Numerik Serie 10

Aufgabe 1

①

x_i	0	1	2	3
y_i	2	1	2	2
h_i	1	1	1	
c_i	0	c_1	c_2	0
b_i				
d_i				
a_i	2	1	2	

gesucht

② c-berechnen

$$A\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

$$A = \begin{pmatrix} 2(h_0+h_1) & h_1 \\ h_1 & 2\cdot(h_1+h_2) \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}$$

$$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} 3\cdot\frac{y_2-y_1}{h_1} - 3\cdot\frac{y_1-y_0}{h_0} \\ 3\cdot\frac{y_3-y_2}{h_2} - 3\cdot\frac{y_2-y_1}{h_1} \end{pmatrix} = \begin{pmatrix} 3+3 \\ -3 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}\cdot\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$$

$\Rightarrow$ $c_1 = -3 - 4c_2$

$c_2 = 6 - 4c_1 \Rightarrow c_2 = 6 - 4(-3-4c_2)$

$= 6 + 12 + 16c_2$

$c_2 - 16c_2 = 18$

$-15c_2 = 18$

$\boxed{c_2 = -\frac{6}{5}}$

$\boxed{c_1} = -3 + 4\cdot\frac{6}{5} = \boxed{\frac{9}{5}}$

③ b berechnen

$$b_i = \frac{y_{i+1}-y_i}{h_i} - \frac{h_i}{3}(c_{i+1}+2c_i)$$

$\boxed{b_0} = \frac{1-2}{1} - \frac{1}{3}\left(\frac{9}{5}+2\cdot 0\right) = -1 - \frac{3}{5} = \boxed{-\frac{8}{5}}$

$\boxed{b_1} = \frac{2-1}{1} - \frac{1}{3}\left(-\frac{6}{5}+2\cdot\frac{9}{5}\right) = 1 - \frac{12}{15} = \boxed{\frac{1}{5}}$

$\boxed{b_2} = \frac{2-2}{1} - \frac{1}{3}\left(0 - 2\cdot\frac{6}{5}\right) = \boxed{-\frac{4}{5}}$

$b_3 =$

④ $d_i = \frac{1}{3\cdot h_i}(c_{i+1}-c_i)$

$\boxed{d_0} = \frac{1}{3}\left(-\frac{9}{5}-0\right) = \frac{9}{15} = \boxed{\frac{3}{5}}$

$\boxed{d_1} = \frac{1}{3}\left(-\frac{9}{5}-\frac{6}{5}\right) = \boxed{1}$

$\boxed{d_2} = \frac{1}{3}\left(0-\frac{6}{5}\right) = \boxed{\frac{2}{5}}$

⑤ Polynom: $s_i(x) = a_i + b_i(x-x_i) + c_i(x-x_i)^2 + d_i(x-x_i)^3$

$s_0(x) = 2 - \frac{8}{5}(x-0) + \frac{3}{5}(x-0)^3$

$s_1(x) = 1 + \frac{1}{5}(x-1) + \frac{9}{5}(x-1)^2 + (x-1)^3$

$s_2(x) = 2 - \frac{4}{5}(x-2) - \frac{6}{5}(x-2)^2 + \frac{2}{5}(x-2)^3$

Bsp. $p_0(1) = 2 - \frac{8}{5} + \frac{3}{5} = 1$ ✓