

1. (a) Convert $(123.625)_{10}$ to hexadecimal. (Hex = Base 16)

$$123/16 = 7 \text{ (rem 11)}$$

$$7/16 = 0 \text{ (rem 7)}$$

$$= 7B \text{ (11 = B in Hex)}$$

$$.625 * 16 = 10$$

$$= A \text{ (10 = A in Hex)}$$

$$(123.625)_{10} = (7B.A)_{16}$$

- (b) Convert $(123.4)_5$ to decimal.

$$(3 \times 5^0) + (2 \times 5^1) + (1 \times 5^2)$$

$$= 3 + 10 + 25 = 38_{10}$$

$$4 \times 5^{-1} = 0.8_{10}$$

$$(123.4)_5 = (38.8)_{10}$$

- (c) Convert $(1011)_3$ to binary and $(18)_{10}$ to a number in the number system with radix 5.

$$1 \times 3^3 + 0 \times 3^2 + 1 \times 3^1 + 1 \times 3^0$$

$$= 27 + 0 + 3 + 1 = (31)_{10}$$

$$31/2 = 15 \text{ (rem 1)}$$

$$15/2 = 7 \text{ (rem 1)}$$

$$7/2 = 3 \text{ (rem 1)}$$

$$3/2 = 1 \text{ (rem 1)}$$

$$\frac{1}{2} = 0 \text{ (rem 1)}$$

$$= (11111)_2$$

$$18/5 = 3 \text{ (rem 3)}$$

$$\frac{3}{5} = 0 \text{ (rem 3)}$$

$$= (33)_5$$

2. Fixed-point representation is used to represent what type of numbers?

Is it possible to use fixed point representation to represent some fractional numbers.

Fixed point representation is used to represent a number whose binary point is fixed at one position. It is possible to use fixed-point representation to represent fractional numbers. Yes

3. Three variable-length representations for the character set {A, B, C, D, E} are given below. Indicate if each of them is a correct representation and explain why.

Representation 1		Representation 2		Representation 3	
A	0	A	1	A	0
B	11	B	01	B	10
C	100	C	001	C	11
D	1010	D	000	D	to be determined
E	1011	E	010	E	to be determined

Representation one is a correct representation because it is one to one and prefix free. The prefix is the previous number of the following number.

Rep. 2 is incorrect since the prefix for B and E are the same.

Rep. 3 will have two three bit codes for D and E and can't start with 0, 10, 11 so yea no prefixes.

4. An 9-bit floating-point number representation system has the following specification:

Among the 8 bits ($B_8 \ B_7 \ B_6 \ B_5 \ B_4 \ B_3 \ B_2 \ B_1 \ B_0$), bit B_8 is the sign bit, and bits $B_7 \ B_6 \ B_5 \ B_4$ is the exponent with a bias value (0 1 0 0). Bits $B_3 \ B_2 \ B_1 \ B_0$ consist of the mantissa. The binary number that is represented is $(-1)^{B_8} \times (0.B_3 B_2 B_1 B_0) \times 10^{B_7 B_6 B_5 B_4 - 0100}$.

(a) What are the largest positive and negative numbers this system can represent?

(b) Find a positive number x which is representable by this system such that $1+x=1$.

Find the largest positive number y which is representable by this system such that $1+y=1$.

a.
$$\begin{aligned} & .M \times 10^{\text{expo-bias}} \\ & (.1111 \times 10^{111-0100} = .1111 \times 10^{1011})_2 = \\ & (.1111 * 10^{0100}) = 1111 \\ & 10^{1011} \times 1/10^{0100} = 10^{1011-0100} = 10^{111} \\ & 1111 \times 10^{111} = 15 \times 128 \end{aligned}$$

For negative do the same shit as a but just multiply by -1

b. a.
$$\begin{aligned} & 1+x=1 \quad 1_2 + (0.0000 \ 0001)_2 = .10000 \ 0001 \times 10^{0101-0100} \\ & .0000 \ 1111 * 10^{0101-0100} = (.00001111 * 10^{0100}) \times (10^{0101} / 10^{0100}) = .1111 \times 10^{0001-0100} \\ & .1111 \times 10^{-011} \end{aligned}$$

$1+y=1$.

