Dynamic Testing and Diagnostics of A/D Converters

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Abstract —**A method is derived to measure the integral and differential nonlinearity of an ADC using a sinewave with unknown amplitude and offset. The uncertainty of the measurement is also estimated. In a second phase, the integral nonlinearity is analyzed, using Walsh Transforms, to identify the nonlinearity at the bit level of the ADC.**

I. Introduction

DURING THE PAST years, the importance of digital signal processing has grown very rapidly. At the same time, the performance of the ADC, which transforms the analog signals to digital ones, has been improved. To have an idea of its real performance, it is necessary to measure its transfer characteristic.

Classically, this was done in a static way. However, a number of imperfections will not be detected by these tests. To improve the knowledge of the behaviour of an ADC, a dynamic test is set up.

The most common dynamic tests used nowadays, are histogram and beat frequency testing [1]. These methods give a good qualitative idea of the ADC performance. The method, presented in this paper, will give a quantitative evaluation of the ADC under test. This gives the possibility to correct the dynamic behaviour of the ADC.

The ADC is excited with a signal with a known probability density function (PDF). A great number of samples are taken and an estimate of the real PDF is made. By comparing the measured PDF with the theoretical one, it is possible to derive the differential and integral nonlinearity. The main idea of this test has already been developed by many authors [2]–[4]. In these articles, a triangular waveform was used because its PDF is very simple. The fundamental drawback of this choice is the distortion of the waveform. However, a sinewave can be generated with a very low distortion, even at high frequencies.

Starting from this idea, an analogous approach was developed as explained in [1] and [5]. The transfer characteristic is calculated and the Integral Nonlinearity (INL) and Differential Nonlinearity (DNL) are derived. To use these results, it is necessary to have an idea about the uncertainty on them. A simple statistical analysis will give the solution.

The concepts of INL and DNL are applicable on all ADC's.

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In a second step, it is possible to make a diagnostics to identify which bits are responsible for the nonlinearity, using the Walsh Transform. These diagnostics are related to the working principle of the ADC. In this paper, Analog-to-Digital converters will be considered which are based upon binary-weighted bit voltages, currents, and the successive approximation principle.

The knowledge of the analysis can be used to correct the transfer characteristic of the ADC.

II. Measuring the INL and the DNL

II.1. Deriving the Transfer Function of the ADC

The PDF of a sinewave $y(t) = A\cdot\sin\omega t + B$ is given by

$$f(y) = \frac{1}{\pi\sqrt{A^2-(y-B)^2}}. \tag{1}$$

During the experiment, a great number of sample points are taken of the sinusoid. The signal can be sampled at random (random sampling method) or at equidistant points (asynchronous sampling method). In the last case, the sample frequency has to be chosen in such a way that the ratio of the sampling frequency and the frequency of the sinusoid is a rational number given by the ratio of two prime numbers. After the completion of the experiment, a vector $\boldsymbol{P}$ can be defined in which the kth element is given by

$$p_k = n_k/N \tag{2}$$

with n_k being the number of samples on the kth level of the ADC and N the total number of samples.

The values p_k can be considered as an estimate of the probability to excite the kth level of the ADC. The probability Q_i to realize a measurement $y < UB_i$, with UB_i the upperbound of the ith level, is

$$\begin{aligned} Q_i = P(y < UB_i) &= \sum_{j=1}^{i} p_j && \text{measured} \\ &= \int_{B-A}^{UB_i} f(y)\,dy && \text{theoretical.} \end{aligned} \tag{3}$$

This integral depends upon the value of the amplitude A and the offset B of the applied sinewave. In practical setups, it is not always possible to know these values. A

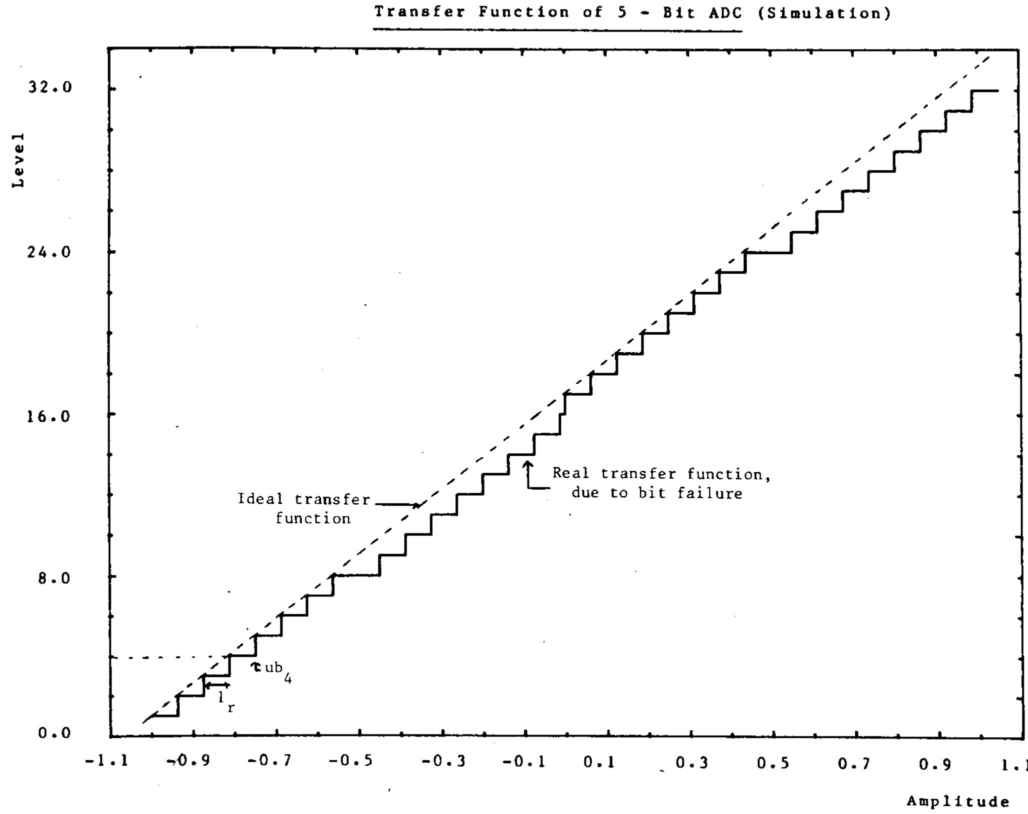


Fig. 1. Transfer function of the simulated ADC.

and B can be eliminated using a linear transformation

$$y = Ay' + B \leftrightarrow y' = (y - B)/A$$
$$ub_i = (UB_i - B)/A \text{ (Fig. 1).} \tag{4}$$

The integral nonlinearity is defined as the difference between the measured transfer characteristic and the best fitted straight line. This line is given by a linear regression analysis

$$\text{INL} = \frac{\text{measured characteristic} - \text{regression characteristic}}{l_r} \text{ LSB.} \tag{7}$$

The shape of the transfer characteristic is not changed by this transformation. Combining (4) and (3) the following relation is derived:

$$ub_i = -\cos \pi Q_i. \tag{5}$$

With this relation, it is possible to make an estimate of the scaled transfer function, using the results of the measurement, stored in $\boldsymbol{P}$. Starting from this transfer function, the DNL and INL are calculated.

