Pan Pearl River Delta Physics Olympiad 2008

2008 年泛珠三角及中华名校物理奥林匹克邀请赛

Part-1 (Total 6 Problems)　卷-1（共 6 题）

(9:00 am – 12:00 pm, 02-14-2008)

Q.1 (4 points) 题 1（4 分） A nucleus-A with mass m_A and initial velocity v_0 along the x-axis collides with a nucleus-B with mass m_B at rest. Some kinetic energy E is absorbed by nucleus-B and converted into nuclear energy during the collision. Since $E << m_B c^2$ where c is the speed of light in vacuum, the change of mass of nucleus-B can be neglected. After the collision nucleus-A moves at an angle $\theta = 90^{\circ}$ to the x-axis. Find the speed of nucleus-A and the velocity of nucleus-B.

一质量为 m_A 的原子核-A 以沿 X-轴方向的初速度 v_0 与一质量为 m_B 的静止原子核-B 相撞。碰撞中一部分机械能 E 被原子核-B 吸收而转变为核能。由于 $E << m_B c^2$，其中 c 是真空光速，所以可忽略原子核-B 质量的变化。碰撞后原子核-A 沿与 X-轴成 $\theta = 90^{\circ}$ 的方向飞出。求原子核-A 的速率，以及原子核-B 的速度。

Q.2 (6 points) 题 2（6 分）

Two large parallel conductor plates are held at voltage difference V at distance d apart. A large dielectric slab of thickness $d/3$ and dielectric constant ε is placed midway in the gap between the plates and is moving parallel to the plates at speed v (<< speed of light).

(a) In the reference frame where the plates are stationary, find the magnetic field in the middle of the upper air gap, in the middle of the slab, and in the middle of the lower air gap.

(b) In the reference frame where the dielectric slab is stationary, repeat (a).

两大导电板间距 d，之间的电压差 V。一厚 $d/3$ 的大介质板，介电常数ε，在两板中间以速度 v（<<光速）沿与板平行的方向运动。

(a) 在导电板静止的参照系，求在上、下空隙中间位置以及介质板中间位置的磁场。

(b) 在介质板静止的参照系，求在上、下空隙中间位置以及介质板中间位置的磁场。

Q.3 (7 points) 题 3（7 分）

An electron has intrinsic angular momentum I called spin, and a permanent magnetic dipole moment $\vec{M} = -\dfrac{ge}{2m}\vec{I}$ associated with the spin, where e is the positive electron charge, m is the electron mass, and g is a number called g-factor. An electron with its spin aligned along its initial velocity in the x-direction enters a region of uniform magnetic field in the z-direction. Show that if g is exactly 2, then the spin is always in the same direction as the velocity of the electron. (The real $g = 2.00232...$. The deviation from 2 can be calculated precisely by quantum electrodynamics.)

电子具有固有的角动量 I，称为自旋，和与自旋相关的磁偶极矩 $\vec{M} = -\dfrac{ge}{2m}\vec{I}$。其中 e 为正电子电荷，m 为电子质量，g 为 g -因子。一电子初始速度和自旋都沿 X-方向，进入一均匀沿 Z-方向的磁场。证明若 g 刚好等于 2，则电子之后的速度和自旋始终在同一方向。（实际的 $g = 2.00232...$。和 2 的差可用量子电动力学准确计算。）

Q.4 (12 points) 题 4（12 分）

Near the Earth's surface, the atmosphere could be considered as ideal gas at constant temperature $T = 300$ K and in a uniform gravitational field $g = 9.8$ m/s^2. The mass per mole of 'air' molecule is $m = 0.029$ kg. The gas constant is $R = 8.31$ J/(K*mol).

(a) For stationary atmosphere, set up a differential equation that relates the pressure $p(h)$ at height h with the gravitational field g, the atmosphere mole number density $\rho(h)$, and mass per mole m. **(1 point)**

(b) Using the ideal gas law and the result in (a), set up a differential equation for the pressure $p(h)$ as a function of height h. **(1 point)**

(c) Assuming that p_0 is the pressure at $h = 0$, solve the differential equation in (b), and find the height where the pressure is $\frac{p_0}{2}$. (Hint: $\int \frac{dx}{x} = \ln|x| + \text{Constant}$) **(4 points)**

(d) In the case of wind with constant velocity v blowing in the atmosphere at all height, the differential equation is approximately $\frac{dp}{dh} + \frac{mv^2}{2}\frac{d\rho}{dh} = -m\rho g$. Find the pressure $p(h)$ at height h. **(3 points)**

