Pan Pearl River Delta Physics Olympiad 2006

Note to graders:

- If the final answer is correct and there are sufficient steps to show the process, give full marks.
- If the final answer is wrong, find the step(s) where mistakes are made and deduct points there only. No repeat deduction of points for wrong answers being used for further steps. This is the principle of error non-progressiveness which we follow here.

Q1 (**8 points**)

Kinetic energy of meson $E_k = \dfrac{4m_1 m_2^{\ 2}}{(m_2+m_1)^2(m_3+m_2)}E$.

Internal energy of meson $E_i = \dfrac{4m_1 m_2 m_3}{(m_2+m_1)^2(m_3+m_2)}E$

Suggested steps:

(1) Use energy and momentum conservation to obtain the speed of m_2 after collision with m_1.

$v_2 = \dfrac{2m_1 m_2}{(m_1+m_2)}E$ (2 points)

The problem then becomes a pair of weights connected by a spring, with one mass suddenly acquires velocity.

(2) Find the center-of-mass speed V_c of m_2-m_3 system (m_3 is still at rest right after collision). The kinetic energy E_k of the m_2+m_3 system is then (m_2+m_3)/2 times the square of V_c. (3 points)

(3) The 'internal energy' E_i can then be found using energy conservation, counting both the meson and the electron after collision, or just counting the energy m_2 has after collision. (3 points)

Q2 (**8 points**) When the moment of inertia of the pulley is the largest, which means putting mass along the edge. Then use energy conservation that the initial potential energy is equal to the kinetic energy of mass m_3 and that of the pulley. (2 points)

Moment of inertia $I = \dfrac{1}{2}M_1R^2 + M_2R^2$ (2 points)

$v = R\omega$ (1 points)

$M_3gh = \dfrac{1}{2}I\omega^2 + \dfrac{1}{2}M_3v^2$ (2 points)

$v = \sqrt{\dfrac{4M_3gh}{M_1+2M_2+2M_3}}$ (1 points)

Q3 (**10 points)**

(a) Pressure is proportional to depth, and the total force is found by integrating the entire depth. The force acting on the strip is

$P = \rho gh$ (1 point)

$F = \int PdA = d\int_0^H \rho ghdh = \dfrac{1}{2}\rho gdH^2$ (2 points)

(b)

Step-1 Find the shape of the liquid surface (2 points)

Method-1: In the rotating reference frame the inertia force is $\omega^2 r$, and the associated potential is $U = \frac{1}{2}\omega^2 r^2$. The liquid surface is an equal-potential surface, and the gravitation potential is *gh*. So the shape is a rotating parabola determined by $gh + \frac{1}{2}\omega^2 r^2 =$ constant.

Method-2: The slope of the surface $\frac{dh}{dr}$ should be such that normal force should balance the gravity in the vertical direction, and give the concentric force for $\omega^2 r$ in the horizontal direction. That leads to the same answer as Method-1.

Method-3

$$F_p = pA \Rightarrow \Delta W_p = -P\Delta V \quad \text{where} \quad \Delta V = 2\pi r \Delta r \Delta h$$

$$F_c = m\omega^2 r \Rightarrow \Delta W_c = m\omega^2 r \Delta r \qquad \text{where } m = 2\pi\rho h r \Delta r$$

$$\Delta W_t = \sum_i \Delta W_i = 0 \qquad (\ )$$

$$\Rightarrow (2\pi\rho h r \Delta r)\omega^2 r \Delta r = \rho g h (2\pi r \Delta r \Delta h)$$

$$\Rightarrow \frac{dh}{dr} \approx \frac{\Delta h}{\Delta r} = \frac{\omega^2}{g} r$$

$$\Rightarrow h = \frac{\omega^2}{2g} r^2 + C \quad \text{where} \quad C \text{ is a constant. } (\ )$$

Step-2 Determine C (2 points)

Consider the Volume during rotation,

$$2\pi \int_0^R h r dr = \pi R^2 H$$

$$2\pi \int_0^R \left(\frac{\omega^2}{2g} r^2 + C\right) r dr = \pi R^2 H$$

$$\Rightarrow C = H - \frac{\omega^2 R^2}{4g}$$

$$\left[h = \frac{\omega^2}{2g}\left(r^2 - \frac{R^2}{2}\right) + H \right] \qquad (\ )$$

For $r = R$, $h = \frac{\omega^2 R^2}{4g} + H$ (1 point)

$$F' = \frac{1}{2}\rho g d \left(\frac{\omega^2 R^2}{4g} + H\right)^2 \qquad \text{(1 point)}$$