II.2. Determination of the DNL and INL

The differential nonlinearity of the ith level is defined as the ratio

$$\text{DNL}_i = \frac{UB_i - UB_{i-1}}{L_R} - 1 \tag{6a}$$

in Least Significant Bit (LSB) with L_R the reference length of a level and with the transformed variables

$$\text{DNL}_i = \frac{ub_i - ub_{i-1}}{l_r} - 1 \qquad \text{LSB}$$
$$\text{with } l_r = \frac{L_R}{A} \qquad \text{(Fig. 1).} \tag{6b}$$

II.3. Study of the Uncertainty on the Measurements

The measurements of the INL and DNL are a result of a stochastic process analysis (the random sampling of a sinusoid). This implies, that even in a noiseless process, there will be an uncertainty in these results. It is very important to have an idea of the uncertainty, to design the experiments and to interpret the results.

In Appendix I, it is proven that the standard deviation σ_{ub_i} and the crosscorrelation $\sigma_{ub_i ub_j}$ are given by

$$\sigma_{ub_i} = \sqrt{\left[\pi^2 Q_i (1 - Q_i) \sin^2 \pi Q_i\right]/N}$$

$$\sigma_{ub_i ub_j} = \sqrt{\left[\pi^2 Q_i (1 - Q_j) \sin \pi Q_i \sin \pi Q_j\right]/N},$$
$$\text{with } Q_j > Q_i. \tag{8}$$

From these results the following expressions are derived:

$$\sigma_{\text{INL}_{1_i}} = \sigma_{ub_i}/l_r \qquad \text{in LSB}$$

$$\sigma_{\text{DNL}_i} = \sqrt{\left[\sigma_{ub_i}^2 + \sigma_{ub_{i-1}}^2 - 2\sigma_{ub_i ub_{i-1}}^2\right]}/l_r. \qquad \text{in LSB.} \tag{9}$$

The maximal uncertainty occurs for $Q_i = 0.5$. The following approximations can then be used:

$$\sigma_{\mathrm{INL_{max}}} = \pi/(2l_r)\cdot 1/\sqrt{N} \qquad \text{in LSB}$$
$$\sigma_{\mathrm{DNL_{max}}} = \sqrt{\pi l_{0.5}}/\left(l_r\sqrt{N}\right) \qquad \text{in LSB}$$
$$\approx \sqrt{\pi/l_r}\cdot 1/\sqrt{N} \qquad \text{if } l_{0.5} \approx l_r \qquad (10)$$

with $l_{0.5}$ the length of the level containing the value $ub = 0$ ($Q = 0.5$) The last result is found by substituting the values $Q_i = 0.5 - \Delta Q_i$ and $Q_j = 0.5 + \Delta Q_j$, with ΔQ_i and $\Delta Q_j \lll 1$, in relations (8) and (9) and using a linear approximation of relation (5) in $Q = 0.5$.

These results are derived for the random sampling method. If the asynchronous sampling method is used, it can be proven that the uncertainties are smaller. In that case the $1/\sqrt{N}$ law becomes a $C^{te}1/N$ law. Indeed, the sampling rate is chosen in such a way that an entire number of periods of the sinusoid is measured for the first time after N sampling points. The number of samples n_k on the kth level is proportional to N. The maximal variation of n_k is 1. Using relation (2) the maximal variation of $p_k = n_k/N$ is given by $|\Delta p_k|$ is $1/N$. From this result, it is easily seen that the uncertainty on the DNL and INL is proportional to $1/N$.

III. Diagnostics on ADC

III.1. Introduction

Using the knowledge of the INL and DNL, it is possible to run diagnostics on the INL to extract information at the bit level. This technique can be used by the ADC hardware designer to qualify his prototype. The INL and the DNL will give him the information about the acceptance of his product. If improvement of the prototype is necessary, the diagnostics of the ADC will provide a straightforward feedback to the ADC designer by indicating which bit(s) is (are) wrong. The diagnostic technique will give also the possibility of compressing the great amount of data from high resolution ADC's, offered by the INL and DNL (2×2^N information items, N-bit ADC), into the essential information on the bit level (N information items).

III.2. Bit Failure, Manifesting in the INL and DNL

To develop a diagnostic method, the effect of bit failure on the INL and DNL must be considered. The effect is illustrated by a simulation of a 5 bit Successive Approximation Register (SAR) ADC (fifth bit: Most Significant Bit, first bit: Least Significant Bit) with a full scale range from -1 to $+1$ V (digitizing step: 1/16 V). Enlarging the weight of the 4th bit (2^3) by 10 percent compared to the ideal situation, a bit failure was introduced. This will result in an INL of 0.8 LSB and a DNL of 0.8 LSB. The asynchronous sampling technique is used to simulate the measurements. 100 000 samples were taken into account to get small uncertainties on the INL and the DNL ($\sigma_{\mathrm{INL}}, \sigma_{\mathrm{DNL}} < 0.08$ LSB). Fig. 1 shows the transfer function of this ADC. The INL (Fig. 2) and the DNL (Fig. 3) do reflect the failure of the ADC. It can be seen that the INL is an integrated form of the DNL. The 95-percent error estimation band gives an idea about the significance of the measurement procedure. It must be noticed that the error band is calculated for a random sampling process.

III.3. Fourier Transform—Diagnostics

It is possible with Discrete Fourier Transform-techniques (DFT) to detect the repetition of the peaks in the DNL and the square waves in the INL. The DFT is applied to the INL and DNL (Figs. 4 and 5).

