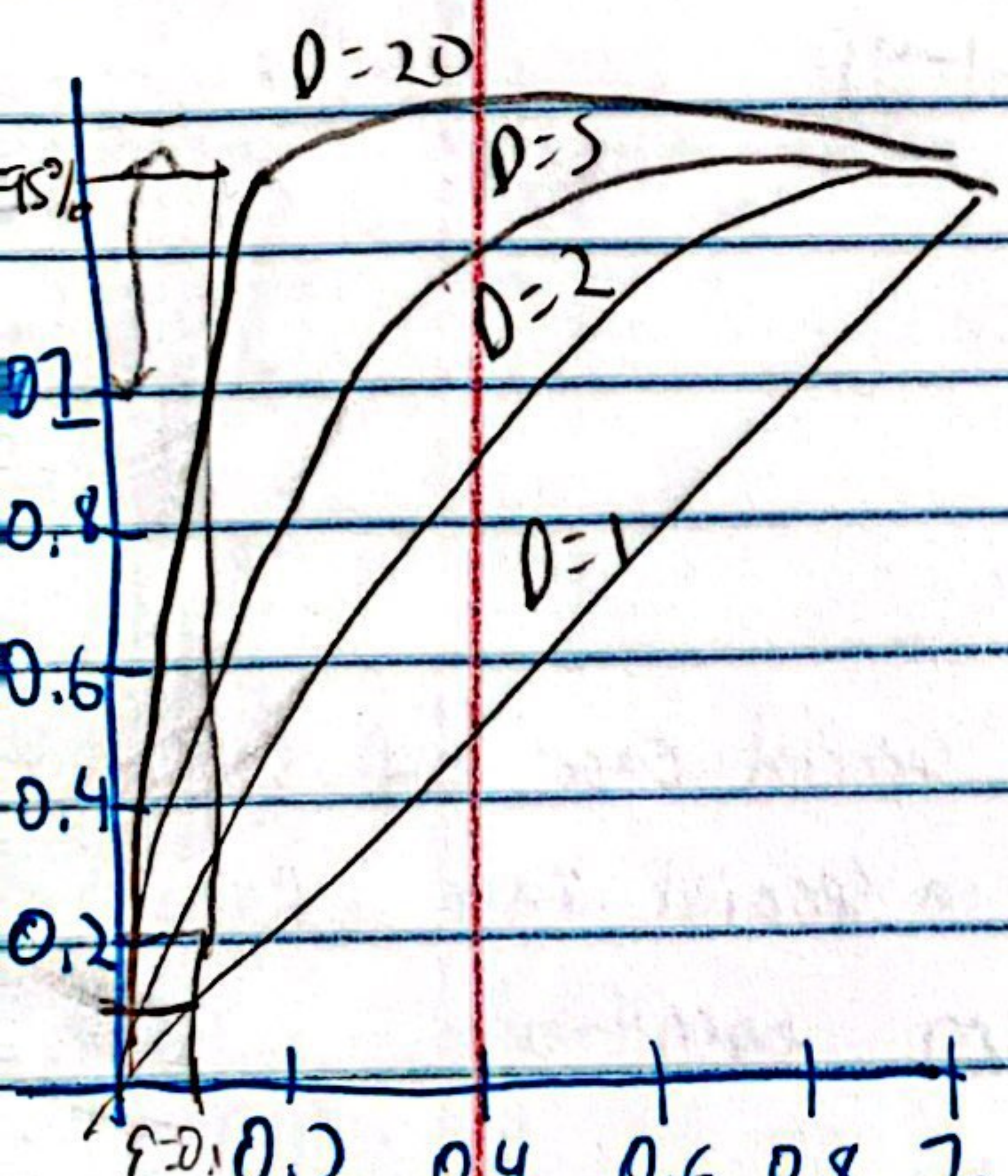
3/21 **Curse of Dimensionality**

→ Any Method that becomes intractable/unmanageable as D grows is said to suffer from curse of Dimensionality

→ Polynomial curve fitting for D-dim data, # of independant coefficients (W's) grows as D^M ← Power Law

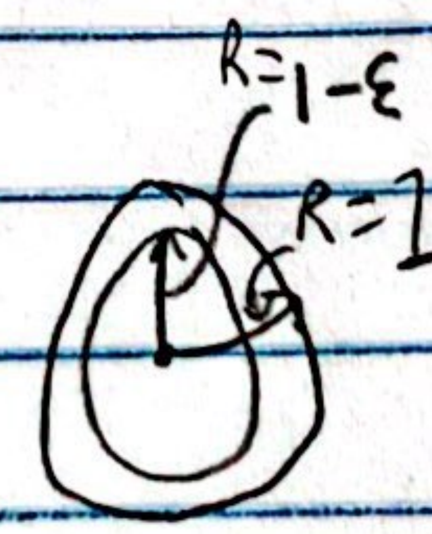
→ Our geometrical intuitions fail when considering high-dim. space. Given a hypersphere in D-Dim. space with radius r=1, what fraction of its volume lies between r=1-ε and r=1 as D grows



