Serie 5 Aufgabe 2

a) Euler-verfahren $h = 0{,}7$

$y(0) = 2$, $x_0 = 0$ $\quad y' = \frac{x^2}{y} \Rightarrow \frac{0^2}{2} = 0 = f(x_0, y_0) = f(0,2)$

$x_1 = x_0 + h = 0 + 0{,}7 = 0{,}7$

$y_1 = y_0 + h \cdot f(x_0, y_0) = 2 + 0{,}7 \cdot 0 = 2$

1. abs. fehler: $y(0{,}7) = \sqrt{\frac{2 \cdot 0{,}7^3}{3} + 4} = 2{,}05637 - 2 = 0{,}05637$

$x_2 = x_1 + h = 0{,}7 + 0{,}7 = 1{,}4$

$y_2 = y_1 + h \cdot f(x_1, y_1) = 2 + 0{,}7 \cdot 0{,}245 = 2{,}1715$ $\quad | f(x_1, y_1) = f(0{,}7, 2) = \frac{0{,}7^2}{2} = 0{,}245$

2. abs. fehler: $y(1{,}4) = \sqrt{\frac{2 \cdot 1{,}4^3}{3} + 4} = 2{,}4144 - 2{,}1715 = 0{,}2429$

$x_3 = x_2 + h = 1{,}4 + 0{,}7 = 2{,}1$

$y_3 = y_2 + h \cdot f(x_2, y_2) = 2{,}1715 + 0{,}7 \cdot 0{,}9026 = 2{,}8033$ $\quad | f(x_2, y_2) = f(1{,}4, 2{,}1715) = \frac{1{,}4^2}{2{,}1715} = 0{,}9026$

3. abs. fehler $\Rightarrow y(2{,}1) = \sqrt{\frac{2 \cdot 2{,}1^3}{3} + 4} = 3{,}18967 \Rightarrow 3{,}18967 - 2{,}8033 = 0{,}38637$

b) Mittelpunkt-verfahren : $h = 0{,}7$ $\quad x_0 = 0$, $y_0 = 2$

$x_{i+h/2} = x_i + \frac{h}{2} \Rightarrow i = 0 : x_{h/2} = x_0 + \frac{h}{2} = 0 + \frac{0{,}7}{2} = 0{,}35$

$y_{i+h/2} = y_i + \frac{h}{2} \cdot f(x_i, y_i) \Rightarrow i = 0 : y_{h/2} = 2 + \frac{0{,}7}{2} \cdot 0 = 2$

$x_{i+1} = x_1 = x_0 + h = 0 + 0{,}7 = 0{,}7$

$y_{i+1} = y_1 = y_0 + h \cdot f(x_{h/2}, y_{h/2}) = 2 + 0{,}7 \cdot 0{,}06125 = 2{,}042875$ $\quad | f(0{,}35, 2) = \frac{0{,}35^2}{2} = 0{,}06125$

$y(0{,}7) = 2{,}05637 - 2{,}042875 = 0{,}013495$ - abs. fehler 1

$x_{h/2} = 0{,}7 + 0{,}35 = 1{,}05$, $y_{h/2} = 2{,}042875 + 0{,}35 \cdot f(0{,}7, 2{,}042875)$ $\quad \frac{0{,}7^2}{2{,}042875} = 0{,}23986$

$2{,}042875 + 0{,}35 \cdot 0{,}23986 = 2{,}126826$

$x_2 = 0{,}7 + 0{,}7 = 1{,}4$

$y_2 = 2{,}042875 + 0{,}7 \cdot f(1{,}05, 2{,}126826)$ $\quad | f(1{,}05, 2{,}1\ldots) = \frac{(1{,}05)^2}{2{,}126826} = 0{,}51838$

$= 2{,}042875 + 0{,}7 \cdot 0{,}51838 = 2{,}405739\,8$

$y(1{,}4) \Rightarrow$ abs. fehler 2 : $2{,}4144 - 2{,}40574 = 0{,}00866$

$x_{h/2} = 1{,}4 + 0{,}35 = 1{,}75$, $y_{h/2} = 2{,}40573\,88 + 0{,}35 \cdot f(1{,}4, 2{,}40573\,98)$ $\quad \frac{1{,}4^2}{2{,}40574} = 0{,}81472$

$= 2{,}40574 + 0{,}35 \cdot 0{,}81472 = 2{,}69089$

$x_3 = 1{,}4 + 0{,}7 = 2{,}1$

$y_3 = 2{,}40574 + 0{,}7 \cdot f(1{,}73, 2{,}69089)$

$= 2{,}40574 + 0{,}7 \cdot 1{,}1122338 = 3{,}2$

abs. fehler 3 : $f(2{,}1) = 3{,}1897 - 3{,}2 = 0{,}010$

Aufgabe 2

c) modifiziertes Euler-Verfahren , $h = 0{,}7$

1. klassisches Eulerverfahren und erste Tangentensteigung in k_1 speichern.

$$\Rightarrow k_1 = f(x_0, y_0) = \frac{0^2}{2} = 0$$

2. zweite Tangentensteigung berechnen und in k_2 speichern.

$$k_2 = f(x_1, y_{eul1}) = \frac{0{,}7^2}{2} = 0{,}245 \qquad y_{eul1} = 2 + 0{,}7 \cdot 0 = 2$$

3. durchschnittliche Steigung: $(k_1 + k_2)/2 \Rightarrow (0 + 0{,}245)/2 = 0{,}1225$

$$x_1 = x_0 + h = 0{,}7$$

$$y_1 = y_0 + h \cdot \left(\frac{k_1 + k_2}{2}\right) = 2 + 0{,}7 \cdot 0{,}1225 = 2{,}0858$$

$$y_{eul2} = 2{,}0858 + 0{,}7 \left(\frac{0{,}7^2}{2{,}0858}\right) = 2{,}2502$$

abs. Fehler 1: = 0,0294

$$x_2 = 1{,}4$$

$$k_2 = 0{,}245$$

$$k_3 = f(x_2, y_{eul2}) = \frac{1{,}4^2}{2{,}2502} = 0{,}87104$$

$$y_2 = 2{,}0858 + 0{,}7 \cdot \left(\frac{k_2 + k_3}{2}\right)$$

$$= 2{,}0858 + 0{,}7 \cdot \left(\frac{0{,}245 + 0{,}87104}{2}\right) = 2{,}47284$$

$$y_{eul3} = 2{,}47284 + 0{,}7 \cdot \left(\frac{1{,}4^2}{2{,}47284}\right) = 3{,}0309$$

abs. Fehler 2: = 0,05844

$$x_3 = 2{,}1$$

$$y_3 = 2{,}48741 + 0{,}7 \cdot \left(\frac{0{,}87104 + 1{,}4552}{2}\right) = 3{,}26$$

$$k_4 = f(x_3, y_{eul3}) = \frac{2{,}1^2}{3{,}0309} = 1{,}4552$$

abs. Fehler 3: = 0,0703