Q1: (4 points)

In the process energy and momentum are conserved, so

碰撞前后动量和能量守恒，因此

$m_A v_0 = m_B v_{Bx} \quad 0 = m_B v_{By} + m_A v_A$ (1 **point**)

$m_A v_0^2 = m_A v_A^2 + m_B v_{Bx}^2 + m_B v_{By}^2 + 2E$ (1 **point**)

Where, v_A is in the y-direction, we can get 其中v_A是沿 Y-方向的速度。由此得

$v_A = \sqrt{\dfrac{m_A(m_B - m_A)v_0^2 - 2Em_B}{m_A(m_B + m_A)}}$, (1 **point**)

$v_{Bx} = \dfrac{m_A}{m_B} v_0$, $v_{By} = -\dfrac{m_A}{m_B} v_A$. (1 **point**)

Q2: (6 **points**)

From Gauss theorem, the area density of the free charge at the surfaces of the conductor plates is $\sigma_f = D$. (0.5 **points**) 根据高斯定理，导电板上自由电荷面密度与板间电位移的关系为$\sigma_f = D$。

In the air gap $E_1 = E_3 = D/\varepsilon_0$, while in the dielectric $E_2 = D/\varepsilon\varepsilon_0$. (0.5 **points**)

在空气中$E_1 = E_3 = D/\varepsilon_0$，在介质内$E_2 = D/\varepsilon\varepsilon_0$，电压为

$$V = \frac{d}{3}E_1 + \frac{d}{3}E_2 + \frac{d}{3}E_3 = \frac{dD}{3\varepsilon_0}\left(2 + \frac{1}{\varepsilon}\right) = \frac{d\sigma_f}{3\varepsilon_0}\left(2 + \frac{1}{\varepsilon}\right),$$

由此得

$\sigma_f = \dfrac{V}{d}\dfrac{3\varepsilon_0\varepsilon}{2\varepsilon + 1}$. (1 **point**)

The bound charge is $\sigma_b = \mp\varepsilon_0(E_1 - E_2) = \mp\dfrac{3V}{d}\dfrac{\varepsilon_0(\varepsilon - 1)}{2\varepsilon + 1}$. (1 **point**)

介质上、下界面的束缚电荷面密度为$\sigma_b = \mp\varepsilon_0(E_1 - E_2) = \mp\dfrac{3V}{d}\dfrac{\varepsilon_0(\varepsilon - 1)}{2\varepsilon + 1}$。

When the slab is moving with speed *v*, the electric currents at two sides are $K = \pm\sigma_b v$. From Ampere law, $B_1 = B_3 = 0$ (1 **point**)

介质板运动时，上、下界面的束缚电流面密度为$K = \pm\sigma_b v$。由安培定理得在空隙间的磁场$B_1 = B_3 = 0$

Inside the dielectric slab the magnetic field is 在介质板里的磁场

$B_2 = \mu_0 K = \mu_0\sigma_b v = \mu_0 v\dfrac{3V}{d}\dfrac{\varepsilon_0(\varepsilon - 1)}{2\varepsilon + 1}$. (1 **point**)

When the parallel conductor plates are moving with speed -*v*, the electric currents at two plates are $K' = \mp\sigma_f v = \mp v\dfrac{V}{d}\dfrac{3\varepsilon_0\varepsilon}{2\varepsilon + 1}$. From the Ampere law, $B_1 = B_2 = B_3 = -\mu_0\sigma_f v$. (1 **point**)

当导电板运动时，上、下板的面电流密度为$K' = \mp\sigma_f v = \mp v\dfrac{V}{d}\dfrac{3\varepsilon_0\varepsilon}{2\varepsilon + 1}$。

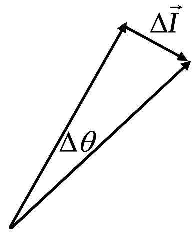
由安培定理得 $B_1 = B_2 = B_3 = -\mu_0\sigma_f v$

Q3: (7 points)

The electron spin (angular momentum) is I, and the associated magnetic dipole moment is $\vec{M}=-\frac{ge}{2m}\vec{I}$.

