

## Project 1

Course: Automatic Control Systems – ASEN 5114-001 – Spring  
2025

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April 7, 2025

*The purpose of this project is to apply frequency domain modeling and design methods to control attitude of a spacecraft mockup. A combination of analytic and empirical modeling will be used to obtain a transfer function representation of the spacecraft dynamics. A controller will then be designed using frequency response methods, then implemented in a simulation of the hardware mockup. Predicted responses will be compared to simulated responses. Colocated versus non-colocated control issues will be explored. The project can be carried out individually, or in small teams. Team projects (up to 3 team members) will submit a single report, and all members of a team project will receive the same grade. The report should include an introduction, separate sections for each of the following parts, and a conclusion pointing out interesting aspects or difficulties. All team members should contribute approximately equally to the conduct of the project and the write-up. Provide a statement of individual contributions in the report. The report is due Monday, April 7 at 11:59 PM.*

1)

[20 pts] Develop an analytic transfer function model to match the empirical frequency response obtained from the spacecraft mockup hardware (data posted on Canvas), for frequencies up through the first resonance/anti-resonance.

The data came in the form of magnitudes (in (rad/s)/V) and phases (in rads) for a set of frequencies (Hz). I started by converting the system to output radians by inserting an integrator block as seen in the diagram below, then converting the readings using the resulting transfer function.

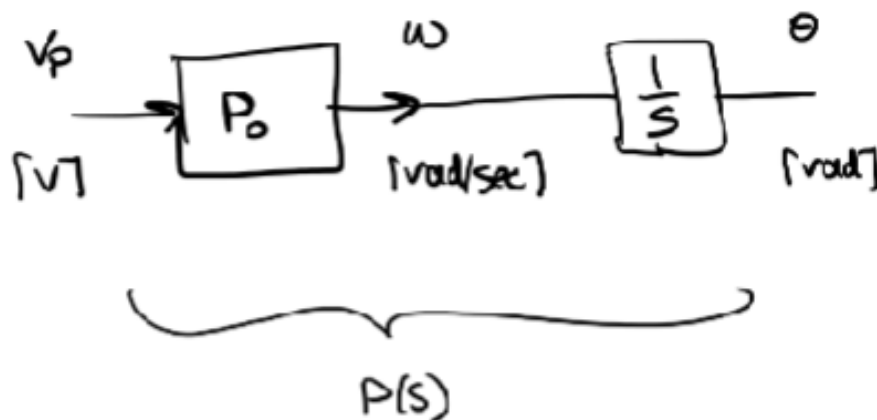


Figure 1: Block diagram of the system.

The transfer function  $P(s)$  is given by:

$$P(s) = P_0(s) \cdot \frac{1}{s}$$

given  $|P_0(j\omega)|$ ,  $\angle P_0(j\omega)$  in radians.

For  $P_0(j\omega)$ :

$$P(j\omega) = P_0(j\omega) \cdot \frac{1}{j\omega}$$

The magnitude in decibels is:

$$|P(j\omega)|_{\text{dB}} = 20 \log_{10} |P_0(j\omega)| - 20 \log_{10} \omega$$

The phase is:

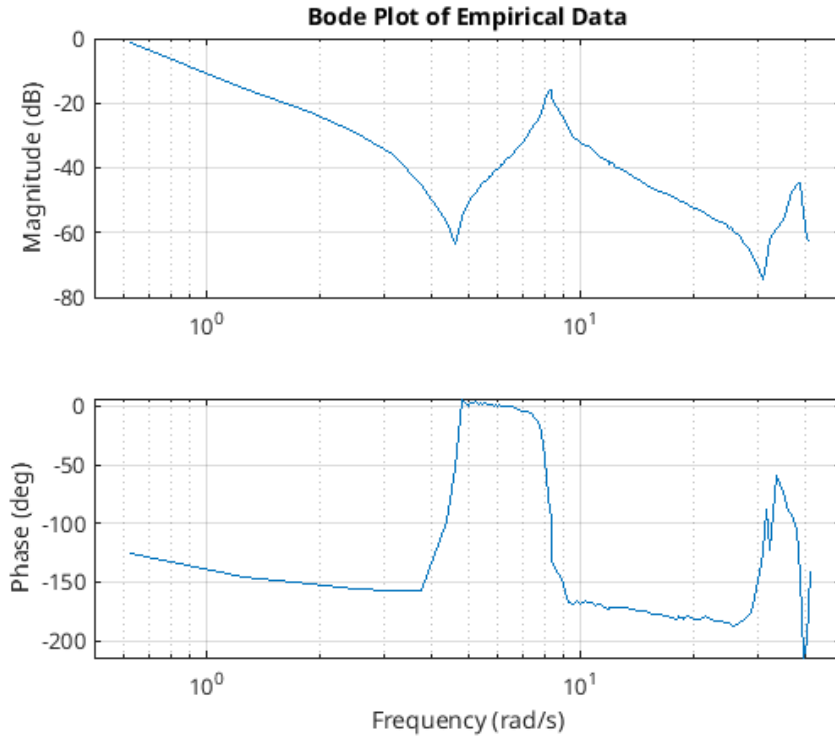
$$\angle P(j\omega) = \angle P_0(j\omega) - \angle \frac{1}{j\omega} = \angle P_0(j\omega) - 90^\circ$$