(e) In (d) the effect of Earth spinning is ignored. In general, any object moving at velocity $\vec{v}$ on Earth will experience an additional 'inertia force field' $\vec{f}_{\text{int}} = 2\vec{\Omega} \times \vec{v}$, where $\vec{\Omega}$ is the angular velocity of the spinning Earth, in addition to the gravitational field g. Verify that the inertia force field can indeed be ignored even in the case of typhoons where the wind speed is up to 500 km/hr. **(1 point)**

(f) Using the wind speed in (e), find the height where the pressure is $\frac{p_0}{2}$. **(2 points)**

在地面附近, 大气层可被当作温度为 $T = 300$ K 的等温理想气体，并处于均匀引力场 $g = 9.8$ m/s^2 之内。 每 mole 的大气质量为 $m = 0.029$ kg。 理想气体常数 $R = 8.31$ J/(K*mol)。

(a) 对于静止的大气层，建立微分方程，把在高度 h 处的压强 $p(h)$ 、重力加速度 g、 大气分子 mole 密度 $\rho(h)$、和每 mole 的大气质量 m 这些物理量联系起来。 **(1 分)**

(b) 利用理想气体方程和(a)的结果, 建立 $p(h)$ 为 h 的函数的微分方程。 **(1 分)**

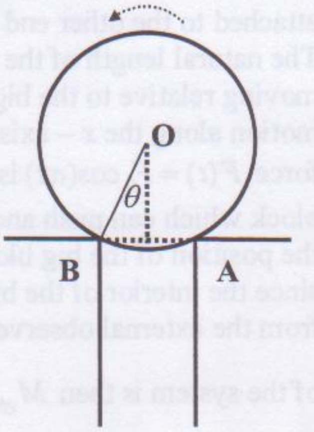
(c) 设在 $h = 0$ 处压强为 p_0，解(b)的微分方程, 并找出压强为 $\frac{p_0}{2}$ 处的高度。(提示: $\int \frac{dx}{x} = \ln|x| + \text{Constant}$) **(4 分)**

(d) 有风时，设大气在所有高度的速度均为 v，则经过简化的微分方程为 $\frac{dp}{dh} + \frac{mv^2}{2}\frac{d\rho}{dh} = -m\rho g$ 。求在高度 h 处的压强 $p(h)$. **(3 分)**

(e) 在(d)中没有考虑地球的自转。一般说来，任何以速度 $\vec{v}$ 在地球表面运动的物体，除了重力 g 外，均受到 '惯性力场' $\vec{f}_{\text{int}} = 2\vec{\Omega} \times \vec{v}$ 的作用，其中 $\vec{\Omega}$ 为地球自转的角速度。 验证甚至在刮 500 km/hr 的台风时地球自转的效应仍可忽略。 **(1 分)**

(f) 用(e)的风速， 求压强为 $\frac{p_0}{2}$ 处的高度。**(2 分)**

Q.5 (11 points) 题 5（11 分）



A uniform solid sphere with mass M, radius R, and moment of inertia $I = \frac{2}{5}MR^2$ around its center is initially rolling without slipping on a horizontal surface. The speed of its center is v. It then encounters a ditch of width d such that $\sin\theta = \frac{d}{2R}$, as shown in the figure. For convenience you may use θ and R to replace d in the following calculations. The initial speed v is smaller than a value $v_{\max}$ such that when the sphere arrives at the near edge of the ditch at point-A, it falls off while keeping in touch with point-A without slipping, until it hits the other edge at point-B.

(a) Find the angular speed of the sphere right before it hits point-B. **(3 points)**

(b) Find the maximum initial speed $v_{\max}$ that the sphere can keep in touch with point-A without slipping before it hits point-B. **(2 points)**

(c) Assuming no slipping when the sphere hits point-B, find the minimum initial speed $v_{\min}$ such that the sphere can get over the ditch. **(4 points)**

(d) To satisfy both the conditions in (b) and (c), the angle θ must satisfy $f(\theta) > 0$. Determine $f(\theta)$. **(2 points)**