The Amount of Extra Force is

$$\Delta F = F' - F = \frac{1}{2}\rho g d \left[\frac{\omega^2 R^2}{2g} H + \left(\frac{\omega^2 R^2}{4g}\right)^2 \right] \qquad \text{(1 point)}$$

Q4 (12 points)

a) Use the ideal gas law,

$$P = R\frac{(n_L T_L + n_R T_R)}{V}, \quad (V = V_L + V_R); \quad V_L = \frac{n_L T_L}{(n_L T_L + n_R T_R)} V \qquad \text{(2 points)}$$

b) Energy conservation $c \equiv n_L T_L(0) + n_R T_R(0) = n_L T_L(t) + n_R T_R(t)$ (1) (1 point)

and $0 = n_L dT_L + n_R dT_R$ (1a)

Equal pressure leads to $n_L T_L(t) V_R(t) = n_R T_R(t) V_L(t)$ (2) (1 point)

and $\dfrac{dT_L}{T_L(t)} + \dfrac{dV_R}{V_R(t)} = \dfrac{dT_R}{T_R(t)} + \dfrac{dV_L}{V_L(t)}$ (2a)

Heat transfer plus work done due to expansion

$$k(T_L(t) - T_R(t))dt = dQ = -\frac{3}{2}RdT_L - P(t)dV_L = -\frac{3}{2}RdT_L - n_L RT_L(t)\frac{dV_L}{V_L(t)} \quad (3)\ (1\ \text{point})$$

Finally we have $V = V_L(t) + V_R(t)$ (4)

From Eqs. (1), (2) and (4) we get $n_L RT_L(t)\dfrac{dV_L}{V_L(t)} = n_L RdT_L$ (5) (1 point)

Also from Eq. (1a) $d(T_L(t) - T_R(t)) = (1 + \dfrac{n_R}{n_L})dT_L$ (6) (1 point)

Put Eqs. (5) and (6) into Eq. (3) $\dfrac{d(T_L - T_R)}{dt} = -\dfrac{2k}{5R}\left(\dfrac{1}{n_L} + \dfrac{1}{n_R}\right)(T_L - T_R) = -\beta(T_L - T_R)$ (2 points)

Where $\beta \equiv \dfrac{2k}{5R}\left(\dfrac{1}{n_L} + \dfrac{1}{n_R}\right)$, and $(T_L(t) - T_R(t)) = (T_L(0) - T_R(0))e^{-\beta t}$. (2 points)

c) Find out how V_L and V_R changes with time.

The conservation of energy supplies another equation

Using Eq. (1)

$$T_R(t) = \frac{1}{n_R + n_L}\left(n_L T_L(0)(1 - e^{-\kappa t}) + T_R(0)(n_R + n_L e^{-\kappa t})\right),$$
$$T_L(t) = \frac{1}{n_R + n_L}\left(n_R T_R(0)(1 - e^{-\kappa t}) + T_L(0)(n_L + n_R e^{-\kappa t})\right) \quad (1\ \text{point})$$

and $V_{L(R)} = \dfrac{n_{L(R)} T_{L(R)}(t)}{(n_L T_L(t) + n_R T_R(t))} V$. (1 point)

Q5 (12 points)

(a) The E-fields in medium-1 and -2 are $\vec{E}_1 = \vec{E}_I + \vec{E}_R$, $\vec{E}_2 = \vec{E}_T$ (1 point)

$\omega\vec{B} = \vec{k} \times \vec{E}$, with $\vec{k} = k\vec{y}_0$ (1 point)

$\vec{B}_1 = \dfrac{k}{\omega}(E_0 - E_r)\vec{z}_0 e^{i(k_1 y - \omega t)}$ (1 point)

$\vec{B}_2 = \dfrac{k}{\omega}E_t \vec{z}_0 e^{i(k_2 y - \omega t)}$ (1 point)

Using the boundary conditions $\vec{E}_1^{//} = \vec{E}_2^{//}$, $\vec{B}_1^{//} = \vec{B}_2^{//}$ at $y = 0$, (1 point)

Dispersion relation $k = \dfrac{n\omega}{c}$, (1 point)

one gets $E_0 - E_R = E_T$, and $E_0 n_1 - E_R n_1 = E_T n_2$, (1 point)

Solving the equations, one obtains $r = \frac{n_1 - n_2}{n_2 + n_1}$ $\quad t = \frac{2n_1}{n_2 + n_1}$ (1 point).