I converted and plotted the resulting magnitudes (in dB) and phases (degrees) for the frequencies (in rad/s) to get the Bode plot of the system using the following MATLAB code:

```

1  dataTable = readtable('data.xlsx');
2  data = table2array(dataTable);
3
4  n = size(data,1);
5  pResponse = zeros(n,3);
6
7  for i = 1:n
8      pResponse(i,1) = 20*log10(data(i,2)) - 20*log10(data(i,1));
9      pResponse(i,2) = data(i,3)*180/pi - 90;
10     pResponse(i,3) = data(i,1)*2*pi;
11 end

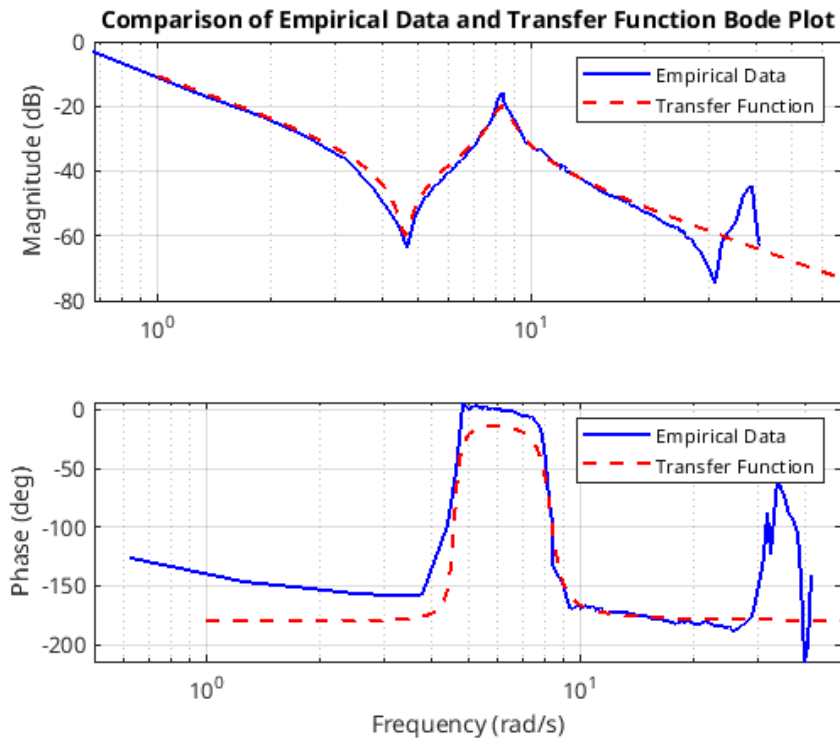
```



In order to match an analytic model to the empirical data, I started with the most notable features, the resonance and antiresonance. I knew that in order to get that resonance behavior you needed a term like a second-order polynomial of the form  $(s^2 + 2\zeta\omega_n s + \omega_n^2)$ , one in the numerator for the antiresonance and one in the denominator for the resonance. I knew that the frequency of the peak of a resonance

corresponds to the square of the natural frequency ( $\omega_n$ ) so I set those to match the data and then tuned the damping ratio ( $\zeta$ ) to get the right peak values and shape of the curve.