In the B-field, the torque on the spin is $\vec{M}\times\vec{B}$ and perpendicular to $\vec{M}$. (1 **point**)

The procession frequency is then determined by $MB\Delta t=\Delta I$, (1 **point**)



$MB\Delta t=I\Delta\theta$ (1 **point**)

$\Rightarrow MB=I\frac{\Delta\theta}{\Delta t}=I\omega$ (1 **point**)

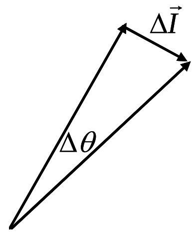
which leads to $\omega=\frac{ge}{2m}B$. (1 **point**)

The negative sign in $\vec{M}=-\frac{ge}{2m}\vec{I}$ ensures that the spin turns in the same direction as the electron trajectory under Lorentz force. $\frac{mv^2}{R}=eBv\Rightarrow\omega'=\frac{eB}{m_e}$ (2 **points**)

电子的自旋（角动量）为 I，与其相关的磁偶极子为 $\vec{M}=-\frac{ge}{2m}\vec{I}$.

在磁场里磁偶极子受的力矩为 $\vec{M}\times\vec{B}$ 与 $\vec{M}$ 垂直。 (1 **point**)

$\vec{M}$ 进动的频率由下式可求：



$MB\Delta t=\Delta I$, (1 **point**)

$MB\Delta t=I\Delta\theta$ (1 **point**)

$\Rightarrow MB=I\frac{\Delta\theta}{\Delta t}=I\omega$ (1 **point**)

得 $\omega=\frac{ge}{2m}B$. (1 **point**)

空间的圆轨迹为：$\frac{mv^2}{R}=eBv\Rightarrow\omega'=\frac{eB}{m_e}$ (2 **points**)

$\vec{M}=-\frac{ge}{2m}\vec{I}$ 中的负号保证了自旋的旋转方向与它在空间的圆轨迹一致。

Q4 (12 points)

(a) Consider a thin layer of air at rest, the pressure difference balances the gravity,
取一薄层空气，上、下面上的压强差刚好与重力平衡，

$$p(h+\Delta h)-p(h)=-m\rho g\Delta h$$

$$\Rightarrow\frac{dp}{dh}=-m\rho g$$ (1 **point**)

(b) Put in the ideal gas law $p=\rho RT$, the differential equation is then
代入理想气体方程 $p=\rho RT$，得微分方程

$$\frac{dp}{dh}=-m\rho g=-\frac{mg}{RT}p.$$ (1 **point**)

(c) From (b), 由(b)解得

$$\frac{dp}{p} = -\frac{mg}{RT}dh \Rightarrow \ln p - \ln p_0 = -\frac{mg}{RT}h \Rightarrow p(h) = p_0 e^{-\frac{mg}{RT}h}.$$ (3 **points**)

Put in the numbers, 代入数值，

$$\frac{mg}{RT} = \frac{0.029\times 9.8}{8.31\times 300} = \frac{1}{8.8}km^{-1}.$$

So the height is 得高度为

$8.8\times \ln 2 = 8.8\times 0.693 = 6.1km$. (1 **point**)

(d) With a constant wind with velocity v we replace the pressure equation by the Bernoulli's equation

有风时，微分方程为

$$\frac{dp}{dh} + \frac{mv^2}{2}\frac{d\rho}{dh} = -mg\rho.$$

Using the ideal gas law, we obtain 代入 $p = \rho RT$ 理想气体方程，得

$$\frac{dp}{dh}\left(1+\frac{mv^2}{2RT}\right) = -\frac{mg}{RT}p.$$ (1 **point**)

Therefore 解得 $p(h) = p_0 e^{-\frac{mg}{\left(RT+\frac{mv^2}{2}\right)}h}$. (2 **point**)

(e) $\vec{\Omega}\times\vec{v} \approx \Omega\times v = 2\pi\times 500000/(24\times 60\times 60\times 60\times 60) \approx 0.01\ m/s^2 << g$. (1 **point**)

(f) The decay length in this case is 代入数值

$$h_0 = \ln 2\left(\frac{RT+\frac{mv^2}{2}}{mg}\right) = 0.693\times\frac{8.31\times 300 + 0.5\times 0.029\times 2.5\times 10^4/3.6}{0.029\times 9.8} = 6.1+0.24 = 6.3\ km$$