一均匀刚球质量 M，半径 R，绕球心的转动惯量 $I = \frac{2}{5}MR^2$，在平面上作纯滚动，球心速度 v。之后遇到一沟，沟宽 d，如图所示 $\sin\theta = \frac{d}{2R}$。为方便起见，在以下的解答你可用 θ 和 R 来代替 d。球的初速度 v 比某 $v_{\max}$ 小，使得当球到达沟的一边 A 点处时，一直与 A 点保持无滑动的接触，直到球碰到沟另一边的 B 点。

(a) 求碰到 B 点前的瞬间球的角速度。**(3 分)**

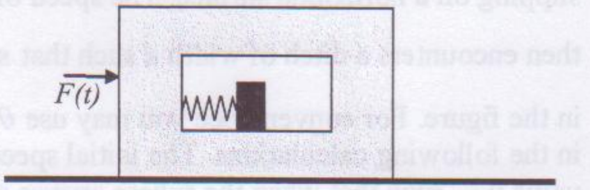
(b) 求球碰到沟另一边的 B 点前一直与 A 点保持无滑动接触的最大初速度 $v_{\max}$。**(2 分)**

(c) 设球碰到 B 点后无滑动，求能使球滚过沟的最小初速度 $v_{\min}$。**(4 分)**

(d) 要使(b) 和 (c)的条件都成立，角度 θ 必须满足 $f(\theta) > 0$。求 $f(\theta)$。**(2 分)**

Q.6 (10 points) 题 6（10 分）

Consider a big block of mass M_1 placed on a smooth horizontal surface with a hollow rectangular cave carved out in the interior, as shown in the figure. Inside the cave are a spring of force constant K with one end attached to the wall, and a smaller block of mass M_2 attached to the other end of the spring which can move on the smooth horizontal cave surface. The natural length of the spring is about half the length of the cave. When the small block is moving relative to the big block it never hits the wall of the cave. Both blocks are confined to motion along the $x-$ axis. A periodic external force $F(t) = F_0 \cos(\omega t)$ is applied to the big block which can push and pull the block. Let the position of the big block be $X_1(t)$, and since the interior of the big block is hidden from the external observers, the effective mass of the system is then $M_{eff} = \dfrac{F(t)}{\ddot{X}_1(t)}$, where $\ddot{X}_1(t) \equiv \dfrac{d^2 X_1}{dt^2}$ is the second derivative of $X_1(t)$ to time.



(a) Assume that the two blocks are in simple harmonic motions at the same frequency as that of the driving force $F(t)$, find the effective mass of the system. **(7 points)**

(b) Find the range of frequencies in which the effective mass is negative. **(3 points)**

如图，一质量 M_1 的大物块放在光滑平面上，大块内有一长方空腔，空腔内有一力常数 K 的弹簧，一端固定在空腔壁上，另一端系有一质量 M_2 的小物块，放在光滑空腔底平面上。弹簧的自然长度约为空腔长度的一半，小物块运动时不会碰到两边的腔壁。所有运动都是沿 $x-$ 轴的一维运动。一周期性外力 $F(t) = F_0 \cos(\omega t)$ 加在大物块上，将它推前、拉后。设大物块的位置为 $X_1(t)$, 因大物块的内部是看不见的，所以系统的有效质量为 $M_{eff} = \dfrac{F(t)}{\ddot{X}_1(t)}$,其中 $\ddot{X}_1(t)$ 为 $X_1(t)$对时间的二次导数，$\ddot{X}_1(t) \equiv \dfrac{d^2 X_1}{dt^2}$ 。

(a) 假设两物块均以与外力 $F(t)$ 相同之频率作简谐运动，求系统的有效质量。**(7 分)**

(b) 求有效质量为负数的频率范围。**(3 分)**

THE END 完

Pan Pearl River Delta Physics Olympiad 2008
2008年泛珠三角及中华名校物理奥林匹克邀请赛
Part-2 (Total 3 Problems) 卷-2（共3题）
(2:30 pm – 5:30 pm, 02-14-2008)

Q1 Point charge in a magnetic field (16 points) 题1 磁场中的带电粒子（16分）

A point charge $-q$ ($q > 0$) with mass m moves without friction inside a region of magnetic field given by $\mathbf{B} = B_0 \dfrac{a}{r}\hat{\mathbf{z}}$ ($a, B_0 > 0$), where $r = \sqrt{x^2 + y^2}$ is the distance from the z-axis. At $t = 0$, the initial position and velocity of the charge is ($x = a, y = 0, z = 0$) and ($v_x = 0, v_y = v_0, v_z = 0$), where $v_0 > 0$.