Then I realized that I needed to add a gain compensator K to move the transfer function up and down and I needed a couple of extra poles to get the -40 dB/decade slope in the magnitude and to bring the phase down to -180 degrees. The resulting magnitude plot looked pretty close but in the empirical data there was this phase lagging behavior that you couldn't see at all in the analytical model:



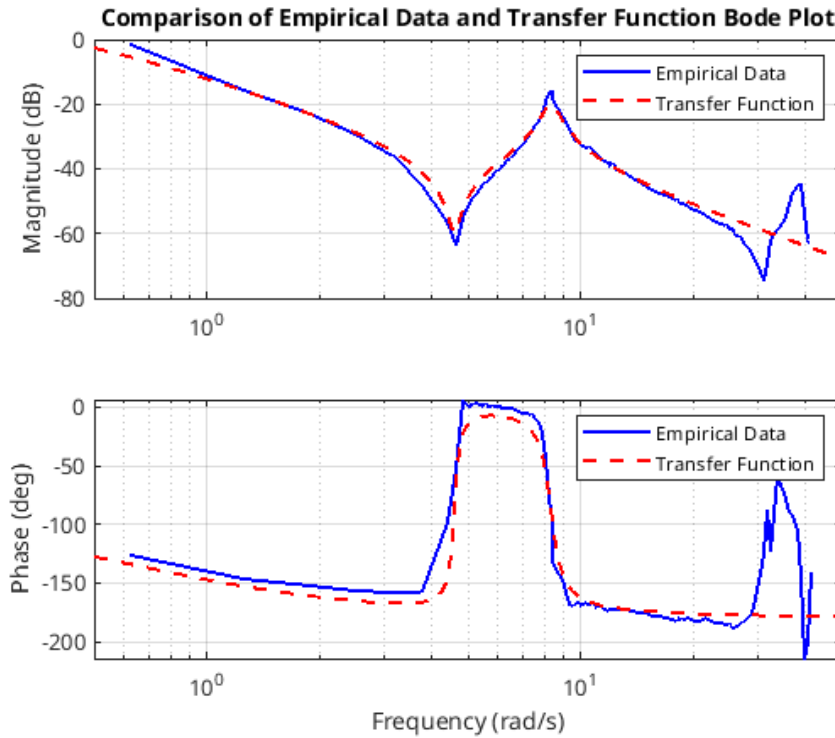
So I decided to change one of the poles to have that  $\frac{1}{s+b}$  lagging behavior and I set b to 0.65 so that it would kick in soon enough to effect low frequency phase while still achieving the -40 dB/decade slope in the magnitude. So put together all of those terms and the resulting transfer function looks like:  $\frac{K*(s^2+2\zeta\omega_n s+\omega_n^2)}{(s^2+2\zeta\omega_n s+\omega_n^2)(s)(s+b)}$ . I tweaked the parameters as described above to get a good fit so the transfer function ended up looking like:  $\frac{K*(s^2+2*0.05*4.60118s+4.60118^2)}{(s^2+2*0.1*8.34686s+8.34686^2)(s)(s+0.65)}$ . And I implemented that in Matlab as such:

```

1 K = 1;
2 zetaZ = 0.05;
3 zetaP = 0.1;
4 omegaZ = 4.60118;
5 omegaP = 8.34686;
6
7 num = K*[1 zetaZ*omegaZ omegaZ^2];
8 den = [1 zetaP*omegaP omegaP^2 0];
9 pole = [1 0.65];
10 den = conv(den, pole);
11 estTF = tf(num,den);

```

The final result looks like this:



Which looks pretty close and all of the important frequencies like the gain crossover frequency and resonance peak frequencies match up closely so that I am satisfied enough that the behavior will be the same.

2)

[20 pts] Examine the two loop transfer functions (analytic and empirical) via Bode plots and Nyquist plots using a proportional controller  $C(s) = K$ . Would the control system be stable for some values of  $K$ ? For all values of  $K$ ?