(2 **points**)

Q5 (11 points):

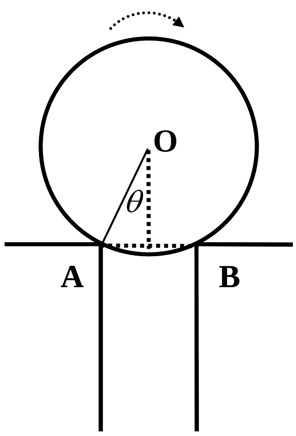
(a) When the ball arrives at point A, it begins to drop down. In this process the potential energy transforms into kinetic energy.

球到 A 点后下落，到 B 点时，势能变化为

$\Delta P = MgR(1-\cos\theta)$ (1 **point**)

The moment of inertia about the edge is 绕球边的转动惯量为

$$\tilde{I} = I + MR^2 = \frac{7}{5}MR^2.$$



Kinetic energy is 动能为

$$K_1 = \frac{1}{2}\tilde{I}\omega^2 = \frac{1}{2}\times\frac{7}{5}MR^2\omega^2 = \frac{7}{10}Mv^2$$ (0.5 **points**)

So 因此 $K_2 = \frac{7}{10}Mv'^2 = MgR(1-\cos\theta) + \frac{7}{10}Mv^2$ (0.5 **points**)

$$\Rightarrow v'^2 = \frac{10}{7}gR(1-\cos\theta) + v^2$$

$$\Rightarrow \omega'^2 = \frac{10}{7R} g(1-\cos\theta) + \left(\frac{v}{R}\right)^2 \quad \text{(1 point)}$$

(b) The critical condition for the ball to keep contact with point A before it touches point B is that: At the moment it touches point B, the centrifugal force equals the gravity component.
要保持与 A 点接触，即球以 A 点作圆周运动，其向心力全由重力提供。

$$\frac{Mv'^2}{R} = Mg\cos\theta \quad \text{(1 point)}$$

$$\Rightarrow \frac{10}{7} gR(1-\cos\theta) + v_{\max}^2 = gR\cos\theta \quad \Rightarrow v_{\max}^2 = \frac{gR}{7}(17\cos\theta - 10) \qquad \text{(1 point)}$$

where $\cos\theta = \dfrac{\sqrt{R^2 - \dfrac{d^2}{4}}}{R} = \sqrt{1 - \dfrac{d^2}{4R^2}}$

(c) In the process of the ball collide with point B, the angular momentum of the ball around point B is unchanged. Before the collision, the angular momentum of the ball around point B is $I\omega' + MR\tilde{v}' = I\omega' + MR(R\omega'\cos 2\theta) = MR^2\omega'\left(\frac{2}{5} + \cos 2\theta\right)$.

After the collision, it is $\tilde{I}\omega'' = \frac{7}{5} MR^2\omega''$.

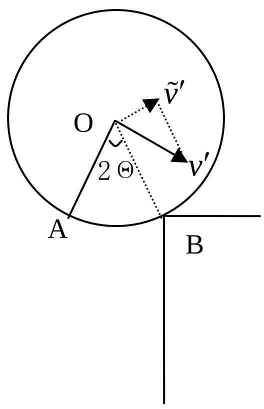
在与 B 点碰撞过程中，球相对于该点的角动量守恒。碰撞前的角动量为 $I\omega' + MR\tilde{v}' = I\omega' + MR(R\omega'\cos 2\theta) = MR^2\omega'\left(\frac{2}{5} + \cos 2\theta\right)$，碰撞后的角动量为 $\tilde{I}\omega'' = \frac{7}{5} MR^2\omega''$。

Hence 因此

$$\frac{7}{5} MR^2\omega'' = MR^2\omega'\left(\frac{2}{5} + \cos 2\theta\right) \Rightarrow \omega'' = \frac{2 + 5\cos 2\theta}{7}\omega' . \text{ (2 points)}$$

So the total energy after collision is 碰撞后的动能为

$$K_3 = \frac{7}{10} MR^2\omega''^2 = \frac{1}{70}(2 + 5\cos 2\theta)^2 Mv'^2 \quad \text{(1 point)}$$