ANSWER: $(0\ 0000\ 0001)$ represents $2^{-4} \times 2^{-4} = 2^{-8} = (0.0000\ 0001)_2$

$1_2 + (0.0000\ 0001)_2 = 1.0000\ 0001 = .1000\ 00001 \times 10^{0101-0100}$

the closest of the above can be represented by $(0\ 0101\ 1000)$, which represents 1.

$1+y = .1000\ 1111 \times 10^{0101-0100}$,

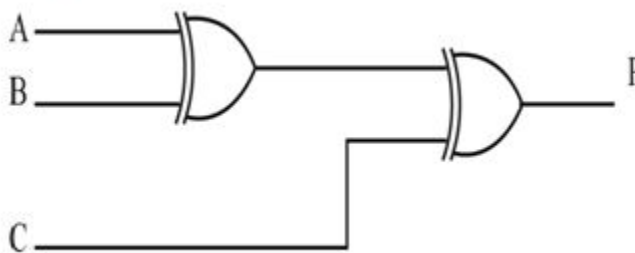
means that $y = .0000\ 1111 \times 10^{0101-0100} = .1111 \times 10^{0001-0100} = (1/2+1/4+1/8+1/16) \times 2^{-3}$,

which is represented by $(0\ 0001\ 1111)$.

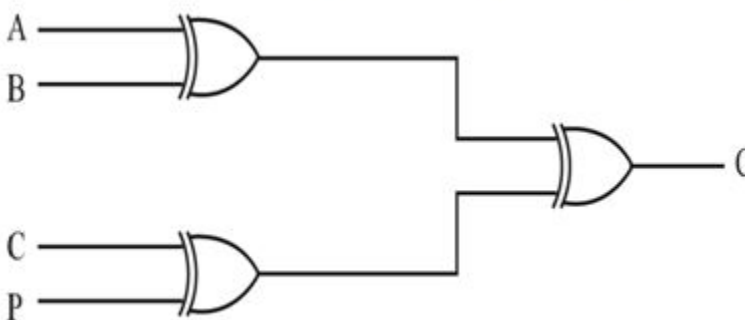
5. Draw the following error detection circuits:

(a) An even **parity generator** that has 3 input bits and 1 output bit. The output bit is the parity bit.

(b) An even **parity checker** that has 4 inputs and 1 output bit. The output is an error indicator.



a.



b.

6. Using the technique of even parity bit to design an error correction code for strings of 9-bit, where each string has at most 1 bit error. Give one code that uses as few parity bits as possible.

Sol: $(m + r + 1) \leq 2^r$ that is $(10 + r) \leq 2^r$. $R = 4$

(m = bits which would be 9)

b1 b2 b3 b4 b5 b6 b7 b8 b9 b10 b11 b12 b13 (9 bits plus R which is 4) (Everything that is 2^n)

$b3 = 1 + 2$

$b5 = 1 + 4$

$b6 = 2 + 4$

$$\begin{aligned}b_7 &= 1 + 2 + 4 \\b_9 &= 1 + 8 \\10 &= 2 + 8 \\11 &= 1 + 2 + 8 \\12 &= 4 + 8 \\13 &= 1 + 4 + 8\end{aligned}$$

On sending, the parity generators generate the following parity bits:

$$b_1 = b_3 \text{ XOR } b_5 \text{ XOR } b_7 \text{ XOR } b_9 \text{ XOR } b_{11} \text{ XOR } b_{13}$$

$$b_2 = b_3 \text{ XOR } b_6 \text{ XOR } b_7 \text{ XOR } b_{10} \text{ XOR } b_{11}$$

$$b_4 = b_5 \text{ XOR } b_6 \text{ XOR } b_7 \text{ XOR } b_{12} \text{ XOR } b_{13}$$

$$b_8 = b_9 \text{ XOR } b_{10} \text{ XOR } b_{11} \text{ XOR } b_{12} \text{ XOR } b_{13}$$

