Pre-Calculus: Functions or Else

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1.

1. 求下列函数的自然定义域：

(1) $y=\sqrt{3x+2}$；

(2) $y=\dfrac{1}{1-x^2}$；

(3) $y=\dfrac{1}{x}-\sqrt{1-x^2}$；

(4) $y=\dfrac{1}{\sqrt{4-x^2}}$；

(5) $y=\sin\sqrt{x}$；

(6) $y=\tan(x+1)$；

(7) $y=\arcsin(x-3)$；

(8) $y=\sqrt{3-x}+\arctan\dfrac{1}{x}$；

(9) $y=\ln(x+1)$；

(10) $y=e^{\frac{1}{x}}$.

2.

下列各题中，函数 $f(x)$ 和 $g(x)$ 是否相同？为什么？

(1) $f(x)=\lg x^2, g(x)=2\lg x$；

(2) $f(x)=x, g(x)=\sqrt{x^2}$；

(3) $f(x)=\sqrt[3]{x^4-x^3}, g(x)=x\sqrt[3]{x-1}$；

(4) $f(x)=1, g(x)=\sec^2 x-\tan^2 x$.

3.

Consider the function f described *implicitly* by

$$y=f(x), \qquad x^2-\frac{y^2}{4}=1 \qquad \text{and} \qquad y\geq 0.$$

i) Find y as a function of x (that is, find an explicit formula for $f(x)$).

ii) What is natural domain of f?

iii) Find all points where the graph of f intercepts the axes.

iv) Calculate

$$\lim_{x\to\infty}(2\sqrt{x^2-1}-2x).$$

What does this limit tell you about the graph of f as $x \to \infty$?

v) Sketch the graph of f.

vi) Sketch the graph of the relation

$$x^2 - \frac{y^2}{4} = 1.$$

(Note that this is not a function.)

In general, any equation of the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1,$$

where a and b are positive real numbers, describes a *hyperbola*.

4.

i) Use the intermediate value theorem to show that the equation

$$2\cos x = x$$

has a positive root. In what interval does a root lie?

ii) (Adapted from Stewart Q45, p.55.) Is there a real number that is exactly 1 more than its cube? (Hint: use the same technique as part i).)

5.

Question 4. Domain and composite functions

4 pts

(a) Find the domain (this is the assumed maximal domain) of the function

$$f(x) = \sqrt{\frac{2x+3}{x+1}}$$ *2 subpts*

(b) Let $f(x) = 2x + 1$. Find a function $g(x)$ such that $(f \circ g)(x) = x^3$. *2 subpts*

6.

Question 6. Continuous functions **5 pts**

Consider the function

$$f(x) = \begin{cases} \sqrt{-x}, & x < 0 \\ 2 - x, & 0 \le x \le 2 \\ (2-x)^2, & x > 2 \end{cases}$$

(a) Sketch f. *1 subpts*

(b) At which points of its domain is the function continuous? *2 subpts*

(c) At which points of its domain is the function discontinuous? *2 subpts*

Appeal to the definition of continuity in justifying your answers.

7.

2. 下列各题中，函数 $f(x)$ 和 $g(x)$ 是否相同？为什么？

(1) $f(x) = \lg x^2, g(x) = 2\lg x$;

(2) $f(x) = x, g(x) = \sqrt{x^2}$;

(3) $f(x) = \sqrt[3]{x^4 - x^3}, g(x) = x\sqrt[3]{x-1}$;

(4) $f(x) = 1, g(x) = \sec^2 x - \tan^2 x$.

解 (1) 不同，因为定义域不同.

(2) 不同，因为对应法则不同，$g(x) = \sqrt{x^2} = \begin{cases} x, & x \geqslant 0, \\ -x, & x < 0. \end{cases}$

(3) 相同，因为定义域、对应法则均相同.

(4) 不同，因为定义域不同.