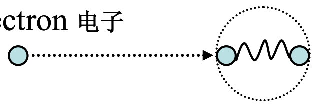
Pan Pearl River Delta Physics Olympiad 2006
2006 年泛珠江三角物理競賽
Part-1 卷-1
(9:00 am – 12:00 pm, 02-09-2006)

Q1 (8 points) 題 1 (8 分)

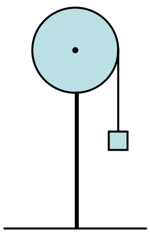
A meson is composed of two quarks with rather complicated interaction between them. To learn the physics inside the meson high energy electrons are usually employed to collide inelastically with the mesons. The process is very difficult to interpret, and to grasp the essential elements of it the Parton model was developed. The model assumes that the incoming electron collides elastically with only part of the meson, i. e., with only one quark. The quark then passes its energy to its parent meson. The essence of the model can be illustrated with the following drastically simplified model. The electron of mass m_1 and kinetic energy E collides elastically with a quark of mass m_2 in a meson. The other quark has mass m_3. The two quarks are connected by a weightless spring. Before the collision the two quarks are at rest and the spring is at its natural length. All motions are along one dimension only, and ignore all relativistic effects. Find the internal energy of the meson as represented by the vibration energy of the spring, and the kinetic energy of the meson as a whole after the collision.



介子由兩個誇克構成，而誇克之間的相互作用相當複雜。研究介子可通過用高能電子與之作非彈性碰撞來進行。由於碰撞過程難於分析，爲掌握其主要內涵，人們發展了一種簡化了的‘分粒子’模型。其主要內容爲：電子只和介子的某部分（比如其中一個誇克）作彈性碰撞。碰撞後的誇克再經過介子內的相互作用把能量和動量傳給整個介子。模型的主要精神可用下面的簡化模型來闡述：一電子質量爲 m_1，動能爲 E，與介子的一個誇克（質量 m_2）作彈性碰撞。介子裏另一個誇克的質量爲 m_3。誇克間以一無質量彈簧相連。碰撞前誇克處於靜止狀態，彈簧處於自然長度。所有運動都是一維的。忽略一切相對論效應。求碰撞後以彈簧振動形式代表的介子內能，和介子作爲一整體所具有的動能。

Q2 (8 points) 題 2 (8 分)

A uniform iron pulley of total mass M_1 and radius R is fixed above the ground. It can spin freely about its center. Additional weights in the form of many small magnets with total mass M_2 can be attached to the pulley. A long thread is winding around its edge, and at its end hangs a weight of mass M_3 at height H from the ground that can be released suddenly. Assume that there is no slip between the thread and the pulley. (a) How should the magnets be placed with circular symmetry on the pulley such that the final speed of M_3 is minimum when it reaches the ground? (b) Find the minimum speed.



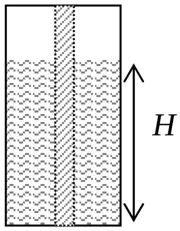
一均勻鐵滑輪質量爲 M_1，半徑爲 R，固定在地面上方。滑輪可以輪心自由轉動。另有總重量爲 M_2 的許多小磁鐵塊可吸附在滑輪上。一長細繩繞著滑輪邊緣，終端掛一質量爲 M_3 的小重塊，離地高度爲 H。細繩與滑輪間無滑動。(a)問小磁鐵塊應怎樣圓對稱地分佈在滑輪上，才能使小重塊被放開後到達地面時的速度爲最小？(b) 求該速度。

Q3 (10 points) 題 3 (10 分)

A cylindrical vessel of radius R is filled with water (density ρ) up to height H.

(a) Calculate the force of water exerting on a narrow vertical strip of width d ($<< R < H$) of the cylinder wall. (3 points)

(b) The vessel is then placed on a platform spinning at angular speed ω ($<\sqrt{gh}/R$). The rotation axis coincides with the central axis of the vessel. The water spins at the same angular speed as the platform. Find the shape of the water surface, and the extra force of water exerting on the strip due to spinning. (g is the gravity acceleration.) (7 points)



一圓柱容器半徑爲 R，裝有高度達 H 的水（密度爲ρ）。

(a) 求水對容器面上一寬度爲 d ($<< R < H$) 的豎直面的壓力。(3 分)