D=1   0   1-ε   1

$$\frac{1-(1-\varepsilon)}{1} = \varepsilon$$

D=2



$$\frac{\pi \cdot 1^2 - \pi(1-\varepsilon)^2}{\pi(1)^2} = 1-(1-\varepsilon)^2$$

$$\text{In General} = 1-(1-\varepsilon)^D$$

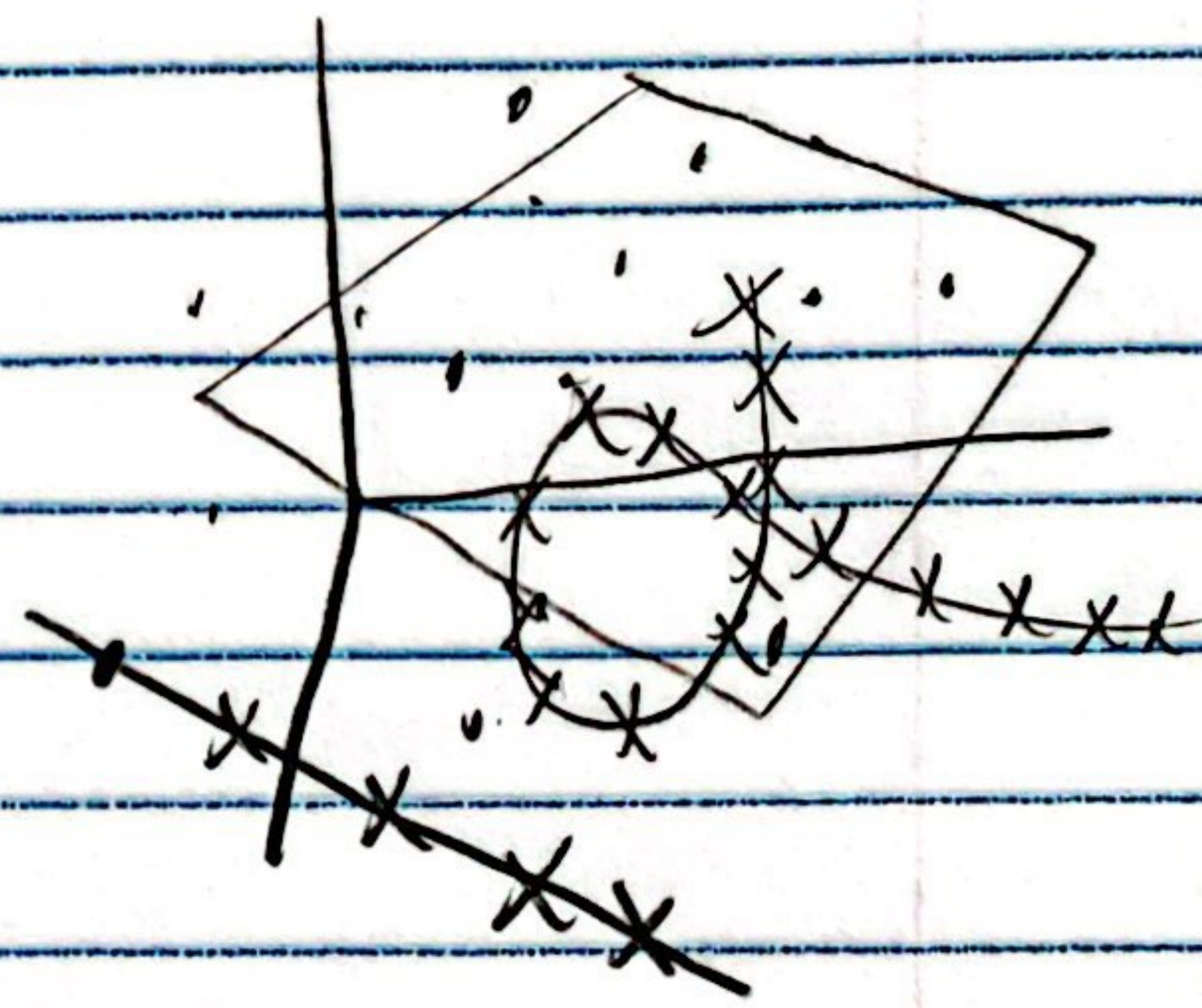
- In density estimation, one needs densely populated regions. if in 1-dim. space one needs N points to be densely populated, then the corresponding D-Dim. space will require $\boxed{N^D}$ points.

Exponential Growth

- Avg. euclidean distance between a point & its nearest neighbor increases with D.

Curse of Dimensionality doesn't prevent us from devising effective techniques in high-dim. spaces. One reason is that real data often resides on a low-dimensional subspace or manifolds.

Manifold Learning:



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