(b) $R = 1$ (1 point)

(c) Use (a) and find the phase of r with the given n_1 and n_2. $r = \frac{1+i}{2}$ (2 point),

Phase shift = 45° (1 point).

Q6 (13 points)

Part-A $PV=nRT$ so the pressure goes to zero. (2 points)

Part-B

(a) $k_n d = 2\pi n$, where n are integers. (2 points)

(b) $E = \frac{\hbar c}{2}\sum_{n<p_c} k_n = \frac{\pi\hbar c}{d}\sum_{n<p_c} n = \frac{\pi\hbar c}{2d} n(n-1) \cong \frac{\pi\hbar c}{2d} n^2$. (2 points)

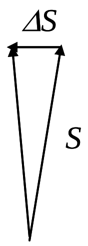
The force is given by $F = -\frac{\partial E}{\partial d} = \frac{\pi\hbar c}{2d^2} n^2$. (2 points)

The system energy decreases with increasing d so the force is pushing outwards. (1 point)

(c) Outside we have $d = \infty$ (1 point), so $F = 0$ (1 point)

(d) $F = \frac{(1.05\times10^{-34}\,Js)(3\times10^{8}\,ms^{-1})\pi[2000(2000-1)]}{4}\frac{1}{(1\times10^{-3}\,m)^2} = 9.89\times10^{-14}\,Nm^2$. (2 points)

Q7 (15 points)



(a) The torque is $\vec{\tau} = \frac{\mu_0}{4\pi a^3}\vec{m}_A \times \vec{m}_B = -\frac{\mu_0(\mu_B S)^2}{4\pi a^3}\vec{x}_0$ which is perpendicular to $\vec{m}_B$ (3 points)

(b) The torque causes $\vec{S}_B$ to spin within the x-y plane, (2 points) and the angular speed is given by $\tau = S\omega$. (2 point)

The angle of the y-axis is $\Delta\theta = \omega\Delta t = \frac{\mu_0}{4\pi a^3}\mu_b^2 S\Delta t$ (1 point)

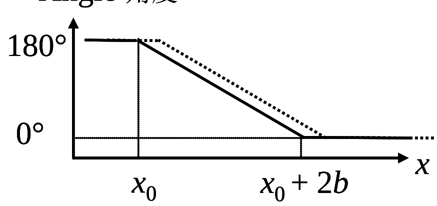


Figure-C

(c) Find how many electrons should pass to cause a particular spin to rotation from 90° - $d\theta$ to 90°and from θ find position of the spin. $(\frac{I}{e}\Delta\theta)dt = d\theta = \frac{\pi}{2b}dx$ (3 points)

$v = \frac{dx}{dt} = \frac{2bI}{\pi e}\Delta\theta = \frac{\mu_0}{2e\pi^2 a^3} bI\mu_b^2 S\Delta t$ (2 points)

(d) $t = 2b/v = \frac{4e\pi^2 a^3}{\mu_0 I \mu_b^2 S\Delta t}$. (2 points)

Q8 (22 points**)**

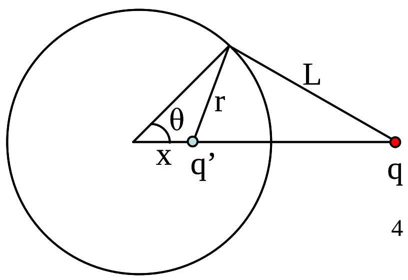
Part-A

Place a point charge q' at distance x from the center, which is at a distance r from a point on the sphere surface. The distance of this point to charge q is L.

Then $L^2 = R^2 + d^2 - 2Rd\cos\theta$ (1)

$r^2 = R^2 + x^2 - 2Rx\cos\theta$ (2)

(1 point)



Zero potential on sphere surface => $\frac{q}{L} = -\frac{q'}{r}$ (3) (1 point)

Putting Eq. (1) and (2) into (3) leads to

$q'^2(R^2 + d^2 - 2Rd\cos\theta) = q^2(R^2 + x^2 - 2Rx\cos\theta)$ (4)

Equation (4) must be true for all angle θ, so

$q'^2(R^2 + d^2) = q^2(R^2 + x^2)$ (5) and

$q'^2 Rd\cos\theta = q^2 Rx\cos\theta$ (6)

Solving Eq. (5) and (6), we get two sets of solutions.

Solution-1: $x = d$ and $q' = -q$. Not the right one because q' ends up outside the sphere.

