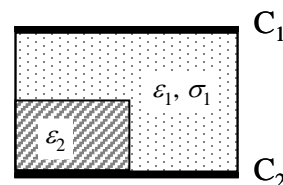


**Pan Pearl River Delta Physics Olympiad 2009**  
**2009 年泛珠三角及中華名校物理奧林匹克邀請賽**  
**Part-1 (Total 5 Problems) 卷-1 (共 5 題)**  
 (9:00 am – 12:00 pm, 02-04-2009)

**Math hints 數學提示:**  $\int x^n dx = \frac{1}{n+1} x^{n+1} + C$ ;  $\int \frac{dx}{x} = \ln x + C$ .

**Q.1 (8 points) 題 1 (8 分)**

Two large parallel square conductor plates  $C_1$  and  $C_2$  of side length  $a$  are separated by two types of media. The cross section is shown in the figure. The distance between the plates is  $d$ . Medium-2 is a rectangular insulator slab with dielectric constant  $\epsilon_2$ , thickness  $d/2$ , and side lengths  $a$  and  $a/2$ , respectively. The rest of the space between the plates is filled with medium-1 with conductivity  $\sigma_1$  and dielectric constant  $\epsilon_1$ . Find the resistance and capacitance between the two plates.



兩塊邊長為  $a$  的正方形平行導電板  $C_1$  和  $C_2$  之間夾著兩種介質，其截面如圖所示。導電板間的距離為  $d$ 。介質-2 為一厚度為  $d/2$ ，邊長分別為  $a$  和  $a/2$  的絕緣長方形板，其介電常數為  $\epsilon_2$ 。導電板之間其餘的空間充滿了介質-1，其介電常數為  $\epsilon_1$ ，電導率為  $\sigma_1$ 。求兩導電板之間的電阻和電容。

**Q.2 (10 points) 題 2 (10 分)**

A galaxy is made of many stars revolving around its center on circular orbits. One way scientists study galaxies is to measure the relation between the revolving speed  $v$  of a star in the galaxy and its distance  $r$  from the center. Expressing the relation as  $v \propto r^n$ , scientists pay special attention to the exponential power  $n$ .

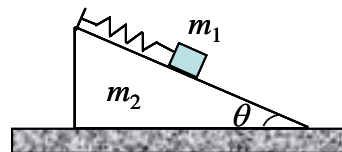
星系由很多繞中心作圓形軌道運行的恒星組成。科學家研究星系的一個方法是測量恒星在星系中的運行速度  $v$  和離星系中心的距離  $r$ 。用  $v \propto r^n$  這樣的關係來表達，科學家們特別關心指數  $n$ 。

- (a) If the gravity force on the star under investigation is mainly from a massive black hole at the galaxy center, derive the value of  $n$ . (4 points)  
 若作用于恒星的引力主要來自星系中心的巨型黑洞，求  $n$  的值。(4 分)
- (b) Suppose there is no black hole at the center and the stars in the galaxy are evenly distributed such that the mass of the galaxy is evenly distributed in a flat disk. Consider a star in the disk at distance  $r$  from the center. The total gravity force on the star from the stars enclosed by its circular orbit can be treated as if they are concentrated at the center. The total gravity force on the star from the stars outside the orbit is zero. (This is like the case of a uniform density sphere.) Derive the value of  $n$ . (4 points)  
 若星系中心無黑洞，星系的恒星分布均勻，整個星系可當作是一個質量均勻分布的扁圓盤。對於一個離中心為  $r$  的恒星來講，來自軌道內的恒星的引力與把它們集中在星系中心的一樣，來自軌道外的恒星的引力合力為零。(類似于均勻質量球的情形) 求  $n$  的值。(4 分)
- (c) What scientists found from real galaxies is that for most of them,  $n$  is larger than 1. Give your brief explanation to this scientific puzzle. (2 points)  
 科學家們發現，很多實際星系的  $n > 1$ 。請簡單解釋。(2 分)

**Q.3 (10 points) 題 3 (10 分)**

As shown, a block of mass  $m_1$  is connected to a spring of force constant  $k$  on the smooth slope (inclination angle  $\theta$ ) of a wedge of mass  $m_2$  placed on a smooth floor. Given a small disturbance to the block and the system starts to oscillate.

During the oscillation motion the block keeps in touch with the slope, and the wedge maintains contact with the floor. Find the oscillation frequency, and check your answer for two special cases of  $\theta = 0$  and  $\theta = 90^\circ$ .