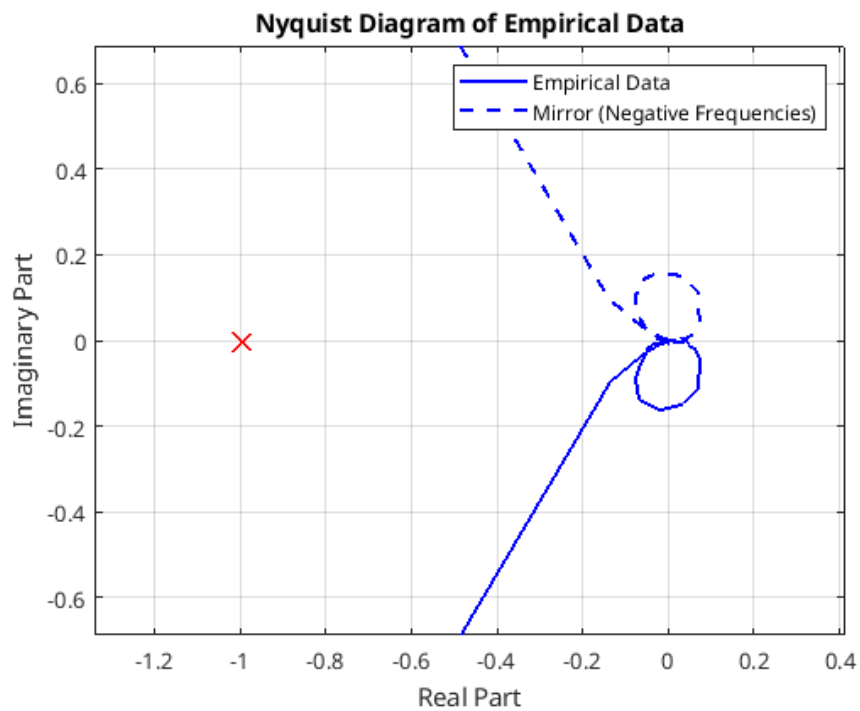
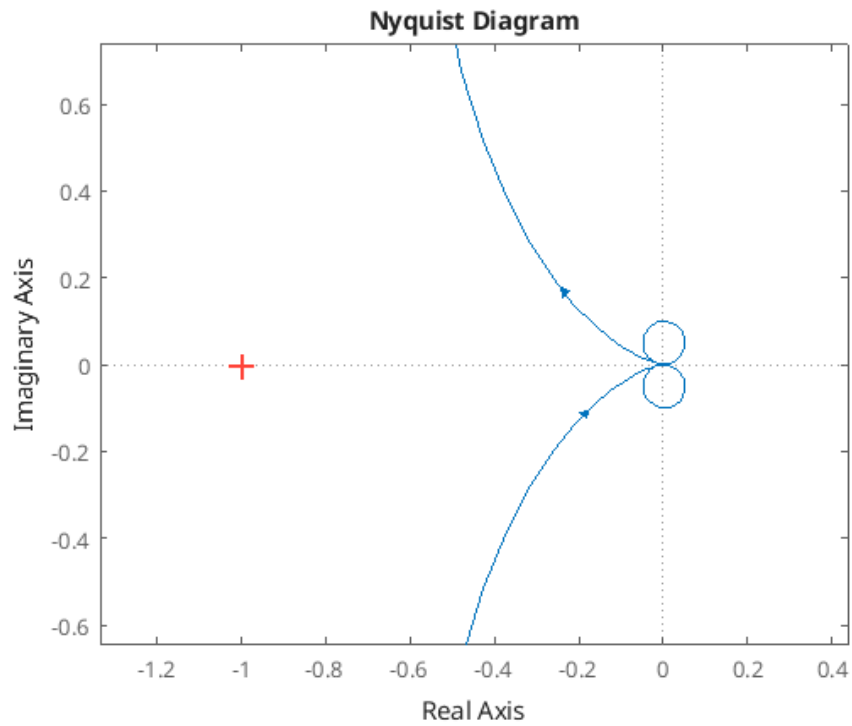
So you can see the Bode plots above, I also plotted the Nyquist plots using the 'nyquist' function in matlab for the analytical and converted the frequency response to complex form for the empirical plot using the following conversion:

$$C(j\omega)G(j\omega) = K|G(j\omega)| \cos(\phi(\omega)) + jK|G(j\omega)| \sin(\phi(\omega))$$

The Matlab code for that conversion/plotting looks like this:

```
1 realPart = K*10.^(pResponse(:,1) / 20) .* cosd(pResponse(:,2));
2 imagPart = K*10.^(pResponse(:,1) / 20) .* sind(pResponse(:,2));
3
4 figure;
5 plot(realPart, imagPart, 'b', 'LineWidth', 1.5);
6 hold on;
7 plot(realPart, -imagPart, 'b--', 'LineWidth', 1.5);
```

The results look like this:



Which look pretty similar which I was happy about. I also drew the Nyquist plot and diagram of the analytic transfer function by hand so that we could see what is going on around infinite to get an idea of the encirclements.