Because the INL can be considered as an integrated form of the DNL, the $1/j\omega$-function (integration effect, represented in the Fourier Transform) is reflected in the INL. The small constant peaks on Fig. 5 are due to small aberrations of the calculated reference level. This aberration introduces also the small constant peaks on Fig. 3 (DNL). Applying a DFT to this kind of signal, a new signal with constant peaks is created, as illustrated in Fig. 5. In actual experiments, the ADC is considered as a black box with no additional information. Therefore, it is necessary to estimate the length of the reference level l_r (equations (6a) and (6b)). This is accomplished by taking the mean of the length $(ub_i - ub_{i-1})$ of all the levels. The reference level is not calculated as $l_r = 2/_2N$, where 2 is the full scale range of the normalized ADC. This is due to the practical realization of the experiment. It is almost not possible to apply a sinewave to the ADC, covering exactly its full scale range without creating saturation effects in the lowest and highest ADC-level. This saturation would cause deformation of the probability density function, which must be avoided. For this reason, only a subrange of the ADC is analyzed. This subrange can be chosen as near as possible to the full scale range. In this case, an aberration of the reference level, only introduces a gain error on the INL, in opposite to the DNL where small parasitic peaks appear. Because of the behavior of the INL, due to bit failure, the sine–cosine functions are not of the appropriate class to analyze the DNL and INL. It becomes difficult to separate the effect of different bit failures when they start to interfere or if bit-intermodulation does occur. The use of the word "bit-intermodulation" describes the behavior of bits, influenced by the appearance of a certain other bit. This phenomenon will be mentioned later.

III.4. Walsh Transforms

The effect of the bit failure on the INL, imposes as the appropriate class of functions the use of the Walsh transforms. In this way, all the problems, coming along with DFT, are avoided. The Walsh functions are adequate to study step-like signals and are defined as follows:

$$Wal(n,t) = \prod_{r=0}^{p-1} (-1)^{n_{p-1-r}(t_r + t_{r+1})} \qquad (11)$$

where

$n_{p-1}, n_{p-2}, \cdots, n_0$	binary representation of n,
$t_{p-1}, t_{p-2}, \cdots, t_0$	binary representation of t,
n	the order of the Walsh function,
t	the argument (often: time representation).

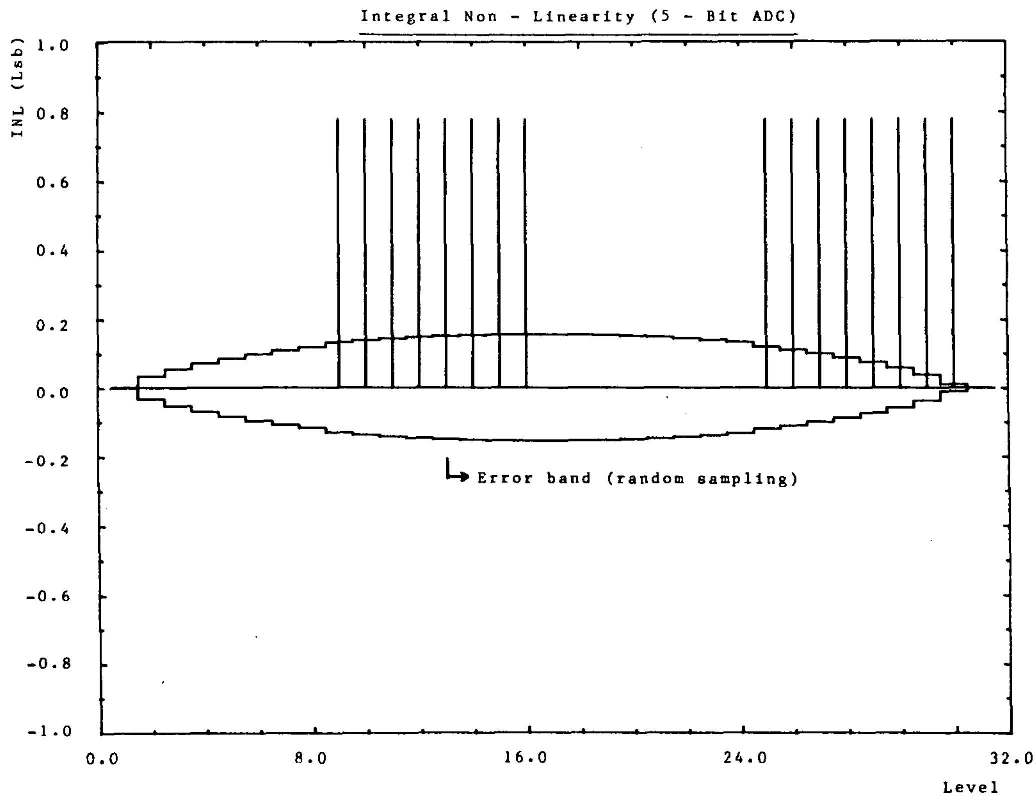


Fig. 2. The integral nonlinearity of the simulated ADC.

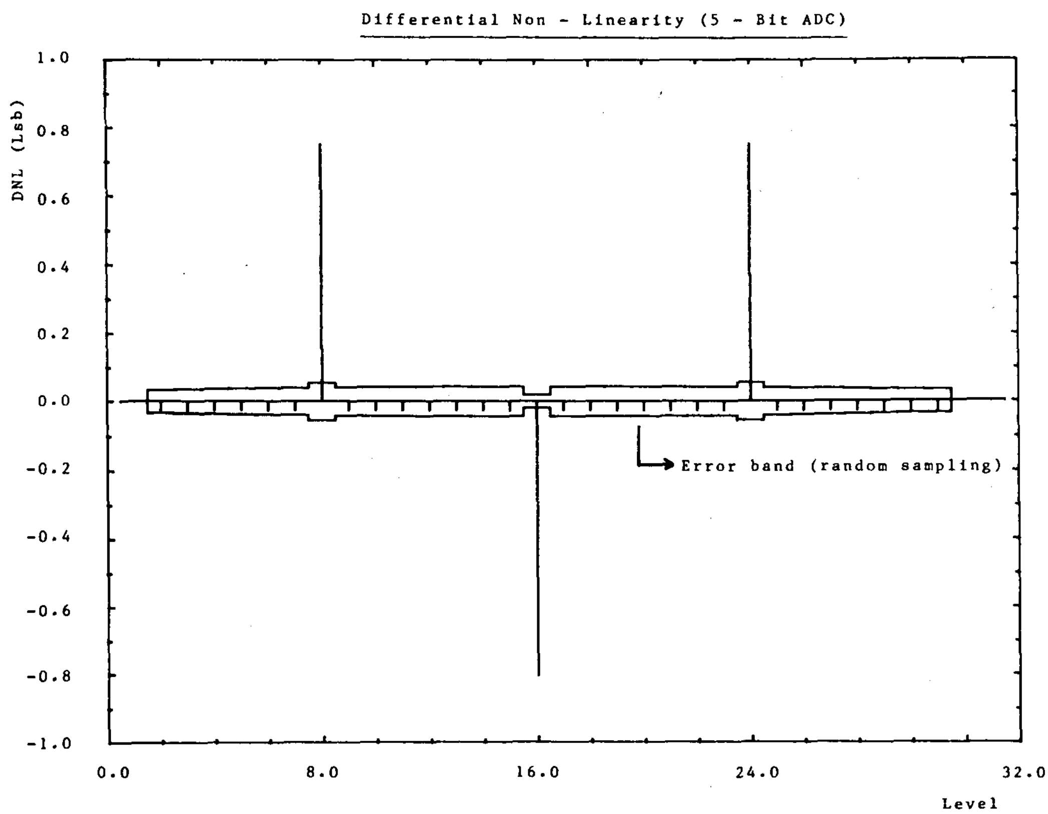


Fig. 3. The differential nonlinearity of the simulated ADC.