如圖所示，一放在光滑地板上質量為  $m_2$  的大物塊的光滑斜面上（傾角為  $\theta$ ）有一力常數為  $k$  的彈簧，彈簧上系有質量為  $m_1$  的小物塊。給小物塊一輕微擾動後，系統開始振蕩。在振蕩期間小物塊始終與斜面保持接觸，大物塊始終與地板保持接觸。求振蕩頻率，并用  $\theta = 0$  和  $\theta = 90^\circ$  這兩個特殊情形檢驗你的答案。

**Q.4 (10 points) 題 4 (10 分)**

An electromagnetic (EM) wave is propagating along the  $\vec{z}_0$  direction in a non-magnetic conducting medium with conductivity  $\sigma$  and dielectric constant  $\epsilon$ . Both constants are real numbers. Its electric field is  $\vec{E}(z, t) = E_0 \vec{x}_0 e^{i(\vec{k}z - \omega t)}$ , where  $E_0$  and  $\omega$  are real constants while

$$\vec{k} = \frac{\omega}{c} \sqrt{\epsilon} + i \frac{\sigma}{2} \sqrt{\frac{\mu_0}{\epsilon \epsilon_0}}.$$

一電磁波在非磁性導電介質中沿  $\vec{z}_0$  方向傳播。介質的導電率為實數  $\sigma$ ，介電常數為實

數  $\epsilon$ 。電磁波的電場為  $\vec{E}(z, t) = E_0 \vec{x}_0 e^{i(\vec{k}z - \omega t)}$ ，其中  $\vec{k} = \frac{\omega}{c} \sqrt{\epsilon} + i \frac{\sigma}{2} \sqrt{\frac{\mu_0}{\epsilon \epsilon_0}}$ ， $E_0$  和  $\omega$  為實數。

- (a) Find the magnetic field of the EM wave. (2 points)

求電磁波的磁場。（2分）

- (b) Find the time-averaged (over one period) Poynting's vector  $\langle \vec{S} \rangle = \frac{1}{\mu_0} \langle (\vec{E} \times \vec{B}) \rangle$ . (2 points)

求經過一個周期的時間平均的 Poynting 矢量  $\langle \vec{S} \rangle = \frac{1}{\mu_0} \langle (\vec{E} \times \vec{B}) \rangle$ 。（2分）

- (c) The quantity  $q = -\frac{d \langle S \rangle}{dz}$  describes the loss of EM wave energy to a unit volume of the medium. Calculate  $q$ . (2 points)

$q = -\frac{d \langle S \rangle}{dz}$  代表電磁波輸送給單位體積的介質的能量。求  $q$ 。（2分）

- (d) Find the power consumed by a unit volume of the medium by directly calculating the Joule Heat averaged over one period of the wave  $\langle \vec{J} \cdot \vec{E} \rangle$ , where  $\vec{J}$  is the current density. (2 points)

求單位體積的介質以焦耳熱形式經過一個周期的時間平均的能量消耗  $\langle \vec{J} \cdot \vec{E} \rangle$ ，其中  $\vec{J}$  為電流密度。（2分）

- (e) Compare the answers in (c) and (d) and explain why they are equal or not equal. (2 points)

比較(c)和(d)的答案，解釋它們相等或不相等的原因。（2分）

**Q.5 (12 points) 題 5 (12 分)**

Consider a **non-ideal** gas with internal energy  $U$  given by  $U = 3PV$ , where  $P$  and  $V$  are the pressure and volume of the gas, respectively.

一非理想氣體的內能為  $U = 3PV$ ，其中  $P$  為氣體的壓強， $V$  為氣體的體積。

- (a) Find the relation between  $P$  and  $V$  in an adiabatic process. (2 points)

求絕熱過程  $P$  與  $V$  的關係式。(2 分)

Experiments show that the temperature  $T$  of the gas depends only on  $P$ , irrespective of its volume. The scale of the temperature can be fixed by setting  $T = 1$  when  $P = 1$ . Consider the following Carnot Process

實驗證明該氣體的溫度只與壓強有關，而與體積無關。溫度的標度可用當  $P = 1$  時  $T = 1$  來定。考慮以下的卡諾過程

$$(P_1, V_1) \xrightarrow{\text{Isothermal}} (P_1, V_2) \xrightarrow{\text{Adiabatic}} (P_2, V_3) \xrightarrow{\text{Isothermal}} (P_2, V_4) \xrightarrow{\text{Adiabatic}} (P_1, V_1)$$