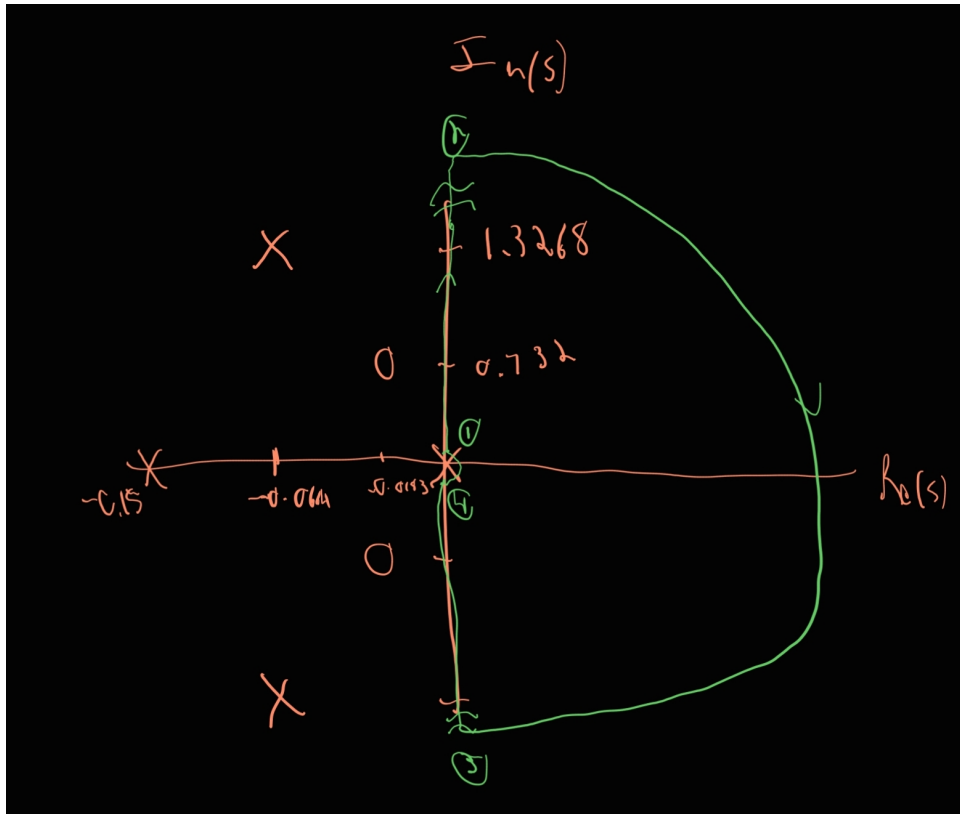


Figure 2: Nyquist Plot of Analytic TF

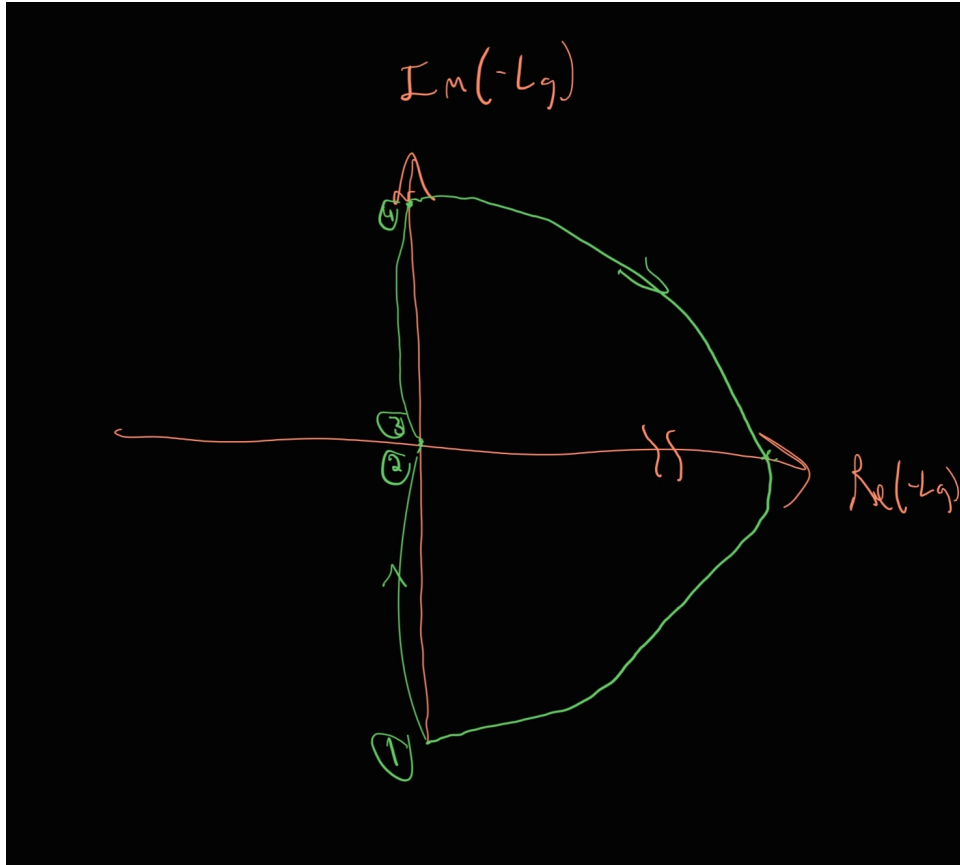


Figure 3: Nyquist Diagram of Analytic TF

As you can see the diagram starts out at point 1 at -90 degrees and negative infinite and then as it goes to point 2 it goes to 0 as the magnitude of the denominator takes over and the 4 poles and 2 zeros sum to -180 degrees as it does so. Then it goes to point 3 and the magnitude stays big but the phase flips to +90, then the magnitude grows to get to point 4 and as point 4 comes back around to point 1 the phase flips again going from positive 90 to negative 90 and you see that -180 curve around infinite in the diagram. This means that there are no encirclements at  $K = 1$  and there are no unstable poles in the right half plane so the system is stable there. As I increase  $K$  you can see how that phase margin is shrinking but will never quite touch that stability criterion.