The requirement for the ball to get over the ditch:
要翻上沟边，动能要克服的势能为

$$K_3 = \frac{1}{70}(2 + 5\cos 2\theta)^2 Mv'^2 > MgR(1-\cos\theta) \quad \text{(1 point)}$$

$$\Rightarrow v^2 > 10gR\left[\frac{7}{(2 + 5\cos 2\theta)^2} - \frac{1}{7}\right](1-\cos\theta)$$

$$\Rightarrow v_{\min}^2 = 10gR\left[\frac{7}{(2 + 5\cos 2\theta)^2} - \frac{1}{7}\right](1-\cos\theta) \text{ (2 points)}$$

(d) We must have 根据题意， $v_{\min}^2 < v_{\max}^2$

既 $\dfrac{1}{10}\cos\theta - \dfrac{7(1-\cos\theta)}{(2 + 5\cos 2\theta)^2} > 0$.

（Numerical result of Max θ: 数值计算得最大角为 0.597797 弧度，或 34°。）

Q6 (10 points)

Suppose the spring is extended, and choose the natural length as the origin of the coordinate of the small block $X_2 = 0$. The Dynamic equation of this system is

设弹簧是拉长的，选弹簧在自然长度时小物块的坐标 $X_2 = 0$。系统的运动方程为

$F + KX_2 = M_1\ddot{X}_1$ (1) (1 **point**) $-KX_2 - M_2\ddot{X}_1 = M_2\ddot{X}_2$ (2) (1 **point**)

Note that 由于 $\ddot{X}_{1,2} = -\omega^2 X_{1,2}$, (1 **point**)

From (2) we get 由(2)得 $X_2 = \dfrac{\omega^2 M_2}{K - \omega^2 M_2} X_1$. (1 **point**)

Then 因此 $F = -M_1\omega^2 X_1 - K\dfrac{\omega^2 M_2}{K-\omega^2 M_2}X_1 = -\omega^2 X_1(M_1 + \dfrac{KM_2}{K-\omega^2 M_2})$ (2 **points**)

(a) Finally 最后得 $M_{eff} = \dfrac{F}{-\omega^2 X_1} = M_1 + \dfrac{KM_2}{K-\omega^2 M_2} = M_1 - \dfrac{KM_2}{\omega^2 M_2 - K}$ (1 **point**)

(b) Negative effective mass 负有效质量:

For negative M_{eff}, we get, after some algebra, $\omega^2 < K(\dfrac{1}{M_1} + \dfrac{1}{M_2})$. However, the term $\dfrac{KM_2}{\omega^2 M_2 - K}$ must be positive. So the final answer is

$\dfrac{K}{M_2} < \omega^2 < K(\dfrac{1}{M_1} + \dfrac{1}{M_2})$ (3 **points**) Missing $\dfrac{K}{M_2} < \omega^2$, (-1 **point**)

要使 $M_{eff} < 0$，经过简单代数运算，得 $\omega^2 < K(\dfrac{1}{M_1} + \dfrac{1}{M_2})$。但是 $\dfrac{KM_2}{\omega^2 M_2 - K}$ 必须是正的。因此得

$\dfrac{K}{M_2} < \omega^2 < K(\dfrac{1}{M_1} + \dfrac{1}{M_2})$ (3 **points**) 漏掉 $\dfrac{K}{M_2} < \omega^2$, (-1 **point**)

Part-II

Q1 (16 points):

(a) By Lorentz force law, $\mathbf{F} = -q\mathbf{v}\times\mathbf{B}$, F has no z-component when $\mathbf{B}$ is along the z-direction. Hence if $v_z = 0$ initially, it always remains zero. (1 **point**)

磁场的力为 $\mathbf{F} = -q\mathbf{v}\times\mathbf{B}$，与 XY 面平行。由于初速度的 Z-分量 $v_z = 0$，所以粒子保持在 XY 面上运动。

(b) The B-field at $r = a$ is just right to keep the particle on a circular orbit of radius a.

