multiplication, similar to how the number one acts as the neutral element for multiplication of real numbers. The identity matrix also plays a crucial role in defining matrix inverses, solving systems of linear equations, and performing transformations.

In machine learning, the identity matrix is often used as the initial value for weight matrices in neural networks. This ensures that the initial weights do not affect the input data during the first round of computations. The use of identity matrices in neural networks helps prevent over-fitting and facilitates better convergence during the training process.

Understanding the identity matrix and its properties is important for working with matrices, transformations, and solving systems of linear equations. It provides a solid foundation for more advanced topics in linear algebra and machine learning.

Implementation in Python

```
import numpy as np

np.eye(3)
# Output:
# array([[1., 0., 0.],
#       [0., 1., 0.],
#       [0., 0., 1.]])

V = np.array([
    [1, 1, 2],
    [0, 0.5, 1],
    [0, 3, 1],
    [2, 1, 0]
])

V.dot(I)
# Output:
# array([[1. , 1. , 2. ],
#       [0. , 0.5, 1. ],
#       [0. , 3. , 1. ],
#       [2. , 1. , 0. ]])

V.dot(I) == V
# Output:
# array([[ True,  True,  True],
#       [ True,  True,  True],
#       [ True,  True,  True],
#       [ True,  True,  True]])
```

Inverse matrix

The inverse of a matrix is a fundamental concept in linear algebra. It is denoted as U^{-1} and represents the matrix that, when multiplied with the original matrix U, yields the identity matrix I.

To calculate the inverse of a matrix U, we need to ensure that U is a square matrix and that it is invertible (i.e., its determinant is non-zero). Here is the general formula for finding the inverse:

$U^{-1} = (1/|U|) * \mathrm{adj}(U)$

In this formula, $|U|$ represents the determinant of matrix U, and adj(U) denotes the adjugate of matrix U. The adjugate of a matrix is the transpose of its cofactor matrix.

Finding the inverse of a matrix is a crucial operation in many areas of mathematics and engineering. It allows us to solve systems of linear equations, perform geometric transformations, and analyze the properties and behavior of matrices. In machine learning, the inverse of a matrix is often used in optimization algorithms and data transformations.

It’s important to note that not all matrices are invertible. If a matrix is not invertible (i.e., it has a determinant of zero), it is called a singular matrix. Singular matrices have zero as an eigenvalue and cannot be inverted.

Understanding the inverse of a matrix and how to calculate it is essential for working with linear systems, transformations, and solving problems in machine learning. It provides a powerful tool for data manipulation and model optimization.

Implementation in Python

```
import numpy as np

# only squared matrices has an inverse matrix
V = np.array([
    [1, 1, 2],
    [0, 0.5, 1],
    [0, 3, 1]
])

# there is a function in numpy available that returns the inverse of a squared ma
V_inv = np.linalg.inv(V)
V_inv
# Output:
# array([[ 1. , -2. ,  0. ],
#        [ 0. , -0.4,  0.4],
#        [ 0. ,  1.2, -0.2]])

# just to check that V_inv.dot(V) == I
Vs_inv.dot(Vs)
# Output:
# array([[1., 0., 0.],
#        [0., 1., 0.],
#        [0., 0., 1.]])
```

Eigenvalues and Eigenvectors

Eigenvalues and eigenvectors are important concepts in linear algebra, particularly when analyzing the properties and behavior of matrices.

An eigenvector of a matrix `U` is a non-zero vector that, when multiplied by `U`, yields a scalar multiple of itself. In other words, the eigenvector remains in the same direction, although its magnitude may change.

Mathematically, for a square matrix `U` and its corresponding eigenvector `v`, the equation for eigenvalues and eigenvectors is given by:

$U * v = \lambda * v$

Here, λ represents the eigenvalue of matrix `U` associated with the eigenvector `v`.

Eigenvalues provide information about how stretching or compression occurs along eigenvectors when a matrix is applied. Eigenvectors, on the other hand, represent the directions that remain unchanged when a matrix transformation is applied.

Eigenvalues and eigenvectors have various applications in linear algebra and machine learning. They are used to analyze the behavior of matrices, perform matrix decompositions (such as eigendecomposition), and understand the dynamics of linear systems. In machine learning, eigenvalues and eigenvectors are particularly relevant in dimensionality reduction techniques (such as Principal Component Analysis) and spectral clustering algorithms.

Understanding eigenvalues and eigenvectors is crucial for analyzing the properties of matrices, performing transformations, and applying advanced techniques in linear algebra and machine learning. They provide insights into the behavior and structure of data, leading to more effective models and algorithms.

Determinants

The determinant is a valuable quantity that carries important information about the properties and behavior of matrices. It is denoted as `|A|` and is calculated for square matrices.

The determinant of a matrix `A` provides information about the scaling factor and orientation of a transformation represented by the matrix. It determines whether the matrix is invertible or singular, and it affects the behavior of the matrix in equations and calculations.

To calculate the determinant of a square matrix, we use various methods depending on the matrix's size. For small matrices (e.g., 2×2 or 3×3), we can use simple formulas. However, for larger matrices, we often employ more efficient algorithms, such as LU decomposition or Gaussian elimination.

The determinant is useful in various applications, such as solving systems of linear equations, determining matrix invertibility, finding eigenvalues, calculating volume or area scale factors, and analyzing the properties of transformations and matrices.

Understanding the determinant is essential for working with matrices, linear systems, and transformations. It provides insights into the behavior and properties of data, enabling more effective analysis and modeling in various fields, including machine learning.

By expanding your knowledge of linear algebra beyond the basic vector and matrix operations, you will have a solid foundation for understanding and applying more advanced mathematical concepts and algorithms. These concepts serve as essential tools for solving complex problems in machine learning and data analysis.

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