(Isothermal = 等溫; Adiabatic = 絕熱)

- (b) Calculate the heat  $Q_1$  absorbed by the gas in the first isothermal process. (1 point)  
求第一個等溫過程氣體吸收的熱量  $Q_1$ 。(1 分)
- (c) Calculate the heat  $Q_2$  absorbed by the gas in the second isothermal process. (1 point)  
求第二個等溫過程氣體吸收的熱量  $Q_2$ 。(1 分)

- (d) The temperature  $T$  can be defined as  $\frac{T_1}{T_2} = -\frac{Q_1}{Q_2}$ . Find the  $P \sim T$  relation. (4 points)

溫度可定義為  $\frac{T_1}{T_2} = -\frac{Q_1}{Q_2}$ 。求  $P$  與  $T$  的關係式。(4 分)

- (e) Find the heat capacity at constant volume of the gas. (2 points)  
求氣體的恒定體積熱容量。(2 分)
- (f) Find the entropy  $S$  of the gas in terms of  $P$  and  $V$ , given that  $S = 0$  when  $T = 0$ . (2 points)  
已知氣體的熵  $S$  在溫度  $T = 0$  時為零。求氣體的熵，並以  $P$  和  $V$  表達。(2 分)

**THE END 完**

**Pan Pearl River Delta Physics Olympiad 2009**  
**2009 年泛珠三角及中華名校物理奧林匹克邀請賽**  
**Part-2 (Total 3 Problems) 卷-2 (共 3 題)**  
 (2:30 pm – 5:30 pm, 02-04-2009)

**Math hints 數學提示:**  $\int x^n dx = \frac{1}{n+1} x^{n+1} + C$ ;  $\int \frac{dx}{x} = \ln x + C$ .

**Q1 Solar System in the Early Days (16 points) 題 1 早期的太陽系 (16 分)**

In the early days of our solar system when the planets were not yet formed, the sun was surrounded by a big ball of gas at rest. The mass of the sun is  $M_s$ . Assume for simplicity that the gas is ideal and is made of single type of molecules of molecular mass  $m$ , and the total mass of the gas is much less than  $M_s$ , you are to find the distribution of the gas at a distance  $r$  ( $\gg$  diameter of the sun) from the sun.

早期的太陽系行星還沒有形成時，太陽被一大團氣體包圍著。已知太陽的質量為  $M_s$ 。為簡單起見設氣體是理想的，并由單種分子質量為  $m$  的分子組成。氣體的總質量遠小于  $M_s$ 。你需要求出離太陽距離  $r$  ( $\gg$  太陽半徑) 處氣體的分布。

- (a) Suppose the gas is all at the same temperature  $T_0$ . The mass density distribution of the gas can be expressed as  $\rho = \rho_0 e^{\alpha/r}$ . Find  $\alpha$ . (3 points)

假設氣體的溫度為  $T_0$  并且是均勻的，氣體的質量密度分布可表達成  $\rho = \rho_0 e^{\alpha/r}$ ，求  $\alpha$ 。  
(3 分)

- (b) There is a major flaw in the mass density expression in (a). Point out the flaw. (2 points)  
上述氣體的質量分布表達式有一重大缺陷。指出缺陷。(2 分)

- (c) In an improved model, suppose the sun emits  $J_0$  amount of thermal energy per second, and there is no energy loss when the thermal energy flows from the sun out to the gas via thermal conduction, find the energy current density (energy through a unit area per second) at a distance  $r$  from the sun. (3 points)

在改良的模型裏，假設太陽每秒釋放出的熱能為  $J_0$ ，當熱能以熱傳導方式從太陽傳給氣體時無損失，求離太陽距離  $r$  處熱能量流強度（每秒穿過單位面積的能量）。(3 分)

- (d) The thermal energy current density  $I(r)$  in (c) is proportional to the temperature gradient. That is,  $I(r) = -\sigma \frac{dT}{dr}$ , where  $\sigma$  is a positive constant called heat conductivity.

