Limits, Differentiation and Derivative

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1.

i) Sketch the graph of the function

$$g(x) = \begin{cases} x - m & \text{if } x < 3 \\ \frac{3}{2} - mx & \text{if } x \geq 3 \end{cases}$$

when $m = 0$ and when $m = -1$.

ii) What is $\lim_{x \to 3^-} g(x)$? What is $\lim_{x \to 3^+} g(x)$? Your answer should depend on m.

iii) Find all real numbers m such that g is continuous at 3.

2.

c) The limit

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

occurs frequently in calculus.

i) Explain why.

ii) Find this limit if $f(t) = \sqrt{t}$ and $x = 4$. (Use the properties of limits.)

iii) Find a formula for this limit for every positive number x. (Use the properties of limits.)

3.

Use implicit differentiation to find $\frac{dy}{dx}$ for

$$x \sin y + y \cos x = \frac{\sqrt{2}\pi}{4}.$$

4.

b) Consider the curve

$$y^2 = x.$$

Find $\frac{dy}{dx}$ using first explicit differentiation and then using implicit differentiation.

5.

c) A particle moves along the x-axis so that its position x at time t is specified by

$$x = t^3 - 4t + 1.$$

Find each of the following and *explain* your answer:

i) the time intervals on which the particle is moving to the right (what does "moving to the right" really mean?);

ii) the time intervals on which the particle is moving to the left;

iii) the time intervals on which the particle is accelerating to the right (what does "accelerating to the right" really mean?);

iv) the time intervals on which the particle is accelerating to the left;

v) the time intervals on which the particle is speeding up (what does "speeding up" really mean?);

vi) the time intervals on which the particle is slowing down;

vii) the acceleration when the velocity is zero;

viii) the average velocity over the time interval $[0, 4]$.

6.

b). A plane flying horizontally at an altitude of 1 km and a speed of 500 km/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 km away from the station.

7.

a). The circumference of a sphere was measured to be 84 cm to the nearest centimetre.

i). Use the differential approximation to estimate the maximum error in the calculated surface area. What is the relative error?

ii). Use the differential approximation to estimate the maximum error in the calculated volume. What is the relative error?

8.

Question 5. Limits

6 pts

Find the limit, if it exists, stating any theorem you use:

(a) $\lim_{x\to 0}\left(x^2\cos\frac{\pi}{x}\right)$ *3 subpts*

(b) $\lim_{x\to\infty}\left(\sqrt{9x^2-2x}-3x\right)$ *3 subpts*