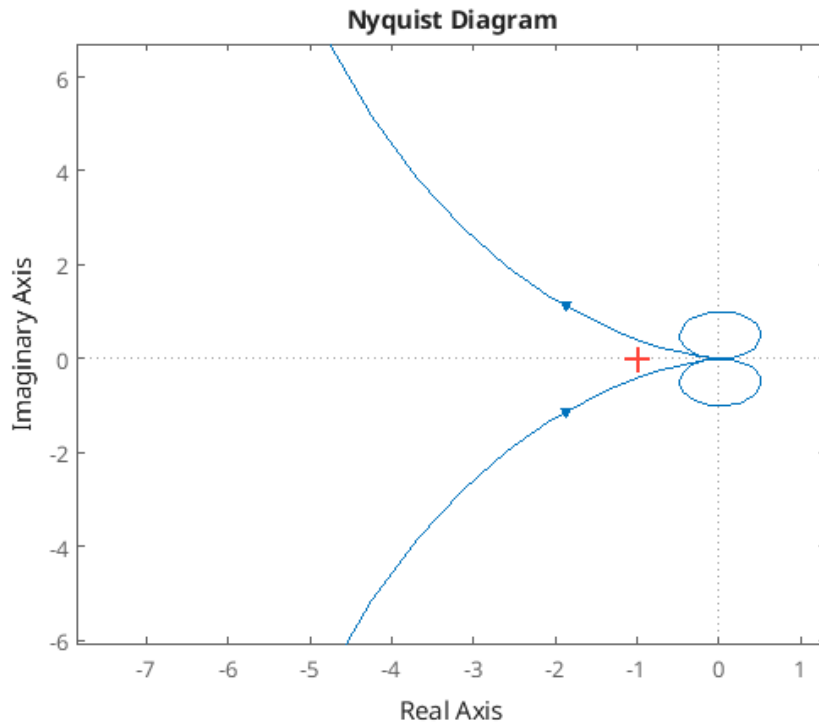


Figure 4:  $K=10$  Nyquist Diagram

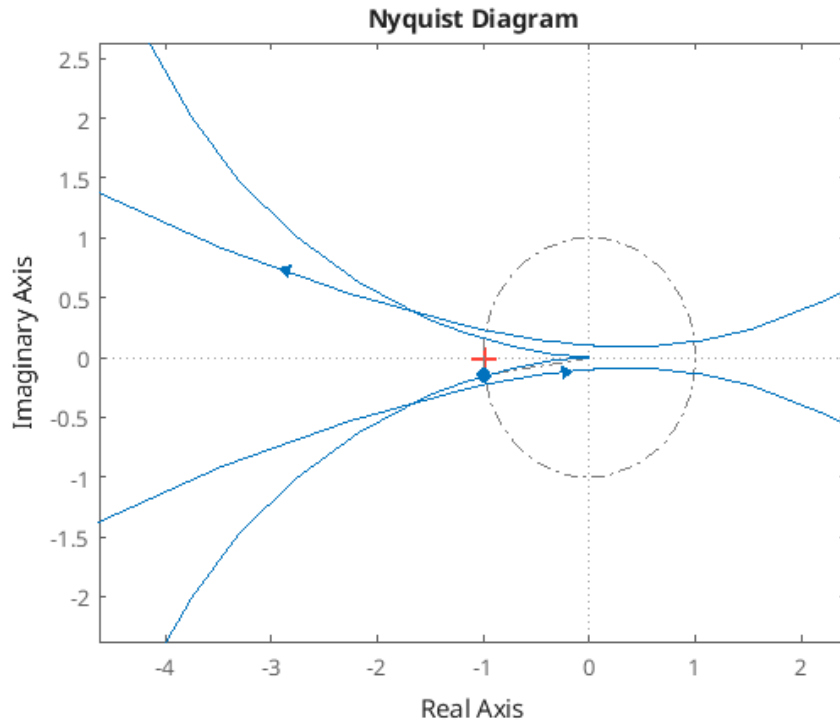
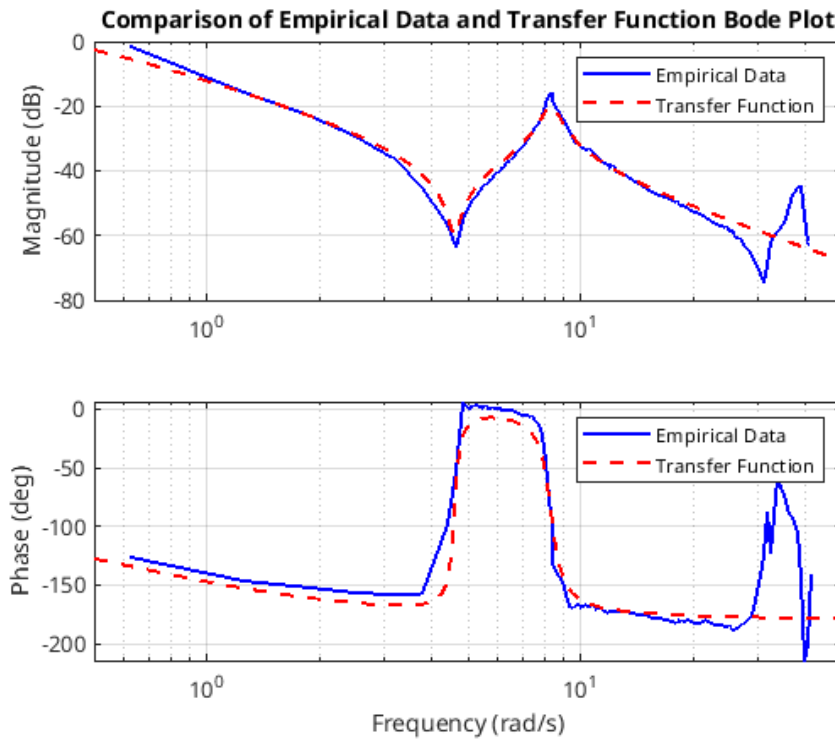


Figure 5: K=100 Nyquist Diagram

This corresponds to that gain crossover frequency moving down the Bode plot as the gain increases and the phase margin getting closer and closer to -180 degrees but never quite touching it.



However, the empirical frequency response does cross over -180 degree phase a couple