在 $r = a$ 处的磁场刚好可以维持粒子以 a 为半径的圆周运动。

$$\frac{mv_0^2}{a} = qv_0B_0 \Rightarrow v_0 = \frac{qB_0a}{m}$$. (2 **points**)

(c) Let the angular momentum of the charge about the origin be L. Then $L(t=0) = mav_0$.

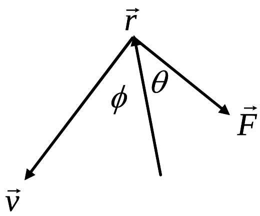
令粒子绕原点的角动量为 L. 则 $L(t=0) = mav_0$。

$$\frac{dL}{dt} = -rF\sin\theta \quad \text{(1 point)}$$

$$= -qrvB\cos\phi = -qrBv_r \quad \text{(1 point)}.$$ $v_r = \frac{dr}{dt}$ is the radial velocity 径向速度分量 $v_r = \frac{dr}{dt}$.

$$= qBr\frac{dr}{dt} \quad \text{(1 point)}$$

$$\Rightarrow \frac{dL}{dr} = qBr \quad \text{(1 point)}$$



One can also obtain the same differential equation by

也可用下列公式

$$\frac{d\mathbf{L}}{dt} = \mathbf{r}\times\mathbf{F} = -q\mathbf{r}\times(\mathbf{v}\times\mathbf{B}) = -q\left[(\mathbf{r}\cdot\mathbf{B})\mathbf{v} - (\mathbf{r}\cdot\mathbf{v})\mathbf{B}\right] = qrv_r\mathbf{B} = qr\frac{dr}{dt}\mathbf{B} \quad \text{(2 points)}$$

$$\frac{dL}{dt} = qBr\frac{dr}{dt} \quad \text{(1 point)}$$

$$\Rightarrow \frac{dL}{dr} = qBr \quad \text{(1 point)}$$

$$L(r) - L(a) = \int_a^r qBrdr = qB_0a(r-a).$$ (1 **point**)

(d) Because B field does not work, the speed of the charge is always v_0. Note that the angular momentum can be $L = \pm mrv_0$

由于磁场不做功，粒子的速率一直为 v_0。注意角动量可以是 $L = \pm mrv_0$。

When $L = mrv_0$, $r = a$, no tangential motion occurs afterwards. (1 **point**)

当 $L = mrv_0$， $r = a$。这只有在初始时有，之后就不再出现了。

When 当 $L = -mrv_0$， $r = a\dfrac{qB_0a - mv_0}{qB_0a + mv_0}$. (1 **point**)

When $qB_0a < mv_0$, the only solution is $r = a$, which corresponds to the initial condition. No tangential motion occurs afterwards. (1 **point**)

当 $qB_0a < mv_0$， 只有初始的 $r = a$, 之后就不再出现了。

当 $qB_0 a > mv_0$, (1 **point**) $r = a\dfrac{qB_0 a - mv_0}{qB_0 a + mv_0} < a$. (1 **point**)

It is possible. 这是个可以出现的情形。

(e) When the charge is moving in the radial direction, L = 0. (1 **point**)
粒子沿径向运动时，L = 0。
Therefore, 因此

$$r = a\left(1 - \frac{mv_0}{qB_0 a}\right). \qquad (1\ \textbf{point})$$

When $qB_0 a > mv_0$, r > 0, the motion can be radial. (0.5 **points**)
当 $qB_0 a > mv_0$, r > 0, 粒子可沿径向运动
When $qB_0 a < mv_0$, r < 0, the motion never becomes radial. (0.5 **points**)
当 $qB_0 a < mv_0$, r < 0, 粒子不可沿径向运动。

Q2 (16 points)

(A.1) The differential equation 微分方程为: $-I\dot{\omega} = \tau + \gamma\omega$ (2 **points**)
(A.2)

$$\frac{d\omega}{\omega + \frac{\tau}{\gamma}} = -\frac{\gamma}{I}dt \Rightarrow \int_{\omega_0}^{0} \frac{d\omega}{\omega + \frac{\tau}{\gamma}} = -\frac{\gamma}{I}\int_0^{t_s} dt \Rightarrow \ln\frac{\frac{\gamma}{\tau}}{\omega_0 + \frac{\tau}{\gamma}} = -\frac{\gamma}{I}t_s \Rightarrow t_s = \frac{I}{\gamma}\ln(1+\frac{\gamma\omega_0}{\tau})\ (2\ \textbf{points})$$