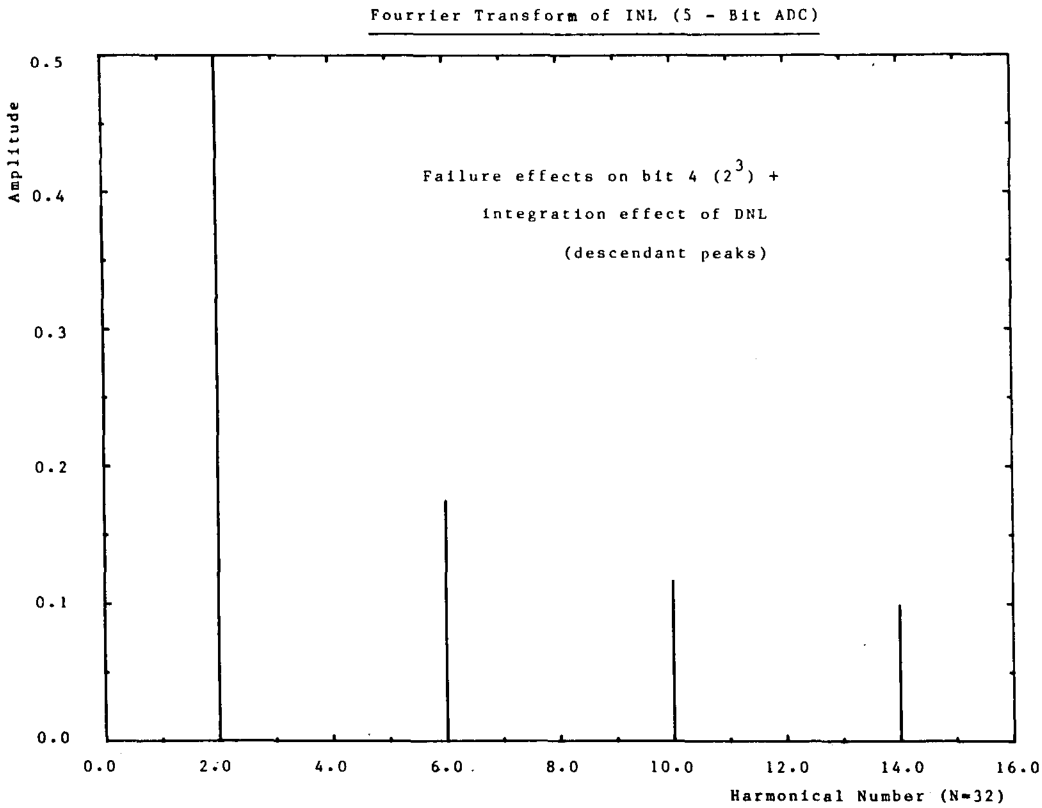


Fig. 4. The Fourier transform of the INL of the simulated ADC.

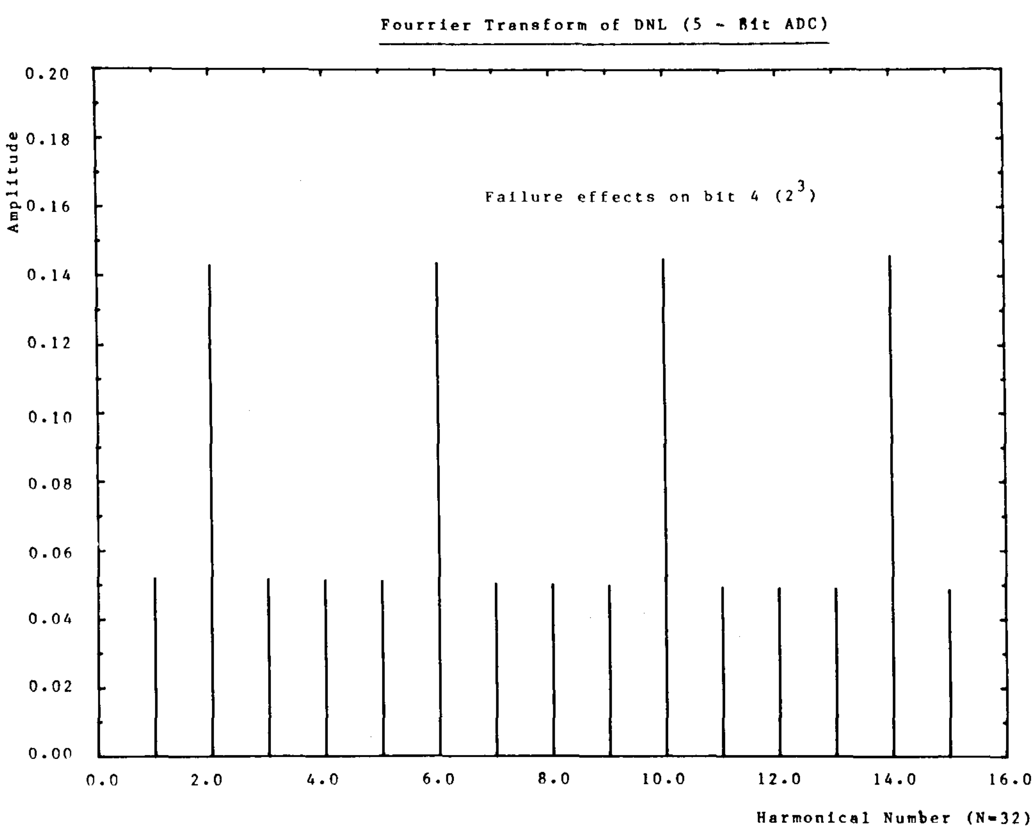


Fig. 5. The Fourier transform of the DNL of the simulated ADC.