(b) 將容器放在以角速度ω ($<\sqrt{gh}/R$) 轉動的轉臺上，轉動軸與容器中心軸重合。水的轉動和轉臺一致。求水面形狀和水對豎直面因轉動而帶來的附加壓力。(g 爲重力加速度。) (7 分)

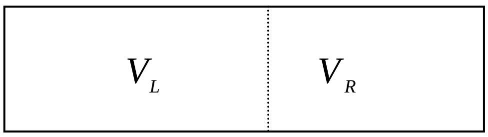
Q4 (12 points) 題 4 (12 分)

A cylinder of fixed volume V is partitioned into left and right compartments separated by a movable piston of negligible mass. The left and right compartments are filled with n_L and n_R moles of the same ideal gas, respectively, at initial temperatures T_L and T_R.

a) What are the volumes of the left and right compartments V_L and V_R initially when the system is at equilibrium? (2 points)

b) Suppose heat can be exchanged between the two compartments but there is no heat exchange between the cylinder and outside world. The amount of heat flow per unit time $\frac{\Delta Q_{L\to R}}{\Delta t}$ from left to right compartment is proportional to the temperature difference between the two compartments, i.e. $\frac{\Delta Q_{L\to R}}{\Delta t} = k(T_L - T_R)$. Find the temperature difference, $T_L - T_R$ as a function of time. (8 points)

c) Find out how V_L and V_R changes with time. (2 points)



一固定體積爲 V 的汽室中間由一可左右無磨擦滑動的輕活塞分開。左 、右汽室分別充有 n_L 、n_R 摩爾初始溫度分別爲 T_L 、 T_R 的相同理想氣體。

a) 求平衡時左 、右汽室的體積 V_L 、V_R 。(2 分)

b) 假設左 、右汽室之間可以作熱交換，但與外界無熱量交換，單位時間熱量的交換 $\frac{\Delta Q_{L\to R}}{\Delta t}$ 與兩邊的溫

差成正比，即 $\dfrac{\Delta Q_{L\to R}}{\Delta t} = k(T_L - T_R)$。求 $T_L - T_R$ 與時間的關係。(8 分)

c) 求 V_L 、V_R 與時間的關係。(2 分)

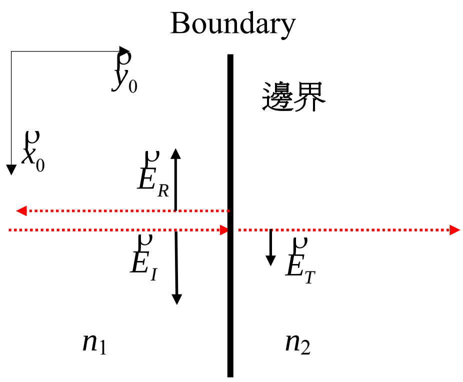
Q5 (12 points) 題 5 (12 分)

A planar boundary at $y = 0$ separates two non-magnetic media with refractive indices n_1 (real) and n_2 (can be complex). An electromagnetic wave $\vec{E}_I = E_0 \hat{x}_0 e^{i(k_1 y - \omega t)}$ is incident onto the boundary, and its reflected wave and transmitted wave are given by $\vec{E}_R = E_r \hat{x}_0 e^{i(-k_1 y - \omega t)}$ and $\vec{E}_T = E_t \hat{x}_0 e^{i(k_2 y - \omega t)}$, respectively.

(a) Use the boundary conditions of electric and magnetic fields to find $r \equiv E_r / E_0$ and $tr \equiv E_t / E_0$ in terms of n_1 and n_2. (8 points)

(b) For $n_1 = 1$, $n_2 = 2.0i$, where $i \equiv \sqrt{-1}$, find the reflection $R \equiv |r|^2$. (1 points)

(c) For $n_1 = 1$, $n_2 = 1.0 + 2.0i$, find the phase shift of the reflected wave relative to the incidence wave. (3 points)



一位於 $y = 0$ 的平面的左、右分別是折射率爲 n_1 (實數)、n_2 (可能是複數)的非磁性介質。一電磁波 $\vec{E}_I = E_0 \hat{x}_0 e^{i(k_1 y - \omega t)}$ 射向介面，其反射波爲 $\vec{E}_R = E_r \hat{x}_0 e^{i(-k_1 y - \omega t)}$，透射波爲 $\vec{E}_T = E_t \hat{x}_0 e^{i(k_2 y - \omega t)}$。