So $A = \dfrac{I}{\gamma}$, (1 **point**); $B = \dfrac{\gamma}{\tau}$, (1 **point**)

(B)

- First, add each pair of blocks at equal distance to the center in order to avoid warbling, keep ω_0 fixed and measure stop time t_{sn}

$t_{sn} = \dfrac{I + n\Delta I}{\gamma}\ln(1+\dfrac{\gamma\omega_0}{\tau}) = \dfrac{n\Delta I}{\gamma}\ln(1+\dfrac{\gamma\omega_0}{\tau}) + \dfrac{I}{\gamma}\ln(1+\dfrac{\gamma\omega_0}{\tau})$. Here n = 1, 2, .., and $\Delta I = mr^2$, where m is the mass of the small block, and r is the distance to the center of the fan measured by the ruler. Plot t_{sn} □ $n\Delta I$, one gets the slope $K = \dfrac{1}{\gamma}\ln(1+\dfrac{\gamma\omega_0}{\tau})$ and the interception $b = \dfrac{I}{\gamma}\ln(1+\dfrac{\gamma\omega_0}{\tau})$. Then $I = b/K$. (4 **points**)

- 首先，保持 ω_0 为常数，将每对小重物放在离轴等距离的两边的叶片上，测量停止时间 t_{sn}。

$t_{sn} = \dfrac{I + n\Delta I}{\gamma}\ln(1+\dfrac{\gamma\omega_0}{\tau}) = \dfrac{n\Delta I}{\gamma}\ln(1+\dfrac{\gamma\omega_0}{\tau}) + \dfrac{I}{\gamma}\ln(1+\dfrac{\gamma\omega_0}{\tau})$，其中 n = 1, 2, .., $\Delta I = mr^2$，m 是每块小重物的质量，r 是小重物离轴的距离。将 t_{sn} □ $n\Delta I$ 作图，得直线的斜率 $K = \dfrac{1}{\gamma}\ln(1+\dfrac{\gamma\omega_0}{\tau})$，与 Y 轴的交点 $b = \dfrac{I}{\gamma}\ln(1+\dfrac{\gamma\omega_0}{\tau})$。求得转动惯量 $I = b/K$。

- Secondly, let the initial angular ω_0 be very slow such that

$t_s = A\ln(1+B\omega_0) \approx AB\omega_0 = \dfrac{I}{\tau}\omega_0$.

From the slope of the $t_s \sim \omega_0$ line we can get the value of τ. (3 **points**)

第二，将初始转速调小，使 $t_s = A\ln(1+B\omega_0) \approx AB\omega_0 = \dfrac{I}{\tau}\omega_0$ 成立。测 $t_s \sim \omega_0$ 并作图。直线的斜率为 $\dfrac{I}{\tau}$。由此得 τ。

- Finally, let the initial angular ω_0 be very fast so that

$$t_s = A\ln(1+B\omega_0) \approx A\ln(B\omega_0) = \frac{I}{\gamma}[\ln(\frac{\gamma}{\tau}) + \ln(\omega_0)].$$

Change ω_0 and plot $t_s \sim \ln(\omega_0)$ which forms a straight line. The slope is I/γ. Since I is already known, γ can be readily obtained. (3 **points**)

最后，将初始转速调大，使 $t_s = A\ln(1+B\omega_0) \approx A\ln(B\omega_0) = \dfrac{I}{\gamma}[\ln(\dfrac{\gamma}{\tau}) + \ln(\omega_0)]$ 成立。测 $t_s \sim \ln(\omega_0)$ 并作图。直线的斜率为 I/γ。由此得 γ。

Q3 (18 points)

Ans:

a) (i) For $R < R_0$ there must be a constant current i is flowing in the system to increase R_{NR} so the total resistance of the system will not be negative.