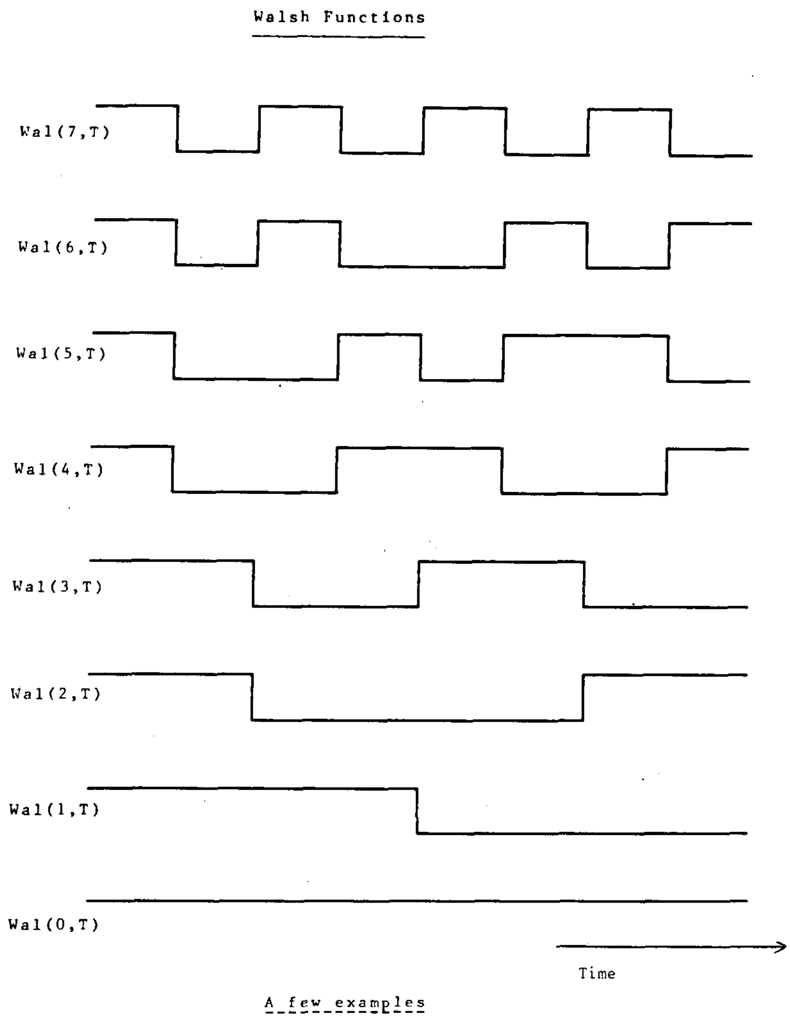


Fig. 6. Example of the Walsh function 0–7.

In Fig. 6, a few Walsh functions (sequence ordered) are illustrated as example.

The Walsh functions are an orthogonal complete set of functions, that results in the Walsh transforms:

$$x_i = \sum_{n=0}^{N-1} X_n \,\mathrm{WAL}(n,i)$$

$$X_n = \frac{1}{N}\sum_{i=0}^{N-1} x_i \mathrm{WAL}(n,i). \tag{12}$$

Implementing the Fast Walsh Transform, the sequence ordered algorithm was chosen. The sequence order, which stands for the number of transitions between -1 and $+1$, can be compared to frequency in Fourier analysis, and to the effects of bit failure on the INL.

III.5. Characterization of Bit Failure

To characterize the bit failure, the general working principle of the ADC has to be considered. This paper studies a Successive Approximation ADC with binary-weighted voltages or currents. The reference voltage, used in such an ADC, can be mathematically represented

$n-$bit ADC:

bit pattern: $p_{n-1}p_{n-2}\cdots p_0$ with $p_i = 0$ or 1

$$V_{\mathrm{ref}} = p_{n-1}W_{n-1}2^{n-1} + \cdots + P_0W_02^0.$$

Ideal linear ADC:

$$W_{n-1} = \cdots = W_0$$
$$= W\text{: quantization step (Weighting Values)}$$
$$\rightarrow V_{\mathrm{ref}\,N} = \left(p_{n-1}2^{n-1} + \cdots + p_0 2^0\right)\cdot W. \tag{13}$$

The reference voltage $V_{\mathrm{ref}\,N}$ is formed by switching in resistors of an $R-2R$-bridge. Inaccuracies on the resistors do cause errors on the weighting value W ($=$ on the quantization step). The bit diagnostics consist of estimating the error δW_i on the weighting value W and the sign of the error (δW_i: positive or negative). Knowing this error contributes to the possible improvement of the performance of the ADC during design and production.

Referring to the simulation, mentioned earlier, enlarging the weight of bit 4 by 10 percent means that $\delta W_3 = 0.1W$.

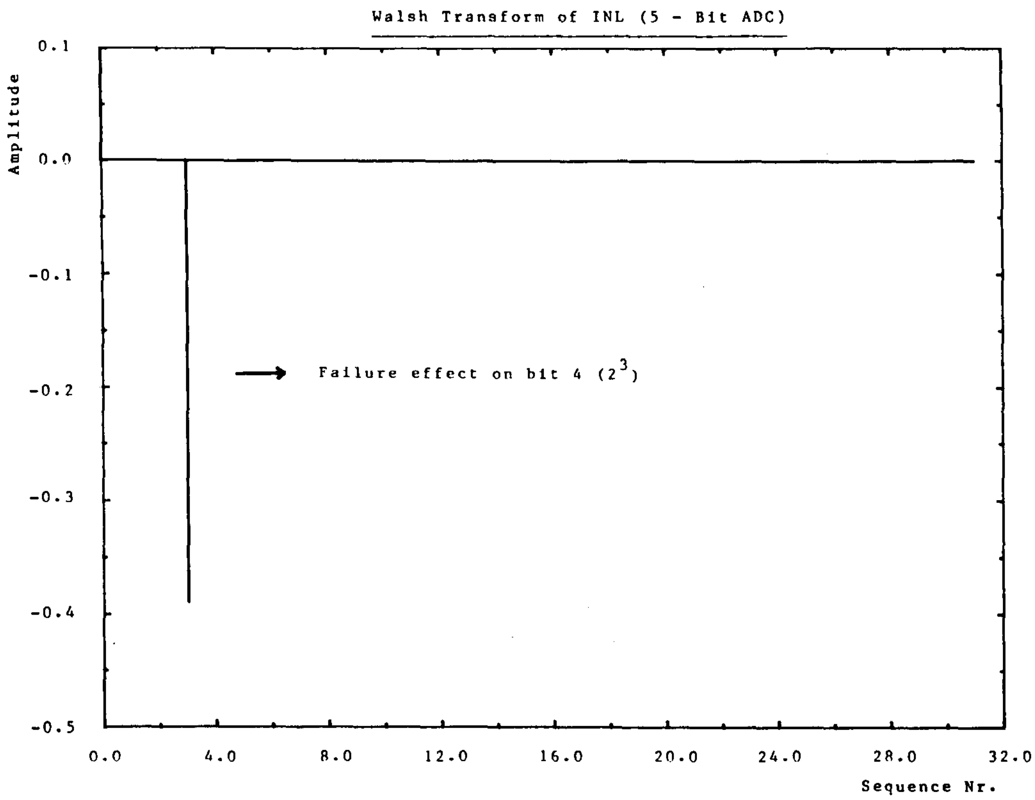


Fig. 7. Walsh transform of the INL of the simulated ADC.