(a) 利用電磁場的邊界條件，導出 $r \equiv E_r / E_0$，$tr \equiv E_t / E_0$。你的答案應只含 n_1 和 n_2。(8 分)

(b) 當 $n_1 = 1$, $n_2 = 2.0i$ 時，其中 $i \equiv \sqrt{-1}$，求反射率 $R \equiv |r|^2$。(1 分)

(c) 當 $n_1 = 1$, $n_2 = 1.0 + 2.0i$ 時，求反射波相對於入射波的位相差。(3 分)

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Part-2 卷-2
(2:00 pm – 5:00 pm, 02-09-2006)

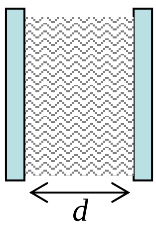
Q6 題 6 Casmir Effect 卡士米爾效應
(13 points **13 分)**

Part-A What is the pressure of ideal gases when the temperature goes to absolute zero? (2 points)

Part-B Casmir proposed in the 1950's that vacuum is actually filled with virtual electromagnetic waves, and the energies stored in the vacuum produces an observable force between two parallel metallic plates separated by distance *d*. The electromagnetic energy stored between the plates is given in Quantum Mechanics by $E = \frac{1}{2}\sum_{n<p} E_n$, where $E_n = \eta c k_n$, *c* is the velocity of light and η is a fundamental constant called Planck's constant, *p* (>> 1) is a large number that depends on the material properties of the plates. $k_n \equiv 2\pi/\lambda_n$, where λ_n is the wavelength of a standing electromagnetic wave that fits into the space between the plates.

(a) Assume that the waves take the form of $\sin k_n x$, what are the possible values of k_n such that at the plate surfaces *x* = 0 and *x* = *d* the standing waves vanish? (2 points)
(b) Show that the vacuum electromagnetic force is of the form a/d^2. What is the constant *a*? (5 points)
(c) What is the force due to the vacuum electromagnetic waves outside the two plates? (2 point)
(d) Find the amplitude of the force when *d* = 1.0 mm. (Hint: If you cannot get *a* in (b), use dimensional analysis to estimate its order of magnitude.) (2 points)

($\eta = 1.05\times10^{-34}$ (Joule∗second); $p = 2000$; $c = 3.0\times10^{8}$ m/s .)



（A）當理想氣體的溫度趨於零時，其壓強爲多少？(2 分)
（B）卡士米爾在 50 年代提出，真空中其實充滿了'虛'電磁波。它們所具有的能量會對一對相隔距離爲 d 的金屬平板產生作用力。 根據量子力學，兩板間儲有的能量爲 $E = \frac{1}{2}\sum_{n<p} E_n$，其中 $E_n = \eta c k_n$，c 爲真空光速，η 爲普朗克常數，p (>> 1)是個很大的數，和金屬板的材料性質有關。 $k_n \equiv 2\pi/\lambda_n$，其中 λ_n 是可在兩板之間形成的駐波的波長。

(a) 設電磁波的波動方程具有 $\sin k_n x$ 這樣的形式，在兩板面 $x = 0$ 和 $x = d$ 處波必須消失，求 k_n 可取的值。(2 分)
(b) 證明真空電磁力具有 a/d^2 這樣的形式，並求常數 a。(5 分)
(c) 板外邊的真空電磁力爲多少? (2 分)
(d) 求當 $d = 1.0$ mm 時力的數值。 (提示: 如果你沒有在（b）部分求得 a，你可用量綱分析來估計力的大小。) (2 分)

($\eta = 1.05\times10^{-34}$ (焦耳*秒); $p = 2000$; $c = 3.0\times10^{8}$ m/s .)

Q7 **題 7** Electric Current Driven Magnetic Domain Movement 電流驅動的磁疇運動
(15 points **15 分)**

Electrons and some atoms have permanent angular momentum called **spins** ($\vec{S}$) that are fixed in amplitude while their directions can be changed by an external torque. Associated with the spins are the magnetic moments that make them as individual magnetic dipoles $\vec{m} = \mu_B \vec{S}$, where μ_B is a constant called magneton. For simplicity, we assume that the electrons and the atoms here have the same spin amplitude S and magneton.