(a) Show that the particle always stays on the xy plane. **(1 point)**

(b) Find the initial speed at which the charge should be launched so that it can perform circular motion around the origin. **(2 point)**

(c) To obtain the motion for arbitrary v_0, set up a differential equation, in the form of $\dfrac{dL}{dr} = \textit{constant}$, for angular momentum L of the charge with respect to the z-axis, and solve the differential equation. If you cannot determine the ***constant***, just assume it is known and solve (d) and (e).
(Hint: $\vec{A}\times\left(\vec{C}\times\vec{B}\right) = \left(\vec{A}\cdot\vec{B}\right)\vec{C} - \left(\vec{A}\cdot\vec{C}\right)\vec{B}$) **(5 points)**

(d) From the result of (c), find the distance of the charge from the origin when it is moving in the tangential direction ($\vec{v}\perp\vec{r}$). Find the minimum v_0 above which the charge can never move in the tangential direction after it is initially launched. **(5 points)**

(e) Find the distance from the origin when the charge is moving in the radial direction ($\vec{v}$ parallel to $\vec{r}$). Find the minimum v_0 above which the charge can never move in the radial direction after it is initially launched. **(3 points)**

一质量 m 的点电荷 $-q$ ($q > 0$) 无阻力地在磁场 $\mathbf{B} = B_0 \dfrac{a}{r}\hat{\mathbf{z}}$ ($a, B_0 > 0$)中运动，其中 $r \equiv \sqrt{x^2 + y^2}$ 为电荷离 z-轴的距离。

在 $t = 0$ 时, 电荷的位置和速度分别为($x = a, y = 0, z = 0$) 和 ($v_x = 0, v_y = v_0, v_z = 0$), 其中 $v_0 > 0$。

(a) 证明粒子的运动始终在 xy 平面上。 **(1分)**

(b) 若粒子以原点作圆周运动，求初速度 。 **(2分)**

(c) 对一般初速度情况下求解。建立粒子绕 z-轴的角动量所满足的微分方程，其形式应为 $\dfrac{dL}{dr} = $ ***常数***，并解之。若无法确定***常数***，你可假设***常数***已知，并用来解答(d)和(e)。(提示：$\vec{A}\times\left(\vec{C}\times\vec{B}\right) = \left(\vec{A}\cdot\vec{B}\right)\vec{C} - \left(\vec{A}\cdot\vec{C}\right)\vec{B}$) **(5分)**

(d) 从(c)的结果, 求当 $\vec{v}\perp\vec{r}$ 时粒子离原点的距离。若粒子在 $t = 0$ 后永远不再有 $\vec{v}\perp\vec{r}$ 的情形出现，求 v_0 的最小值。 **(5分)**

(e) 求当 $\vec{v}$ 平行于 $\vec{r}$ 时粒子离原点的距离。若粒子在 $t = 0$ 后永远不再有 $\vec{v}$ 平行于 $\vec{r}$ 的情形出现，求 v_0 的最小值。 **(3分)**

Q2 Electric Fan (16 points) 题 2 电扇（16 分）

Consider an electric fan with moment of inertia I around its central axis initially rotating about the axis at constant speed ω_0 driven by a motor. The fan will slow down once the motor is turned off and finally stops due to the friction forces from two sources. One is the fixed torque τ due to the friction between its rotating central axis and its holder. The other is the air resistance on the fan blades which is proportional to the instantaneous rotating speed $\omega(t)$, so the air resistance torque is $\gamma\omega(t)$, where γ is a constant.

(a) Theory

(a.1) Write down the differential equation for the instantaneous rotating speed $\omega(t)$ of the fan. **(2 points)**

(a.2) Given the initial speed ω_0, the time it takes for the fan to stop (t_s) can be expressed as $t_s = A \cdot \ln(1 + B\omega_0)$. Determine the constant A and B in terms of I, τ, and γ. **(4 points)**

(b) Design of experiment

Suppose the initial speed of the fan can be so slow that $\ln(1 + B\omega_0) \approx B\omega_0$, or can be so fast that $B\omega_0 >> 1$. The initial speed of the fan can be set and read from the meter on the fan controller. You are also given the following items: a ruler, a stop watch, and several pairs of small known mass blocks (about 1/10 the mass of the blades) that can be firmly attached to the blades of the fan, but their air resistance can be neglected. Design an experiment to determine τ, γ, and I. You should state clearly what data are to be collected and processed, what plot(s) should be drawn, and how to extract the parameters from the plot(s) to reach the final answers. **(10 points)**