当 $R < R_0$，需有电流使 R_{NR} 增加，使系统的总电阻不为负。

$$\left(R - R_0\left(1 - \left(\frac{i}{i_o}\right)^2\right)\right)i = 0$$ (Kirchhoff's law) (0.5 **points**)

$$\Rightarrow i = \pm\sqrt{\frac{R_0 - R}{R_0}}\,i_o$$ (0.5 **points**)

Voltage drop across $R_{NR} = -R_0\left(1 - \left(\dfrac{i}{i_o}\right)^2\right)i = -Ri$ = - voltage drop across R. (0.5 **points**)

电压为 $iR_{NR} = -R_0\left(1 - \left(\dfrac{i}{i_o}\right)^2\right)i = -Ri$

For $R > R_0$, the total resistance > 0, so the solution is i = 0. There is no voltage drop. (0.5 **points**)

若 $R > R_0$，则系统的总电阻为正，电流为零。

(ii) L has no resistance. So a constant current $i = \pm i_o$ flows through the system and $R_{NR} = 0$. There is no voltage drop anywhere. (1 **point**)

电感无直流电阻，所以 $R_{NR} = 0$。电流为 $i = \pm i_o$，但无电压。

(iii) In this case i = 0 and charge $q = \pm q_o$ is needed to maintain the circuit in equilibrium (R_{NR} = 0). (1 **point**)
Therefore a minimum voltage $V_o = \pm q_o / C$ is needed to maintain the system at equilibrium. There is no voltage drop across R_{NR}. (1 **point**)
直流电流 i = 0。电容上的电荷为 $q = \pm q_o$，R_{NR} = 0。所需最小电压为 $V_o = \pm q_o / C$。R_{NR} 上无电压。

b) (i) For $R < R_0$ the Kirchhoff's Law becomes 当 $R < R_0$，有

$$\left(R - R_0\left(1 - \left(\frac{i}{i_o}\right)^2\right)\right) i = V_0$$

Writing $i = \sqrt{\frac{R_0 - R}{R_0}} i_o + j = i' + j$ where j is small,

代入 $i = \sqrt{\frac{R_0 - R}{R_0}} i_o + j = i' + j$，其中 j 为一级小量，

we obtain to linear order in j 保持 j 的一级小量，得

$$V_0 = \left[R - R_0\left(1 - \left(\frac{i' + j}{i_o}\right)^2\right)\right](i' + j) \approx 2R_0 \sqrt{\frac{R_0 - R}{R_0}} \frac{j}{i_0}(i' + j) \approx 2(R_0 - R) j,$$ (1 **point**)

so $j = \frac{V_0}{2(R_0 - R)}$ for both AC and DC. (1 **point**)

最后的（无论是 AC 或 DC）$j = \frac{V_0}{2(R_0 - R)}$。

For $R > R_0$, the original current is zero. $R_0\left(\left(\frac{j}{i_o}\right)^2 - 1\right) \approx -R_0$.

So we obtain $j = \frac{V_0}{(R - R_0)}$. (1 **point**)

当 $R > R_0$，原来的电流为零。因此 $R_0\left(\left(\frac{j}{i_o}\right)^2 - 1\right) \approx -R_0$，得 $j = \frac{V_0}{(R - R_0)}$。

<<Notice that for $R = R_0$, we obtain 当 $R = R_0$ 时，得

$$R_0\left(\frac{j}{i_o}\right)^2 j = V \Rightarrow j = \left(\frac{V}{R_0} i_o^2\right)^{\frac{1}{3}}.$$ >>

(ii) In this case we obtain 在此情形，我们有

$$-R_0\left(1-\left(\frac{i_o+j}{i_o}\right)^2\right)(i_o+j)+L\frac{d(i_o+j)}{dt}=V$$, (1 **point**)

Note that 但是 $\frac{di_0}{dt}=0$.

$$-R_0\left(1-\left(\frac{i_o+j}{i_o}\right)^2\right)(i_o+j)=R_0\left(1+\frac{2j}{i_o}-1\right)(i_o+j)=2R_0 j$$

We then obtain 由此得微分方程 $2R_0 j+L\frac{dj}{dt}=V$. (1 **point**)

For AC voltage $V(t)=V_0\sin\omega t$, the solution of this equation is $j=j_o\sin(\omega t+\delta)$ where $\tan\delta=-\frac{L\omega}{2R_o}$ and $j_o=\frac{V_0}{2R_o\cos\delta-L\omega\sin\delta}$. (1 **point**)