(a) Refer to Figure-A below. Dipole-A is at $\vec{r} = 0$ and along $\hat{z}_0$, and Dipole-B is at $\vec{r} = a\hat{x}_0$ and along $\hat{y}_0$. Find the torque of Dipole-A on Dipole-B. (3 points**)**

(b) Find the direction of Dipole-B after a short time Δt, assuming that the direction of Dipole-A remains unchanged. (5 points**)**

(c) Figure-B is a schematic of a magnetic domain in (x_0, $x_0 + 2b$) that contains many atoms (Dipole-B's). The directions of the dipoles are pointed in an ordered fashion, as shown. Dipole-B's are at the fixed sites of equal distance along the x--axis (within the space of $2b$ there are 10^4 dipole-B's). An electron (Dipole-A) flies by and its action on the Dipole-B is the same as your answer in (b). Figure-C shows the initial angular profile of the dipoles. The dipoles at $x_0 + b$ are along $\hat{y}_0$, the dipoles at $x \leq x_0$ are along $-\hat{x}_0$, and that at $x \geq x_0 + 2b$ are along $\hat{x}_0$. Dipoles along $-\hat{x}_0$ can no longer rotate. Dipoles along $\hat{x}_0$ cannot rotate either, except for the ones close to the domain edge which can rotate freely as the ones pointing in other directions. Given the electric current carried by the electrons as I, find the speed of the movement of the domain center when the current is just turned on. (Take the electron charge e as known) (5 points**)**

(d) Find the time it takes for the whole domain to pass the $x_0 + 2b$ point. (2 points**)**

(Hint: Magnetic field $\vec{B}(\vec{r})$ at position $\vec{r}$ due to a magnetic dipole $\vec{m}$ at coordinate origin is given by $\vec{B}(\vec{r}) = \dfrac{\mu_0}{4\pi r^3}\left[\dfrac{3(\vec{m}\cdot\vec{r})\vec{r}}{r^2} - \vec{m}\right]$. Torque $\vec{N}$ of magnetic field $\vec{B}$ on a dipole $\vec{m}$ is $\vec{N} = \vec{m} \times \vec{B}$.)

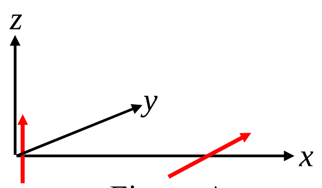


Figure-A

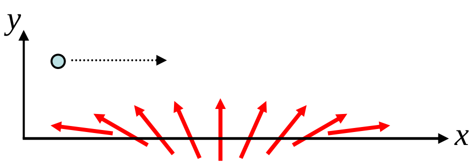


Figure-B

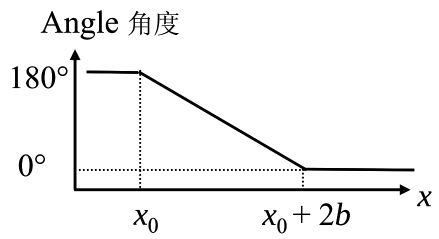


Figure-C

電子和某些原子具有叫做自旋的固定角動量 $\vec{S}$，其大小是不變的，但方向可由外力矩改變。 由自旋產生的磁矩爲磁偶極子 $\vec{m} = \mu_B \vec{S}$，其中 μ_B 爲常數。爲簡單起見，我們假設電子和原子的自旋和磁偶極子都是一樣的。

(a) 參照上圖-A。磁偶極子-A 在 $\vec{r} = 0$ 處，沿方向 $\hat{z}_0$，磁偶極子-B 在 $\vec{r} = a\hat{x}_0$，沿方向 $\hat{y}_0$。求磁偶極子-A 對磁偶極子-B 的力矩。(3 分)

(b) 假設磁偶極子-A 的方向固定，求過了一短時間 Δt 後，磁偶極子-B 的方向。(5 分)