一电扇绕其中心轴的转动惯量 I，初始时在马达驱动下以角速度 ω_0 绕中心轴转动。马达关了后，由于两方面的摩擦阻力电扇的转动会慢下来，直到停止。阻力之一为中心轴与外套之间的固定摩擦阻力力矩 τ。另一个是电扇叶片的空气阻力力矩，与瞬时角速度 $\omega(t)$成正比，因此可表达为$\gamma\omega(t)$, 其中 γ 为常数。

(a) 理论

(a.1) 写出电扇瞬时角速度的微分方程 **(2 分)**

(a.2) 给定初始角速度 ω_0，从停止驱动到电扇停下的时间 t_s 可表达成 $t_s = A \cdot \ln(1 + B\omega_0)$。用 I、τ、γ 来表达系数 A、B。**(4 分)**

(b) 实验设计

设电扇的初始角速度很慢，使$\ln(1 + B\omega_0) \approx B\omega_0$ 得以成立；或很快，使 $B\omega_0 >> 1$ 得以成立。电扇的初始角速度可由马达的控制器设置并读出。另有标尺一把，计时器一个, 和几对已知质量（约为电扇叶质量的 1/10）、可粘在电扇叶上的小重块。小重块的空气阻力可忽略。设计实验步骤以确定 τ、γ、和 I。你必须清楚表述需要测量哪些实验数据，如何作数据处理，如何作图，从图上得到哪些参数，最后得到结果。**(10 分)**

Q3 Negative Resistance Instability & Zero Resistance State (18 points)
题 3 负电阻不稳定性和零电阻状态 (18 分)

Two years ago it was discovered that the resistance of a semiconductor device containing two dimensional electron gas becomes negative when it is in a constant magnetic field and under strong microwave radiation. Because a system including the device and other circuit elements such as normal resistors, capacitors, inductors, and voltage sources will be unstable (violating the second law of thermodynamics) if the *total* resistance of the system R_{sys} is negative, the system reorganizes itself into a new state with zero total resistance. The resistance of the device can be expressed as a current and charge dependent resistance

$$R_{NR} \equiv R(i,q) = R_0\left(\left(\frac{i}{i_o}\right)^2 + \left(\frac{q}{q_o}\right)^2 - 1\right),$$

where $R_0 > 0$ so that the resistance is negative when the current passing through the system i or charge storing in the system q is small, and becomes positive when there is large enough current or charge building up in the system. Here R_0, i_o, q_o are constants that depend only on the internal structure of the device, the strength and frequency of microwave radiation, and the strength of the magnetic field. When R_{NR} is connected to other normal circuit elements like capacitor, normal resistor, inductor, and voltage source, the usual Kirchhoff's circuit laws still apply, and we seek steady-state solutions only.

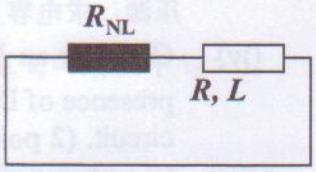
两年前人们发现在微波照射下，在直流磁场中的由二维电子气构成的半导体器件的电阻可成负值。由于含有这种器件和其它常规电阻、电感、电容、电源的系统的总电阻R_{sys}不能为负，否则会违反热力学第二定律，所以系统会自动调节，以达到一新的零电阻状态。器件的电阻与电流、电荷有关的表达式为

$$R_{NR} \equiv R(i,q) = R_0\left(\left(\frac{i}{i_o}\right)^2 + \left(\frac{q}{q_o}\right)^2 - 1\right),$$

其中$R_0 > 0$。因此当系统的电流 i 或电荷 q 太小时，其电阻为负。而当系统的电流或电荷够大时，其电阻变为正值。R_0, i_o, q_o 为常数，只与器件的本身结构，以及磁场、微波频率和照射强度有关。设对于 R_{NR} 和其它常规电阻、电感、电容、电源形成的电路，常规 Kirchhoff 电路原理仍然适用，并只考虑系统已到达稳定状态下的解。）

a) We start with simple systems. 我们从简单的系统开始。

(i) When R_{NR} is in series with a normal resistor with resistance R, find the DC current and the voltage drop across R and R_{NR}. (Consider both cases when $R > R_0$ and $R < R_0$.) **(2 points)**