(Or $\frac{dj}{dt}=-i\omega j$, and $j_o=\frac{V_0}{2R_o-iL\omega}$)

令 $V(t)=V_0\sin\omega t$，解为 $j=j_o\sin(\omega t+\delta)$，代入微分方程得 $\tan\delta=-\frac{L\omega}{2R_o}$，$j_o=\frac{V_0}{2R_o\cos\delta-L\omega\sin\delta}$。

（或用 $\frac{dj}{dt}=-i\omega j$，代入微分方程得 $j_o=\frac{V_0}{2R_o-iL\omega}$。两种解等价。）

(iii) For small additional voltage source there is additional small amount of charge *q'*. The equation becomes

有小电源时，原来的电荷会增加一小量 *q'*，原来的方程变为

$$-R_0\left(1-\left(\frac{q_o+q'}{q_o}\right)^2\right)\left(\frac{d(q_o+q')}{dt}\right)+\frac{q_o+q'}{C}=V+V' \qquad \text{(1 point)}$$

Note that $\frac{dq_o}{dt}=0$, and $\frac{q_o}{C}=V$. The above equation then becomes

$R_0(1+\frac{2q'}{q_0}-1)\frac{dq'}{dt}+\frac{q'}{C}=V'\Rightarrow\frac{q'}{C}=V'$.

It is true for both AC and DC. (1 **point**)

由于 $\frac{dq_o}{dt}=0$，$\frac{q_o}{C}=V$，上述方程简化成 $R_0(1+\frac{2q'}{q_0}-1)\frac{dq'}{dt}+\frac{q'}{C}=V'\Rightarrow\frac{q'}{C}=V'$。

AC 或 DC 都适用。

(iv) For DC voltage at equilibrium $i=0$ and the situation is same as (b(ii)). A minimum voltage $V_o=\pm q_o/C$ is needed to maintain the system at equilibrium. (2 **points**)

和(b(ii))相同。$V_o = \pm q_o / C$

(v) In the presence of an additional AC voltage, Kirchhoff's Law becomes

多一个 AC 电源，方程为

$$R_0\left(\left(\frac{q_o+q'}{q_o}\right)^2+i_o^{-2}\left(\frac{d(q_o+q')}{dt}\right)^2-1\right)\left(\frac{d(q_o+q')}{dt}\right)+\frac{q_o+q'}{C}+L\frac{d^2(q_o+q')}{dt^2}$$ (1 **point**)

$$=V+V'$$

Note $\frac{q_o}{C}=V$, and $\frac{dq_o}{dt}=0$, the first term becomes

由于$\frac{q_o}{C}=V$，$\frac{dq_o}{dt}=0$，上述方程第一项简化成

$$R_0\left(\left(\frac{q_o+q'}{q_o}\right)^2+i_o^{-2}\left(\frac{d(q_o+q')}{dt}\right)^2-1\right)\left(\frac{d(q_o+q')}{dt}\right)$$

$$=R_0\left(\frac{2q'}{q_o}+i_o^{-2}\left(\frac{dq'}{dt}\right)^2\right)\left(\frac{dq'}{dt}\right)=0$$

So we obtain 最终得 $\frac{q'}{C}+L\frac{d^2q'}{dt^2}=V'$, (1 **point**)

Therefore, for $V'(t)=V_0\sin\omega t$. We obtain $q'(t)=\frac{V_0}{\left(C^{-1}-L\omega^2\right)}\sin\omega t$. (0.5 **points**)

And $j(t)=\frac{\omega V_0}{\left(C^{-1}-L\omega^2\right)}\cos\omega t$ (0.5 **points**)

用交流形式解代入，得 $q'(t)=\frac{V_0}{\left(C^{-1}-L\omega^2\right)}\sin\omega t$，$j(t)=\frac{\omega V_0}{\left(C^{-1}-L\omega^2\right)}\cos\omega t$ 。

Or 或

$$<< \quad q'=\frac{V_0}{\left(C^{-1}-L\omega^2\right)},\quad j=\frac{dq'}{dt}=\frac{-i\omega V_0}{\left(C^{-1}-L\omega^2\right)} >>$$