Suppose: error on bit $i+1$ (2^i)

$$\rightarrow W_i = W + \delta W_i$$

$$V_{\text{ref}} = V_{\text{ref}\,N} + p_i \delta W_i 2^i = V_{\text{ref}\,N} + \delta V. \tag{14}$$

The deviation of $V_{\text{ref}\,N}(=\delta W_i 2^i)$ will influence the point of switching in bit $i+1$, compared to the applied voltage. If δW_i is positive, the decision to switch in bit $i+1$ will be delayed, if δW_i is negative, the decision will be accelerated. Due to the deviation, the voltage, corresponding to the beginning of the levels involved, will be shifted over $\delta W_i.2^i = \delta V$. This results in an INL of $\delta V/W$, each time the bit is evaluated:

$$\frac{\delta V}{W} = \frac{\delta W_i^{2^i}}{W} \quad \text{LSB}. \tag{15}$$

The relative error on the weighting value can be represented by

$$\frac{\delta W_i}{W} \cdot 100. \tag{16}$$

Applying the Walsh Transform to an INL with step $\delta V/W$, will result in a value $\delta V/2W$ on sequence number $2^{N-i}-1$ (N-bit ADC). The factor 2 results from the Walsh function which toggles between -1 and 1 with amplitude 1. If a failure occurs on bit $i+1$, during the ADC-test, the Walsh Transform will give a value a_i on the sequence number $2^{N-i}-1$ (N-bit ADC), as stated before.

Refering to (14) and (15), this value can also be calculated as a function of the error δW_i and the bit-weighting factor W

$$a_i = \frac{\delta V}{2W} = \frac{\delta W_i 2^i}{2W}. \tag{17}$$

The relative error, on the weighting value, derived from the Walsh transform, can be calculated as follows:

$$\%_i = \frac{\delta W_i}{W} \cdot 100 = \frac{2 \cdot a_i}{2^i} \cdot 100. \tag{18}$$

Until now, the sign of a_i has not been considered. It gives information about the direction of deviation from the ideal ADC. The sign depends on the form of the Walsh functions. Comparing the Walsh functions of Fig. 6 with the INL of the 5-bit ADC simulation (Fig. 2), where δW_3 was positive, one concludes that the Walsh function $\text{Wal}(3,t)$ with a negative coefficient describes the INL completely. So, if the Walsh coefficient is negative, the error δW on the weighting value W is positive and vice versa.

This technique can be demonstrated by processing the information, obtained by the simulation. In Fig. 7, the Walsh Transform of the INL of the example in Section III.1 is shown. Analyzing this Walsh Transform, results in the following conclusions:

$N = 5$ ($5-$Bit ADC)

Consider Fig. 7 $\rightarrow$ Sequence Number $= 2^{N-i}-1 = 3$

$\rightarrow i = 3$

$\rightarrow$ Bit failure on bit $4(2^3)$

$\rightarrow a_3 = -0.39$

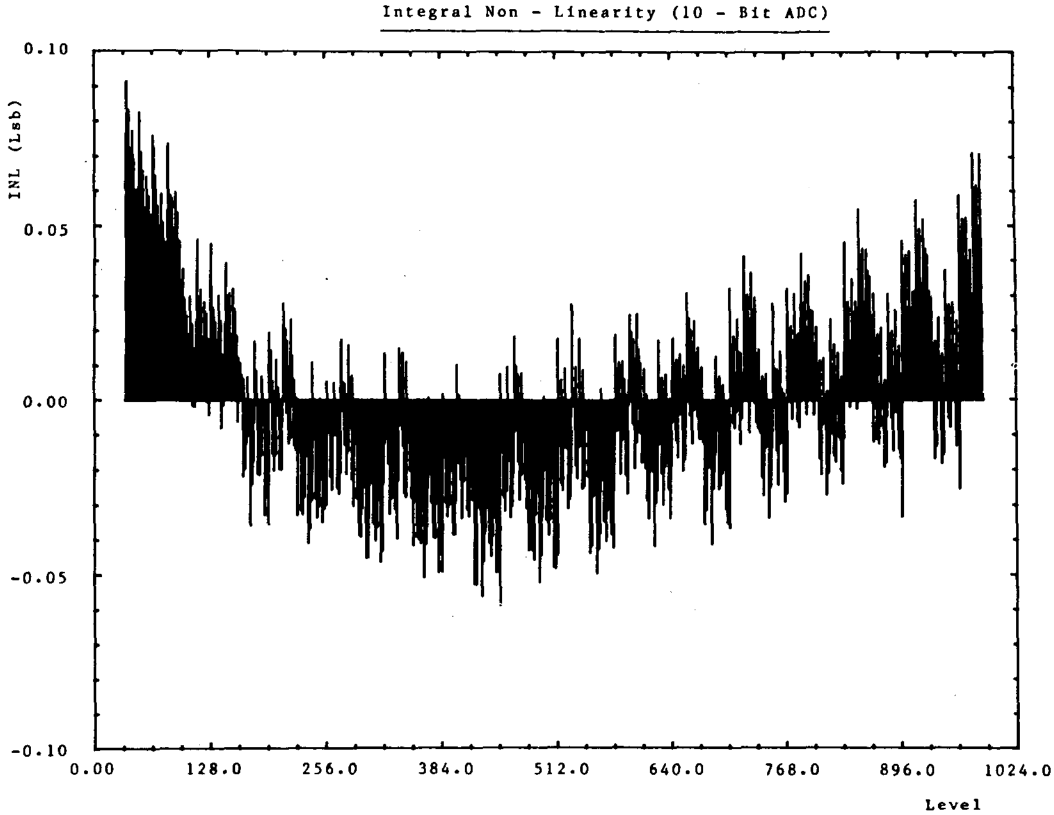


Fig. 8. The integral nonlinearity of the tested ADC (10 bit).

