Anna Kalin, Thomas Stelter

SO3

Aufgabe 1)

$f = \ln(x^2) = 2\ln(x)$ $\quad f' = \frac{2}{x} = 2 \cdot x^{-1}$ $\quad f'' = -2 \cdot x^{-2} = -\frac{2}{x^2}$

$f''' = 4x^{-3}$ $\quad f^{(4)} = -12x^{-4} = -\frac{12}{x^4}$

$\rightarrow \max_{x \in [1,2]} |f''(x)| = 2$ $\quad \hookrightarrow \max |f^{(4)}(x)| = 12$

Summierte Rechteckregel: $\frac{h^2}{24} \cdot 2 \leq 10^{-5} \rightarrow h \leq \sqrt{12 \cdot 10^{-5}} \rightarrow h \leq 0.011$

$\rightarrow n = 1/h = 91$

Summierte Trapezregel: $\frac{h^2}{12} \cdot 2 \leq 10^{-5} \rightarrow h \leq \sqrt{6 \cdot 10^{-5}} \rightarrow h \leq 0.008$

$\rightarrow n = 1/h = 129$

Summierte Simpsonregel: $\frac{h^4}{2880} \cdot 12 \leq 10^{-5} \rightarrow h \leq \sqrt[4]{240 \cdot 10^{-5}} \rightarrow h \leq 0.221$

$\rightarrow n = 1/h = 5$

Aufgabe 2)

$T_{00} = \pi\left(\frac{\cos(0) + \cos(\pi^2)}{2}\right) = 0.153$

$T_{10} = \frac{\pi}{2}\left(\frac{\cos(0) + \cos(\pi^2)}{2} + \cos\left(\pi/2^2\right)\right) = -1.151$

$T_{20} = \frac{\pi}{4}\left(\frac{\cos(0) + \cos(\pi^2)}{2} + \cos\left(\frac{\pi}{4}^2\right) + \cos\left(\frac{\pi}{2}^2\right) + \cos\left(\left(\frac{3}{4}\pi\right)^2\right)\right) = 0.650$

$T_{30} = \frac{\pi}{8}\left(\frac{\cos(0) + \cos(\pi^2)}{2} + \cos\left(\frac{\pi}{8}^2\right) + \cos\left(\frac{\pi}{4}^2\right) + \cos\left(\left(\frac{3}{8}\pi\right)^2\right) + \cos\left(\frac{\pi}{2}^2\right) + \cos\left(\left(\frac{5}{8}\pi\right)^2\right) + \cos\left(\left(\frac{3}{4}\pi\right)^2\right) + \cos\left(\frac{3}{8}\pi^2\right)\right) = 0.603$

$T_{40} = \frac{\pi}{16}\left(\frac{\cos(0) + \cos(\pi^2)}{2} + \cos\left(\frac{\pi}{16}^2\right) + \cos\left(\frac{\pi}{8}^2\right) + \cos\left(\left(\frac{3}{16}\pi\right)^2\right) + \cos\left(\frac{\pi}{4}\right)^2 + \cos\left(\left(\frac{5}{16}\pi\right)^2\right) + \cos\left(\left(\frac{3}{8}\pi\right)^2\right) + \cos\left(\left(\frac{7}{16}\pi\right)^2\right) + \cos\left(\frac{\pi}{2}^2\right) + \cos\left(\left(\frac{9}{16}\pi\right)^2\right) + \cos\left(\left(\frac{5}{8}\pi\right)^2\right) + \cos\left(\left(\frac{11}{16}\pi\right)^2\right) + \cos\left(\left(\frac{3}{4}\pi\right)^2\right) + \cos\left(\left(\frac{13}{16}\pi\right)^2\right) + \cos\left(\left(\frac{7}{8}\pi\right)^2\right) + \cos\left(\left(\frac{15}{16}\pi\right)^2\right)\right) = 0.575$

Romberg: $T_{01} = \frac{4 \cdot T_{10} - T_{00}}{4-1} = \frac{4 \cdot (-1.151) - 0.153}{3} = -1.586$

$T_{11} = \frac{4 \cdot T_{20} - T_{10}}{4-1} = 1.25$

$T_{21} = \frac{4 \cdot T_{30} - T_{20}}{4-1} = 0.587$

$T_{31} = \frac{4 \cdot T_{40} - T_{30}}{4-1} = 0.565$

$T_{02} = \frac{4^2 \cdot T_{11} - T_{01}}{4^2 - 1} = 1.459$

$T_{12} = 0.543$

$T_{22} = 0.564$

$T_{03} = 0.529$

$T_{13} = 0.564$

$T_{04} = 0.564$