On the receiving side, the parity checkers check the following:

$$q_1 = b_1 \text{ xor } b_3 \text{ xor } b_5 \text{ xor } b_7 \text{ xor } b_9 \text{ xor } b_{11} \text{ xor } b_{13}$$

$$q_2 = b_2 \text{ xor } b_3 \text{ xor } b_6 \text{ xor } b_7 \text{ xor } b_{10} \text{ xor } b_{11}$$

$$q_3 = b_4 \text{ xor } b_5 \text{ xor } b_6 \text{ xor } b_7 \text{ xor } b_{12} \text{ xor } b_{13}$$

$$q_4 = b_8 \text{ xor } b_9 \text{ xor } b_{10} \text{ xor } b_{11} \text{ xor } b_{12} \text{ xor } b_{13}$$

Then, if $(q_4 \ q_3 \ q_2 \ q_1) = (0 \ 0 \ 0 \ 0)$, then no error.

If $(q_4 \ q_3 \ q_2 \ q_1) = (0 \ 0 \ 0 \ 1)$, then bit b_1 is flipped.

If $(q_4 \ q_3 \ q_2 \ q_1) = (0 \ 0 \ 1 \ 0)$, then bit b_2 is flipped.

If $(q_4 \ q_3 \ q_2 \ q_1) = (0 \ 0 \ 1 \ 1)$, then bit b_3 is flipped.

... ..

If $(q_4 \ q_3 \ q_2 \ q_1) = (1 \ 1 \ 0 \ 1)$, then bit b_{13} is flipped.

7. Design a combinatorial circuit on the decoding site (i.e. receiver) for 7-bit binary strings equipped with Hamming coding. The input to the circuit is a 7-bit string $(I_7 \ I_6 \ I_5 \ I_4 \ I_3 \ I_2 \ I_1)$, where I_1, I_2 and I_4 are the three even parity bits. Suppose that each 7-bit string $(I_7 \ I_6 \ I_5 \ I_4 \ I_3 \ I_2 \ I_1)$ has at most error on one bit. The output of the circuit has 4 bits B_7, B_6, B_5 and B_3 , and for $k = 3, 5, 6, 7$, $B_k = I_k$ if the input bit I_k is correct, otherwise bit $B_k = I_k'$.

Sol:

$$3 = 1 + 2$$

$$5 = 1 + 4$$

$$6 = 2 + 4$$

$$7 = 1 + 2 + 4$$

Parity generator:

$$b1 = b3 \text{ xor } b5 \text{ xor } b7$$

$$b2 = b3 \text{ xor } b6 \text{ xor } b7$$

$$b4 = b5 \text{ xor } b6 \text{ xor } b7$$

Receiving side:

$$q1 = l1 \text{ xor } l3 \text{ xor } l5 \text{ xor } l7$$

$$q2 = l2 \text{ xor } l3 \text{ xor } l6 \text{ xor } l7$$

$$q3 = l4 \text{ xor } l5 \text{ xor } l6 \text{ xor } l7$$

$(q3 \ q2 \ q1) = (0 \ 0 \ 0)$, no error, $B_k = l_k$ for $k = 3, 5, 6, 7$

$= (0 \ 1 \ 1)$, error, then $b3 = l3'$

$$b5 = l5$$

$$b6 = l6$$

$$b7 = l7$$

$= (1 \ 0 \ 1)$, error, then $b3 = l3$

$$b5 = l5'$$

$$b6 = l6$$

$$b7 = l7$$

$= (1 \ 1 \ 1)$, error, then $b3 = l3$

$$b5 = l5$$

$$b6 = l6$$

$$b7 = l7'$$

$= (1 \ 1 \ 0)$, error, then $b3 = l3$

$$b5 = l5$$

$$b6 = l6'$$

$$b7 = l7$$

magic