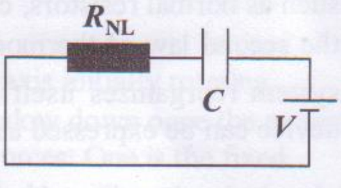


R_{NR}和一常规电阻R串连，求电路的直流电流和R、R_{NR}上的电压。(须考虑$R > R_0$ 和 $R < R_0$这两种情况。) **(2 分)**

(ii) When R_{NR} is in series with a normal inductor with inductance L, find the DC current and the voltage drop across L and R_{NR}. **(1 point)**

R_{NR}和一常规电感L串连，求电路的直流电流和L、R_{NR}上的电压。**(1 分)**

(iii) When R_{NR} is in series with a normal capacitance C and a DC voltage source V, find the minimum voltage V needed to make $R_{sys} = R_{NR} = 0$. Also find the DC current and the voltage drop across C and R_{NR}. (Hint: The charge q on the capacitor is the charge stored in the system that determines the value of R_{NR}.) **(2 points)**

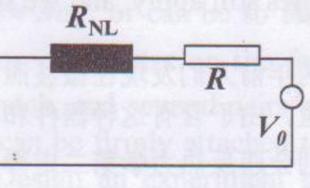


R_{NR} 和一常规电容 C、直流电压源 V 串连。求使 $R_{sys}=R_{NR}=0$ 所需的最小电压，并求这时的直流电流和 C 、R_{NR} 上的电压。（提示：电容上的电量 q 就是决定 R_{NR} 的储存于系统的电荷量）**(2 分)**

b) Now a small AC or DC voltage source V_0 is added in the above circuits. In the following calculations of the contribution of V_0 to the additional current and/or charge on top of the original ones, keep only their first order. In the cases when the small voltage source is AC, you should set up differential equations first.

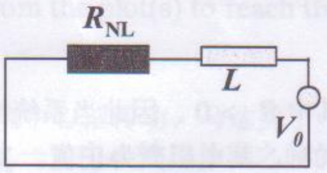
现在以上的电路中加一直流或交流的小电源 V_0。在以下的计算由 V_0 所带来的额外电流或电荷的过程中，只需保留它们的一次项。当小电源为交流时，先建立微分方程。

(i) When R_{NR} is in series with a normal resistor with resistance R and a small DC voltage source V_0, find the DC current. You should consider both the cases $R>R_0$ and $R<R_0$, separately. **(3 points)**



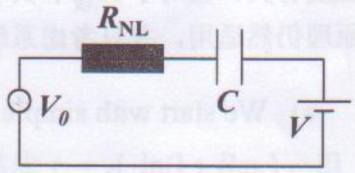
当 R_{NR} 和一常规电阻 R、小直流电压源 V_0 串连，求直流电流。（分别考虑 $R>R_0$，和 $R<R_0$ 这二种情况。）**(3 分)**

(ii) When R_{NR} is in series with a normal inductor with inductance L and a small AC voltage source with amplitude V_0 and frequency ω, find the current through L and R_{NR}. **(3 points)**



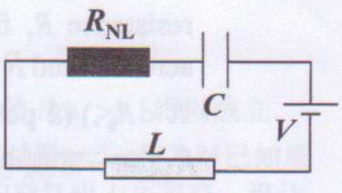
当 R_{NR} 和一常规电感 L、频率为 ω 幅度为 V_0 的小交流电压源串连，求电流。**(3 分)**

(iii) A small AC voltage source with amplitude V_0 and frequency ω is added to the circuit in part-a(iii), find the charge on the capacitor. **(2 points)**



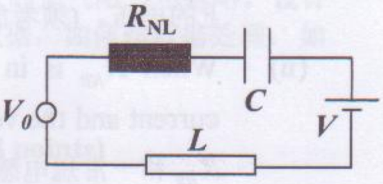
a(iii)部分的电路里加上频率为ω幅度为 V_0 的小交流电压源，求电容上电荷。**(2 分)**

(iv) Consider an LCR circuit with R_{NR} as the resistor in the presence of DC voltage source V, find the current in the circuit. **(2 points)**



在 LCR 电路中将常规电阻用 R_{NR} 代替，并加上直流电压源 V，求直流电流。**(2 分)**

(v) A small AC voltage source with amplitude V_0 and frequency ω is added to the circuit in (iv). Find the current in the circuit. **(3 points)**



在以上(iv)的电路中加上频率为ω幅度为 V_0 的小交流电压源，求交流电流。**(3 分)**

THE END 完