Solution-2: $q' = -qR/d$, and $x = R^2/d < R$. Correct. (2 points)

Part-B

(a) $q_0 = 4\pi\varepsilon_0 RV$ (1 point)

(b) $-q_0$ and at a distance h_0 on the other side of the plane. (2 points)

(c) The contribution of q_1 and q_2 is to make the sphere surface zero potential. The answer in Part-A can be used here. $h_2 = h_0 - \frac{R^2}{h_0 + h_0}$, $q_2 = \frac{R}{2h_0}q_0$ (2 points)

(d) $h_{2(n+1)} = h_0 - \frac{R^2}{h_0 + h_{2n}}$, (1 point) $q_{2(n+1)} = \frac{R}{h_0 + h_{2n}}q_{2n}$, (1 point) $q_{2n+1} = -q_{2n}$ (1 point)

(e) Sum over all charges on both sides of the plane. $F = \frac{-1}{4\pi\varepsilon_0}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{q_{2m}q_{2n}}{(h_{2m} + h_{2n})^2}$ (2 points)

(f) Using (a) and (d), define $k_0 \equiv h_0 / R$,

$k_{2(n+1)} \equiv \frac{h_{2(n+1)}}{R} = k_0 - \frac{1}{k_0 + k_{2n}}$, (1 point)

$q_{2n} = \frac{1}{k_0 + k_{2n-2}}q_{2(n-1)} = \frac{1}{k_0 + k_{2(n-1)}}\frac{1}{k_0 + k_{2(n-2)}}\frac{1}{k_0 + k_{2(n-3)}}....q_0$ (1 point)

Using (e) $F = -4\pi\varepsilon_0 V^2 \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{1}{(k_0 + k_{2(n+1)})}\frac{1}{(k_0 + k_{2(m+1)})}\frac{1}{(k_{2m} + k_{2n})^2} \equiv -4\pi\varepsilon_0 V^2 g(k_0)$. (1 point)

The force is proportional to V^2, and depend only on the ratio of $k_0 = R/h_0$. So the answer is 4.4 x 10^{-12} N. (1 point)

(g) Since all the image charges above the surface must be inside the spheres, the distance between any charge outside the sample and those inside should be large than $2(h_0 - R)$. Then $h_0 + h_{2n-2} > 2(h_0 - R)$. So $|q_{2n}| = \frac{R}{h_0 + h_{2n-2}}|q_{2n-2}| < \frac{R}{2(h_0 - R)}|q_{2n-2}| = \frac{1}{100}|q_{2n-2}|$, and only the $n = 1$ terms should be kept. (1 point)

The total force is the sum of q_0q_1, q_0q_3, and q_2q_1, or (01), (03), and (12) for short. $F = -\frac{\pi\varepsilon_0 V^2}{51^2}(1 + \frac{1}{51})$. (1 point)

(h) The terms in the expansion that are of the order of $\left(\frac{R}{2(h_0 - R)}\right)^2 = (10^{-2})^2$ are (05), (23), (41),

$|F_{error}| = 4\pi\varepsilon_0 V^2\left[\frac{10^{-4}}{4}k_2^2 + \frac{10^{-4}}{4(1/k_0 + 1/k_4)^2} + \frac{10^{-4}}{4(1/k_1 + 1/k_5)^2}\right] < 3\pi\varepsilon_0 V^2 10^{-4}\frac{1}{(50)^2}$

So the relative error is at most 3×10^{-4}. (2 points)

What are left in (g) in the force calculation is

$$
\begin{aligned}
|F_{error}| &< 4\pi\varepsilon_0 V^2\left[\frac{10^{-4}}{4h_2^2}+\frac{10^{-4}}{4(h_0+h_4)^2}+\frac{10^{-4}}{4(h_1+h_5)^2}+\sum_{m=2}^{\infty}\sum_{n=2}^{\infty}\frac{1}{(h_{2m}+h_{2n})^2}\left(\frac{R}{2(h_0-R)}\right)^{m+n}\right] \\
&< 4\pi\varepsilon_0 V^2 10^{-4}\left[\frac{3}{4(h_0-R)^2}+\sum_{m=2}^{\infty}\sum_{n=2}^{\infty}\frac{1}{(2h_0-2R)^2}\left(\frac{R}{2(h_0-R)}\right)^{m+n}\right] \\
&< 4\pi\varepsilon_0 V^2 10^{-4}\frac{1}{4(h_0-R)^2}\left(3+\left(\sum_{n=0}^{\infty}\left(\frac{1}{100}\right)^n\right)^2 \right.\cong 3\pi\varepsilon_0 V^2 10^{-4}\frac{1}{(h_0-R)^2}
\end{aligned}
$$

So the relative error is at most 3×10^{-4}.