(c) 上圖-B 描述的是一個在(x_0, $x_0 + 2b$)空間裏的含有很多原子(磁偶極子-B)的磁疇。磁偶極子-B 占滿了 x--軸上的固定等距位置，它們的方向以一定的規律排列。(在 $2b$ 長度內可含有 10^4 個磁偶極子-B)。一電子(磁偶極子-A)飛過，它對每個磁偶極子-B 的作用和(b)部分的答案相同。上圖-C 爲開始時磁偶極子方向分佈。在 $x_0 + b$ 的磁偶極子沿 $\hat{y}_0$ 方向，在 $x \geq x_0 + 2b$ 的磁偶極子沿 $\hat{x}_0$ 方向，在 $x \leq x_0$ 的磁偶極子沿 $-\hat{x}_0$ 方向。沿 $-\hat{x}_0$ 的磁偶極子不能再轉動，沿 $\hat{x}_0$ 的磁偶極子除了靠近磁疇邊緣的那些以外也不

能轉動。指向其他方向的磁偶極子和磁疇外面一點點的沿 $\vec{x}_0$ 的磁偶極子可自由轉動。給定電子的電流爲 I， 求電流剛開始時磁疇中心的速度。(當電子電荷 e 爲已知) (5 分)

(d) 求整個磁疇通過 $x_0 + 2b$ 點所需的時間。(2 分)

(提示: 在 $\vec{r}$ 處由位於座標原點的磁偶極子 $\vec{m}$ 產生的磁場爲 $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi r^3}\left[\frac{3(\vec{m}\cdot\vec{r})\vec{r}}{r^2} - \vec{m}\right]$。在磁場 $\vec{B}$ 裏磁偶極子 $\vec{m}$ 受的力矩 $\vec{N}$ 爲 $\vec{N} = \vec{m}\times\vec{B}$ 。)

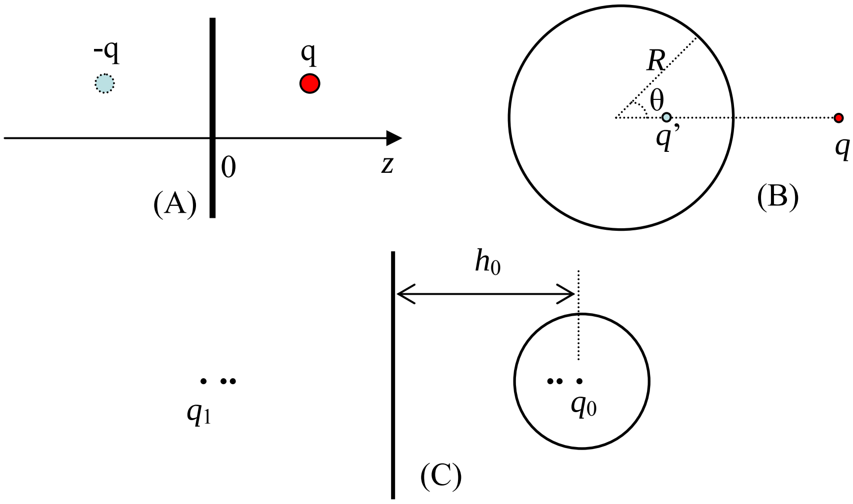
Q8 Force between a conductor sphere and a conductor plane 導體球與導體板之間的力
(22 points **22 分)**

The uniqueness theorem of the theory of electric field ensures that virtual charges could be placed ***outside*** the closed space where the electric field is to be determined to mimic the real boundary conditions. For example, in Figure-A below, to find the electric field when a point charge q is placed in front of a large planar conductor at zero potential, a virtual, or 'image' charge $-q$ is placed at the mirror-image position so that the combined contribution of the two charges is to make the potential everywhere on the plane zero. The force on q by the plane is then the same as the force by the image charge.

Part-A
Consider the force of a conductor sphere of radius R held at zero potential on a point charge q at a distance d (> R) from the sphere center. The image charge q' should be inside the sphere (outside the space outside the sphere) and on the line joining the sphere center and q, as shown in Figure-B. The combined contribution of q and q' is to make the potential everywhere on the sphere surface zero. Find the position and the value of q'. (4 points)

Part-B
As shown in Figure-C, a problem often encountered in atomic force microscopy is to determine the force of the sample on the probe tip, which is a conductor sphere of radius R held at electric potential V at distance h_0 from a large conducting sample surface at zero potential. In the following we will apply the 'image charge' method step by step to find the force.

