ASEN 5519 - Algorithmic Motion Planning
Fall 2024

Mini Project

Assigned November 20; Due November 23

INSTRUCTIONS:

1. This (mini) project, excluding the extra credit portion, must be completed in **no more than 5 hours**.
2. You may use material outside of the class, provided that appropriate citations are included in your solutions.
3. You may **not** seek or receive assistance in any form (e.g., brainstorming) from anyone, including fellow students and faculty.
4. The solutions must be **typed** and submitted via Gradescope.
5. Your submission must include a cover page with your name and student ID number.
6. For extra credit problems, you must submit your code along with line-by-line instructions on how to run it.

Scenario (imaginary). A company has generously donated a small robot to CU Boulder's CAES to be used for delivering mail to faculty offices during non-business hours when the building is closed. You have been selected to lead the motion planning team for this robot. Your team's task is to design a motion planning algorithm under the following assumptions: (i) full maps of the building floors are available, (ii) the indoor GPS system and local sensors are fully accurate, with no sensor uncertainty.

The robot is in the shape of a rectangle with length $L = 2$ and width $W = 1$ with second-order car dynamics:

$$\dot{x} = v\cos\theta, \qquad \dot{y} = v\sin\theta, \qquad \dot{\theta} = \frac{v}{L}\tan\varphi, \qquad \dot{v} = u_1, \qquad \dot{\varphi} = u_2,$$

where x and y determine the position, θ is the orientation, v is the speed, and φ is steering angle of the front wheels of the car. The control inputs u_1 and u_2 are the acceleration and steering turn rate, respectively. The physical constraints of the car are:

$$\theta \in [-\pi, \pi], \quad v \in [-\frac{1}{6}, \frac{1}{2}], \quad \varphi \in [-\frac{\pi}{6}, \frac{\pi}{6}], \quad u_1 \in [-\frac{1}{6}, \frac{1}{6}], \quad u_2 \in [-\frac{\pi}{6}, \frac{\pi}{6}].$$

The robot experiences bounded motion disturbances that are not explicitly modeled in the dynamics. To mitigate this, it is equipped with a closed-loop controller that ensures the robot can follow a nominal trajectory while maintaining its state within a ball of radius r around the trajectory.

Given an initial condition for the car and a destination set defined by $x_d \in [x_d^-, x_d^+]$, $y_d \in [y_d^-, y_d^+]$, $\theta_d \in [\theta_d^-, \theta_d^+]$, and $v_d \in [v_d^-, v_d^+]$, the motion planning algorithm must find a valid trajectory that:

- respects all physical and control constraints,
- ensures that, by following the trajectory using the provided controller, the robot can move from the initial state to a state within the destination set, and
- avoids collisions with obstacles in the environment.

(a) What space will you choose for motion planning for this mail-delivery robot? Define this space and state your reasons for this choice.

(b) Describe the advantages and disadvantages of each method of motion planning listed below for your problem.

  - i. Gradient descent planner with a potential function
  - ii. Wave-front planner
  - iii. Probabilistic roadmap planner
  - iv. Randomized tree-based planner

(c) Design an appropriate motion planner for the robot. Provide a detailed pseudocode for the planner, similar in style to those presented in the lecture notes, and explain the key lines. Ensure that all necessary elements are clearly defined. If the main pseudocode relies on functions as subroutines, include well-defined pseudocode for those subroutines as well.

(d) Discuss the completeness property of your planner.

(e) Can you turn your planner into an optimal one? Justify your answer.

(f) The robot is required to return to its charging station at the end of each day. However, due to heavy rain and roof leakage, puddles have formed on the building floor. While the robot can traverse through puddles without difficulty, it must dry off on a carpeted area before proceeding to the charging station. Formalize this requirement using Linear Temporal Logic. Make sure that all necessary elements are clearly defined.

(g) Design a motion planner for the robot that generates a valid trajectory, assuming the locations of the puddles, carpets, and charger are known. Begin by describing your approach in simple terms. Next, present a detailed pseudocode for the planner and explain each line thoroughly. Make sure that all components and subroutines are clearly defined and well-explained.

(h) Discuss the completeness property of this planner.

Extra Credit 1. Implement your planner in part (f) and plot three generated motion plans in the following environment:

- Initial condition: $x = 0$, $y = 0$, $\theta = 0$, $\varphi = 0$, and $v = 0$.
- Destination set: $x_d, y_d \in [8, 9]$, $\theta_d = [\frac{4\pi}{9}, \frac{5\pi}{9}]$, and $v_d \in [-\frac{1}{20}, \frac{1}{20}]$
- Workspace bounds: $x, y \in [-1, 11]$
- Workspace obstacles: all rectangles with vertices:

$$\begin{aligned}
O_1 &: (3,0),\ (5,0),\ (5,2),\ (3,2);\\
O_2 &: (7,3),\ (9,3),\ (9,5),\ (7,5);\\
O_3 &: (1,4),\ (4,4),\ (4,6),\ (1,6);\\
O_4 &: (5,7),\ (6,7),\ (6,10),\ (5,10).
\end{aligned}$$

- Uncertainty radius is $r = 0.25$

Extra Credit 2. Implement your planner in part (g) and plot three generated motion plans in the environment defined above with the same initial condition, charger region is the destination set, and rectangular carpet and puddle regions with vertices:

$$\begin{aligned}
\text{Puddle} &: (0,7),\ (4,7),\ (4,11),\ (0,11);\\
\text{Carpet} &: (10,0),\ (11,0),\ (11,10),\ (10,10).
\end{aligned}$$