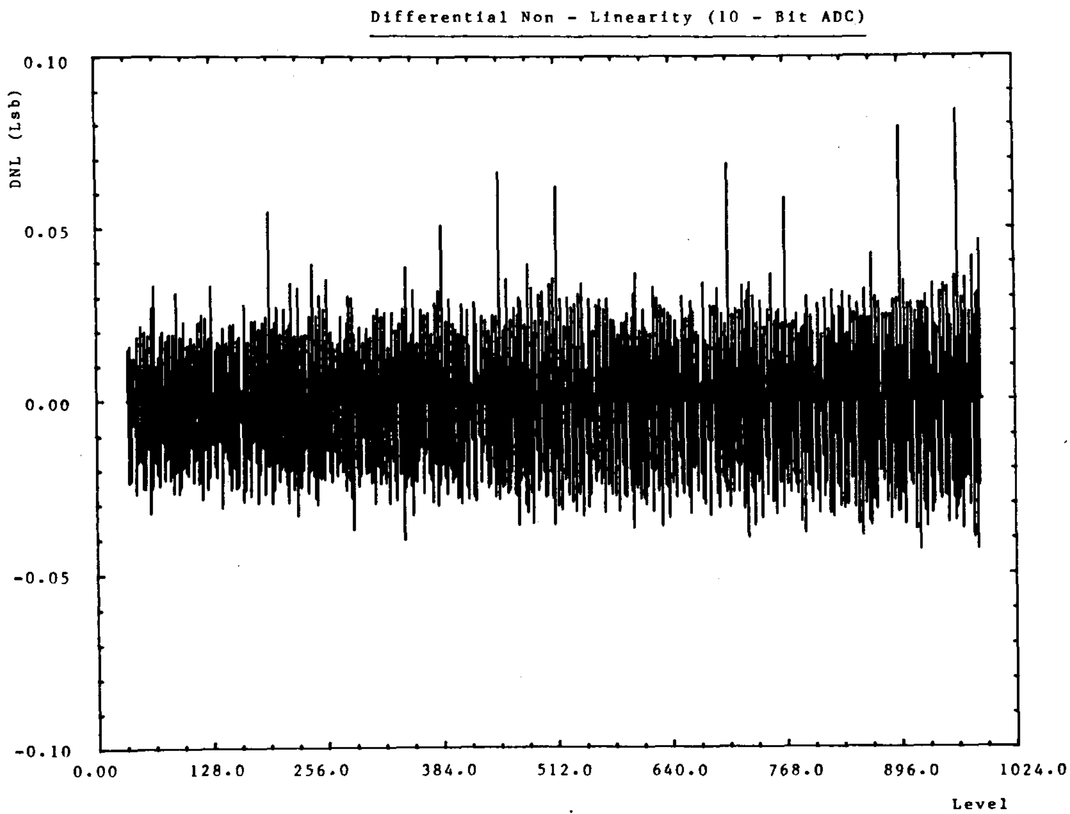
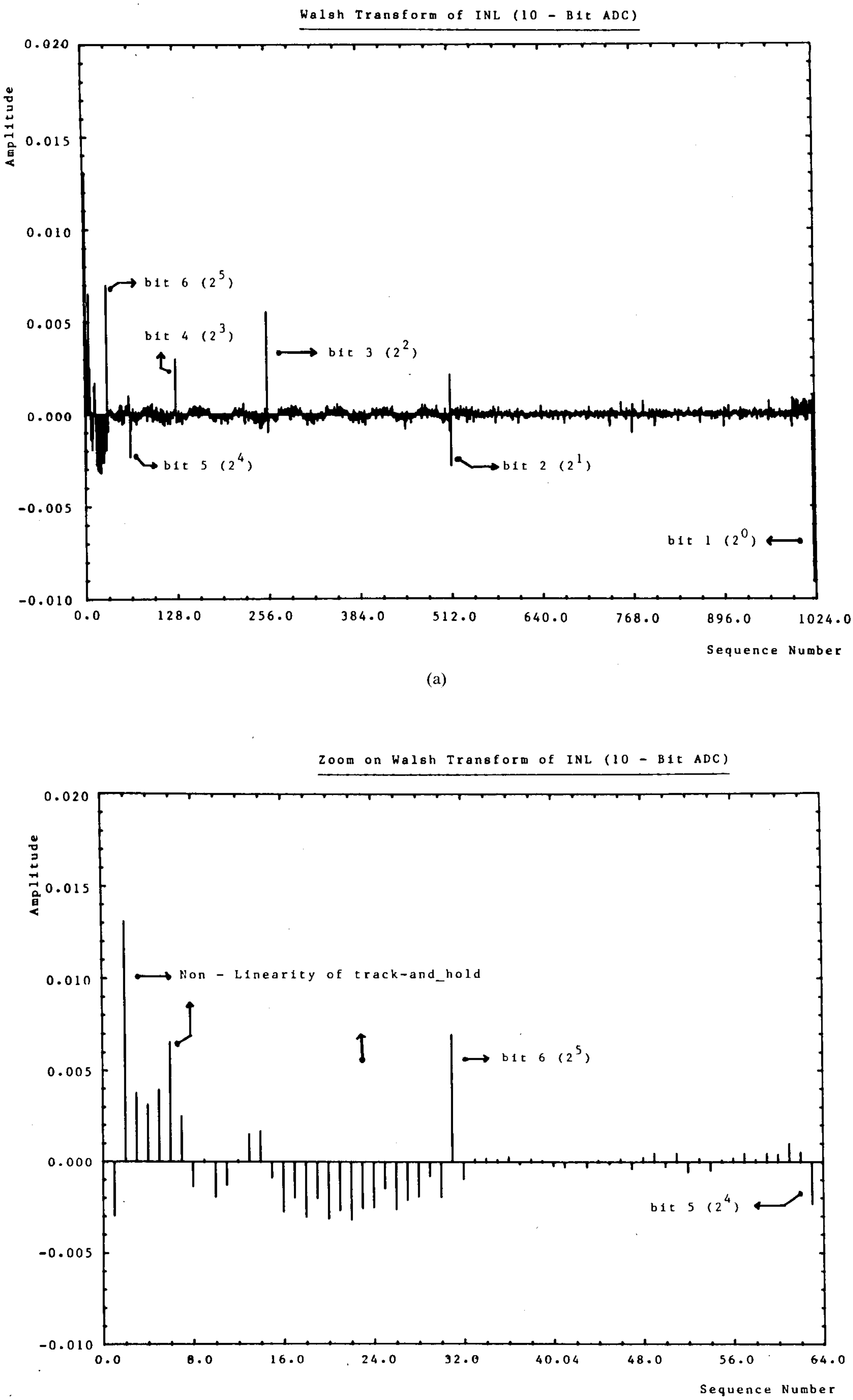


Fig. 9. The differential nonlinearity of the tested ADC (10 bit).