(a) We start by putting a point charge q_0 inside the sphere such that the sphere surface is an equal-potential one with voltage V, while ignoring the effect of the conductor plate. Determine q_0 in terms of V and R. (1 point)
(b) Determine the value and position (h_1) of the image charge of q_0 and call it q_1, such that the combined contribution of them is to make the sample surface a zero-potential plane. (2 points)
(c) The presence of q_1 now makes the conductor sphere surface no longer equal-potential. Put another point charge q_2 inside the sphere such that the combined effect of q_0, q_1 and q_2 is to make the potential on the sphere surface equal to V again. Determine q_2 and its position h_2. (2 points)
(d) Repeat (b) to determine the image charge q_3 of q_2, and repeat (c) to find the image charge q_4 of q_3. Derive the general expression between h_{2n} and $h_{2(n+1)}$, q_{2n} and $q_{2(n+1)}$, and q_{2n+1} and $q_{2(n+1)+1}$, $n = 0, 1, 2...$ (3 points)
(e) Derive the total force of the sample on the sphere ***<u>in the form of a summation over an infinite series</u>***. (2 points)
(f) Suppose the force in (e) is 1.1×10^{-12} N when $V = V_0$, $R = 1.0 \times 10^{-8}$ m, and $h_0 = 5.0 \times 10^{-8}$ m, find the force when $V = 2V_0$, $R = 1.0$ m, and $h_0 = 5.0$ m. (4 points)
(g) Given $R/h_0 = 1/51$, to determine the force in (e) up to ~1% accuracy, what are the terms that should be kept? (2 points)
(h) Estimate the order of magnitude of the relative error caused by the terms neglected in (g). (2 points)



利用電場理論的唯一性定理，我們可在一閉合空間外用一虛擬電荷分佈來產生和原來相同的電勢邊界條件，以便求出閉合空間內的電勢/電場。如圖-A 所示，一點電荷 q 放在零電勢的大導電平面前，爲求電場，可以在平面左邊放一‘鏡像’電荷-q，兩電荷的合電勢使平面的電勢仍然爲零。平面對 q 的力等於-q 對 q 的力。

（A）

我們先考慮一半徑爲 R 的零電勢導體球對放在離球心距離爲 $d\ (> R)$ 的點電荷 q 的力的問題。如圖-B 所示，q 的鏡像電荷 q' 在球裏面（球以外空間的外面）球心與 q 的連線上。q 和 q' 的合電勢使球面上處處電勢爲零。求 q' 的值和位置。（4 分）

（B）

如圖-C 所示，原子力顯微鏡經常遇到的問題是要確定樣品對掃描探頭的力。掃描探頭爲一半徑爲 R 電勢爲 V 的導體球，離導電樣品表面距離爲 h_0。樣品電勢爲零。下面我們利用鏡像電荷方法一步步地把力最終求出來。

(a) 首先，當樣品不在，放一點電荷 q_0 在球心。求使球面的電勢爲 V 時 q_0 的值。(1 分)
(b) q_0 和它的鏡像電荷 q_1 必須使樣品的表面處處電勢爲零。求 q_1 的值和位置 h_1。(2 分)
(c) q_1 的出現使導體球面不再是等勢面。在球裏放上點電荷 q_2，使 q_0, q_1 和 q_2 的合電勢場在球面處處爲 V。求 q_2 的值和位置 h_2。 (2 分)
(d) 重複（b）求 q_2 的鏡像電荷 q_3，重複（c）求 q_3 的鏡像電荷 q_4。導出 h_{2n} 與 $h_{2(n+1)}$, q_{2n} 與 $q_{2(n+1)}$，以及 q_{2n+1} 與 $q_{2(n+1)+1}$ 之間的一般關係式。 $n = 0, 1, 2...$ (3 分)
(e) 求樣品對球的力。答案可以無窮級數相加形式表達。(2 分)
(f) 已知當 $V = V_0$, $R = 1.0 \times 10^{-8}$ m, $h_0 = 5.0 \times 10^{-8}$ m 時，樣品對球的力爲 1.1×10^{-12} N, 求當 $V = 2V_0$, $R = 1.0$ m, and $h_0 = 5.0$ m 時的力。(4 分)
(g) 已知 $R/h_0 = 1/51$，如要求用(e)的答案來計算力時可達到大約 1%的精確度，需要保留哪幾項？(2 分)
(h) 估計(g)中因忽略其餘項所引起的相對誤差的數量級。(2 分)

END 完