of times and so you would expect that there would be a high gain where you would see a nyquist diagram encirclement and therefore some instability. Below you can see where I set K to 1000 for the empirical transfer function and how that creates an encirclement.



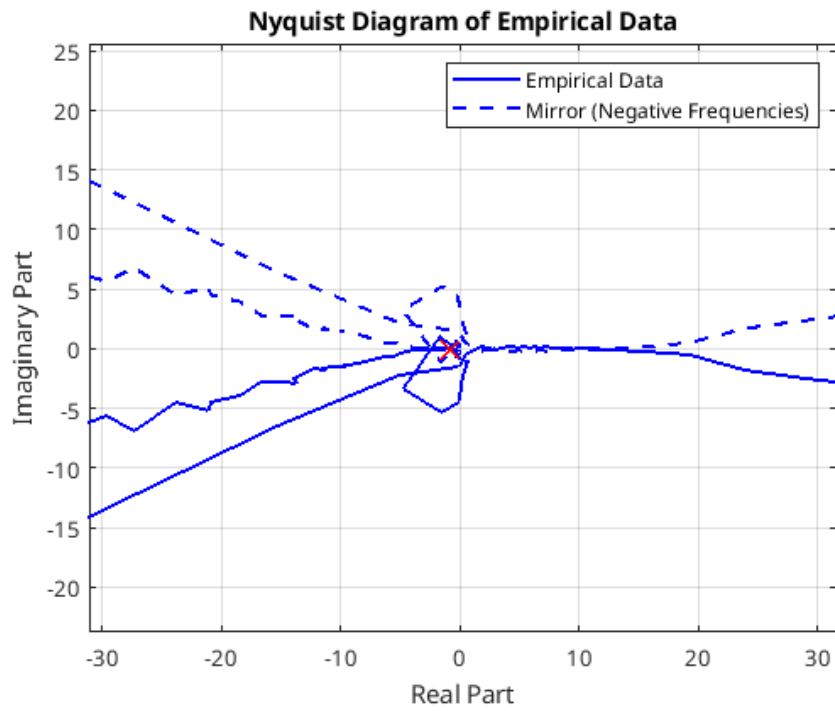


Figure 6:  $K=1000$  Empirical Nyquist Diagram

Also, any negative values of  $K$  will result in an encirclement of the -1 point and therefore an unstable system.