The minus sign comes from the fact that heat always flows from high  $T$  region to low  $T$  region. Find the temperature at distance  $r$  from the sun. (3 points)

上述熱能量流強度  $I(r)$  與溫度梯度成正比，既  $I(r) = -\sigma \frac{dT}{dr}$ ，其中  $\sigma$  為熱導率，是正數，式中的負號是因為熱流總是從高溫區域流向低溫區域的。求離太陽距離  $r$  處溫度。  
(3 分)

- (e) The pressure can now be expressed as  $P = P_0 (r/r_0)^{-\beta}$ . Find  $\beta$  and the mass density distribution. (3 points)

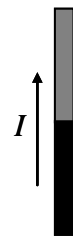
氣體的壓強可表述為  $P = P_0 (r/r_0)^{-\beta}$ ，求  $\beta$  以及質量密度分布。(3 分)

- (f) From (d) one can see that beyond certain distance  $r_0$  from the sun the temperature is below the ice temperature. Estimate  $r_0$  in terms of the radius of a planet orbit of the present solar system. (2 points)

由(d)可知超過某距離  $r_0$  氣體的溫度會低于水的冰點。 $r_0$  大概與現有的哪個行星的軌道半徑相近？(2 分)

**Q2 Spin Torque (16 points) 題 2 自旋力矩 (16 分)**

An electron has permanent angular momentum called **spin** ( $\vec{S}$ ) that is fixed in amplitude  $h/4\pi$ , where  $h$  is the Planck Constant, while its direction can be changed by external interactions. Consider a thin wire with electric current  $I$ , the lower half of which is made of a magnetized conductor and the upper half non-magnetic conductor. Due to quantum mechanics, the electron spins can only point to one of the two directions, namely in the electric current direction or opposite to it. In the magnetized section of the wire there are  $\alpha (> 50\%)$  portion of the electrons with spins in the current direction, while in the non-magnetic section there are equal numbers of electrons with spins in the two directions. The extra number of electrons with spins in the current direction in the magnetic section will instantly flip to the opposite direction once they cross the boundary and enter the non-magnetic section. As a result, a net torque is exerted on the wire.

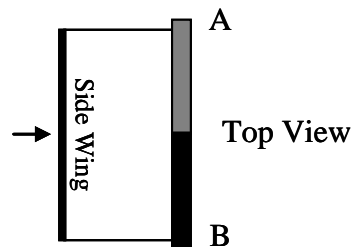


電子具有固有的角動量  $\vec{S}$ ，叫做自旋，其方向可在與外界的相互作用下改變，其大小為固定的  $h/4\pi$ ，其中  $h$  為普朗克常數。一導線載有電流  $I$ ，導線的下半部分由磁化了的導體組成，上半部分由非磁性導體組成。由量子力學得知，電子的自旋只能與電流同方向，或與電流反方向。磁化導體內有  $\alpha (> 50\%)$  部分的電子的自旋與電流同方向。非磁性導體內自旋與電流同方向和反方向的電子具有相同數量。磁化導體內多出的自旋與電流同方向的電子流進非磁性導體後會立刻改變它們的方向，從而產生力矩作用在導線上。

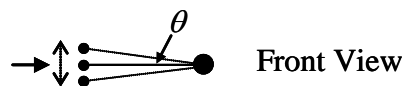
- (a) Find the torque in terms of  $h$ ,  $I$ ,  $\alpha$ , and electron charge  $e$ . (4 points)

以  $h$ ,  $I$ ,  $\alpha$ , 和電子電荷  $e$  來表達力矩。(4 分)

- (b) To measure the torque, a conducting side wing is attached rigidly to the wire but is electrically insulated from it, as shown in the figure. The wire is suspended between two fixed points A and B. Under constant current  $I_0$  the spin torque will twist the entire wire along the AB axis up to a small angle  $\theta_0$ , until it is balanced by the elastic restoring torque  $\tau_e = -\kappa\theta_0$ , where  $\kappa$  is a positive constant, at the two fixed points. Given that the moment of inertia of the entire structure with respect to the AB axis is  $J$ , and there is also a friction



torque of  $-\eta \frac{d\theta}{dt}$ , where  $\eta$  is a positive constant, find the angle  $\theta$  as a function of time  $t$  after the current is suddenly turned off at  $t = 0$ . (6 points)



為了測量力矩，我們在導線上裝上一導電邊翼(side wing)，如俯視圖(top view)和正視圖(front view)所示。邊翼與導線的連接是剛性的，但相互間不導電。導線的兩端固定在 A、B 兩點，中間懸空。有恒定電流  $I_0$  時，自旋力矩將導線以 AB 為軸扭轉一小角  $\theta_0$ ，直到被 A、B 點的彈性力矩  $\tau_e = -\kappa\theta_0$  抵消為止，其中  $\kappa$  是正常數。已知整個結構以 AB