(b)

Fig. 10. (a) The Walsh transform of the INL of the tested ADC. (b) Zoom on Fig. 10(a).

$$\mathrm{INL}_3 = -2a_3 = 0.78$$

compare with given value: 0.8 LSB

$$\%_3 = -2a_3/2^3 \cdot 100 = 9.8$$

(compare with 10-percent error on weighting value)

These values correspond within the uncertainty to the given parameters in the simulation.

III.6. Application on a 10-bit SAR–ADC.

The previous techniques have been applied to a real 10 bit SAR–ADC, preceded by a track-and-hold circuit and used in a data acquisition channel.

Figs. 8 and 9 show the INL and DNL. The INL-characteristic shows a nonstochastical deviation, due to the nonlinearity of the track-and-hold. This was confirmed by a spectral analysis, made after the track-and-hold.

In Fig. 10(a), the resulting Walsh transform, applied to the INL is illustrated. The influence of the bit failure is easily seen. The bits, which are wrong, are indicated on the figure. To find these bits, the results of the Walsh Transform of the INL are scanned by looking at the components with sequence number $2^{N-i}-1$. If only bit failure occurs, as described in Section III.5 (= single-bit failure), the Walsh Transform results in components with only sequence number $2^{N-i}-1$ (= odd number). Again looking at Fig. 10(a), one sees important components with an even number (sequence number 2 and 6). These are caused by the non-linearities of the track-and-hold, which also reflects the parabolic evolution (= even function) of the INL. This effect, however, will not disturb the odd components, describing single-bit failure.

Investigating bits 2, 3, and 5 on Fig. 10(a), another effect is illustrated. The components, corresponding to these bits, are accompanied by components with sequence number $2^{N-i}-2$. This effect can probably be explained by bit-intermodulation. The meaning of bit-intermodulation in SAR–ADC's is that an error occurs on a lower bit because a higher bit is switched on. This phenomenon will be subject to further investigations building a fitting model and extracting quantitive information.

IV. Conclusion

A method is presented to measure and analyze the characteristics of an ADC. It is shown that it is possible to extract the information concerning the nonidealities from the bit-level. Using these techniques, the analysis can be done on 2 levels.

The first level (= INL- and DNL-analysis) is used by the designer of high-performance Digital Signal Processing-applications to evaluate the heart of his system, the ADC.

While the designer will use the second level (= Walsh analysis + information extraction on bit level) to evaluate and correct his ADC-hardware.

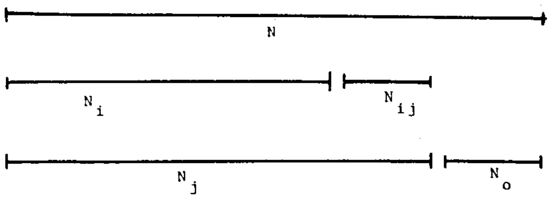


Fig. 11. Study of the cross correlation of N_i and N_j.

Appendix I

To estimate the uncertainty on the DNL and INL it is necessary to know the probability distribution of the cummulative probability Q_i. The variance and cross-correlation of ub_i is derived using linear approximations.

To realize the value Q_i it is necessary to have N_i measurements with a value $< UB_i$ and $(N-N_i)$ measurements with a value $> UB_i$. The distribution of Q_i is a binomial distribution, which can be very well approximated by a normal distribution [7]

$$\begin{aligned} P(Q_i') &= C_N^{N_i} P(\text{meas} < UB_i)^{N_i}(1-P(\text{meas} > UB_i))^{N-N_i} \\ &= C_N^{N_i} Q_i^{N_i}(1-Q_i)^{N-N_i} \end{aligned} \tag{19}$$

with Q_i' the estimated value of Q_i.

The mean and standard deviation is given by [7]

$$\mu_{Q_i'} = Q_i, \qquad \sigma_{Q_i'} = \sqrt{Q_i(1-Q_i)/N}. \tag{20}$$

This means that Q_i' is an unbiased estimate of Q_i. For this reason no difference is made in notation for the measured and true value of Q_i in the other parts of the text.

It's also necessary to know the covariance between Q_i and Q_j. Considering Fig. 11 the following relations can be made:

$$\sigma^2_{N_iN_j} = E\left[dN_i\, dN_j\right] \tag{21}$$

with dN_i, dN_j the deviations of N_i, N_j to the expected values $E[N_i], E[N_j]$

$$N_j = N_i + N_{ij} \to dN_j = dN_i + dN_{ij}.$$

Equation (21) becomes

$$\sigma^2_{N_iN_j} = \sigma^2_{N_i} + \sigma^2_{N_iN_{ij}}. \tag{22}$$

On the other hand, we have the following relation:

$$N_i + N_{ij} + N_0 = N = c^{te} \to dN_0 = -dN_{ij} - dN_i$$

or

$$\sigma^2_{N_0} = \sigma^2_{N_i} + \sigma^2_{N_{ij}} + 2\sigma^2_{N_iN_{ij}}.$$

From this relation, $\sigma^2_{N_iN_{ij}}$ is derived and substituted in (22)

$$\sigma^2_{N_iN_j} = \left[\sigma^2_{N_i} + \sigma^2_{N_0} - \sigma^2_{N_{ij}}\right]/2$$

which can be further reduced, using

$$\sigma^2_{N_i} = NQ_i(1-Q_i)$$

$$\sigma^2_{N_0} = NQ_0(1-Q_0)$$

$$\sigma^2_{N_{ij}} = NQ_{ij}(1-Q_{ij})$$

with

$$Q_0 = P(\text{meas} > ub_j)$$
$$Q_{ij} = P(UB_i < \text{meas} < UB_j) = 1 - Q_i - Q_j.$$

Finally, (23) results in

$$\sigma^2_{N_i N_j} = NQ_i Q_0 = NQ_i(1 - Q_j)$$

and

$$\sigma^2_{Q_i Q_j} = Q_i(1 - Q_j)/N. \tag{24}$$

To derive the variance σ^2_{ub} the relation (5) $ub_i = -\cos \pi Q_i$ is used. Differentiation gives

$$dub_i = \pi \sin \pi Q_i \, dQ_i$$

and

$$\sigma^2_{ub_i} = E[dub_i \, dub_i] = \pi^2 \sin^2 \pi Q_i \sigma^2_{Q_i}$$
$$= \pi^2 \sin^2 \pi Q_i Q_i (1 - Q_i)/N.$$

In the same way it is found that

$$\sigma^2_{ub_i ub_j} = E\left[dub_i \, dub_j\right] = \pi^2 \sin \pi Q_i \sin \pi Q_j Q_i (1 - Q_j)/N.$$

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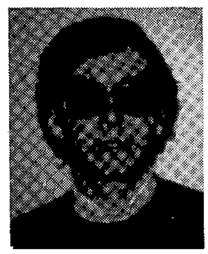

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