為軸的轉動慣量為  $J$ ，摩擦力矩為  $-\eta \frac{d\theta}{dt}$ ，其中  $\eta$  為正常數。在時間  $t = 0$  時電流被突然切斷，求  $\theta$  之後的演化。(6 分)

- (c) When an alternating electric current  $I = I_0 e^{i\omega t}$  is applied, the wire will twist back and forth. The side wing of length  $L$  at a distance  $d$  from the wire follows the twisting motion of the wire. A magnetic field  $B$  is applied perpendicular to the side wing, as shown by the solid arrow in the figure. The field will not change the spins. Find the electromotive potential between the two ends of the side wing. (6 points)

將導線通上交流電  $I = I_0 e^{i\omega t}$ ，則導線會來回扭動，帶動邊翼上下擺動。已知邊翼的長度為  $L$ ，離導線的距離為  $d$ ，外加恒穩磁場場強為  $B$ ，方向與邊翼垂直，如圖中實綫箭頭所示。該磁場不會改變電子的自旋。求邊翼兩端之間的電動勢。（6 分）

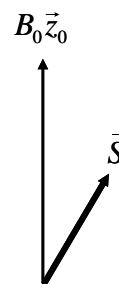
### Q3 Nuclear Magnetic Resonance (18 points) 題 3 核磁共振 (18 分)

Like the electrons, most nuclei have permanent angular momentum called **spins** ( $\vec{S}$ ) that are fixed in amplitude while their directions can be changed by an external torque. Associated with the spins are the magnetic moments that make them as individual magnetic dipoles  $\vec{m} = \mu \vec{S}$ , where  $\mu$  is a constant. Here we will study the spin dynamics using classical mechanics.

大多數原子核象電子那樣具有叫作自旋的固有角動量  $\vec{S}$ ，其大小固定，方向隨外力矩的作用而改變。由自旋產生的磁偶極子為  $\vec{m} = \mu \vec{S}$ ，其中  $\mu$  為常數。下面我們用經典力學研究自旋的運動。

- (a) A nucleus with spin  $\vec{S}$  is placed in a static magnetic field  $B_0 \vec{z}_0$  in a laboratory, as shown in the figure. Find the motion of the spin of the nucleus as seen in the laboratory reference frame. (4 points)

如圖，一自旋為  $\vec{S}$  的原子核處於實驗室的靜磁場  $B_0 \vec{z}_0$  中。求在實驗室參考系觀察到的自旋的運動。（4 分）



- (b) In a rotating reference frame with angular velocity  $\vec{\omega}_0$  the nucleus spin appears stationary. Find such reference frame. (2 points)

在一個以角速度  $\vec{\omega}_0$  旋轉的參考系觀察到的自旋是固定不動的。找出該參考系。（2 分）

- (c) The correct answer to (b) indicates that in a rotating reference frame with angular velocity  $\vec{\omega}$  there is an additional magnetic field  $\vec{B}_\omega$  seen by the spin. Find the additional magnetic field. (2 points)

上述(b)的正確答案說明，在一個以角速度  $\vec{\omega}$  旋轉的參考系裏，自旋會感受到一個額外的磁場  $\vec{B}_\omega$ 。求額外磁場。（2 分）

- (d) Back in the laboratory reference frame a second magnetic field is applied. The second field with strength  $B_1$  is always perpendicular to  $\vec{z}_0$  but its direction rotates in the XY plane at constant angular speed  $-\omega_1$ . Switching to the reference frame that rotates with the second magnetic field, find the total magnetic field seen by the nucleus. (3 points)
- 在實驗室參考系，我們加上另一個磁場。該磁場強度為  $B_1$ ，其方向與  $\vec{z}_0$  垂直，並在 XY 平面內以均勻角速度  $-\omega_1$  旋轉。在隨著該磁場旋轉的轉動參考系裏，求自旋感受到的總磁場。（3 分）

- (e) When  $\omega_0 = \omega_1$ , find the minimum time it takes to flip the spin from  $+\vec{z}_0$  to  $-\vec{z}_0$ . (This situation is called the Nuclear Magnetic Resonance) (4 points)

當  $\omega_0 = \omega_1$  時，求將自旋從  $+\vec{z}_0$  方向轉到  $-\vec{z}_0$  方向所需的最短時間。（這情形稱為核磁共振）（4 分）

- (f) When  $\omega_0 \neq \omega_1$ , find the motion of the spin observed in the rotating frame. (3 points)

當  $\omega_0 \neq \omega_1$  時，求在轉動參考系裏觀察到的自旋運動。（3 分）

THE END 完