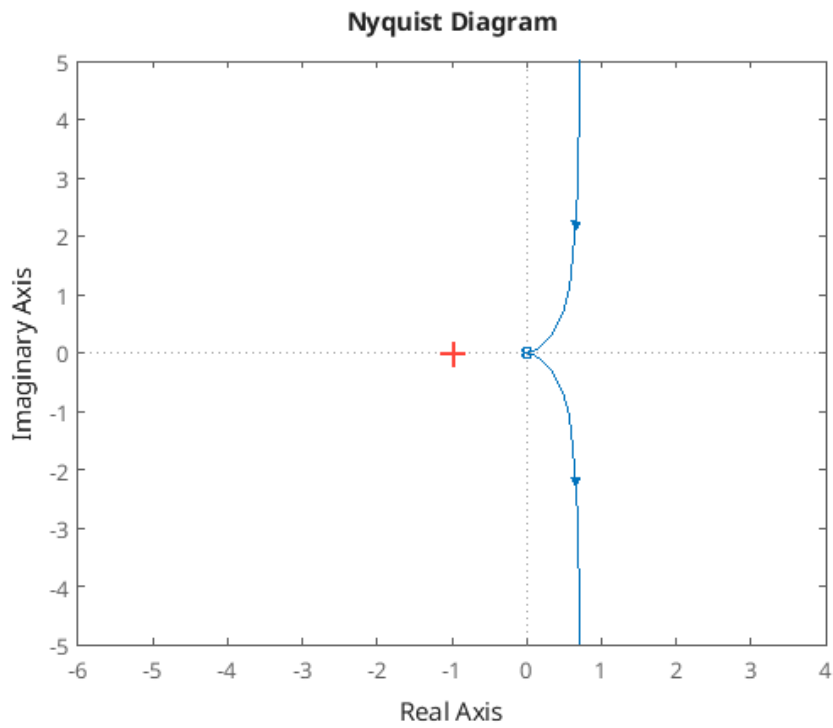


Figure 7:  $K=-1$  Nyquist Diagram

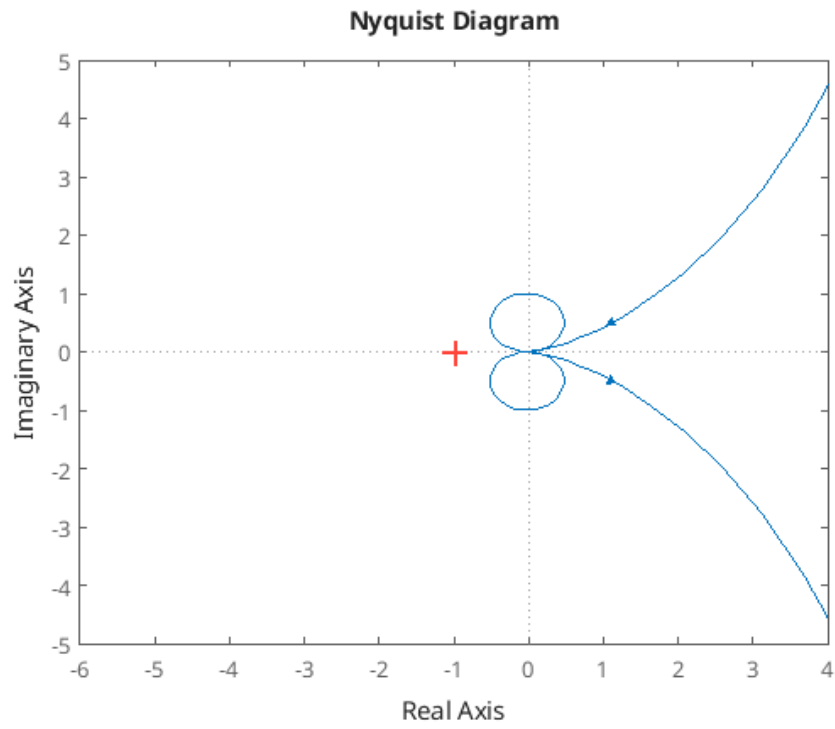


Figure 8:  $K=-10$  Nyquist Diagram

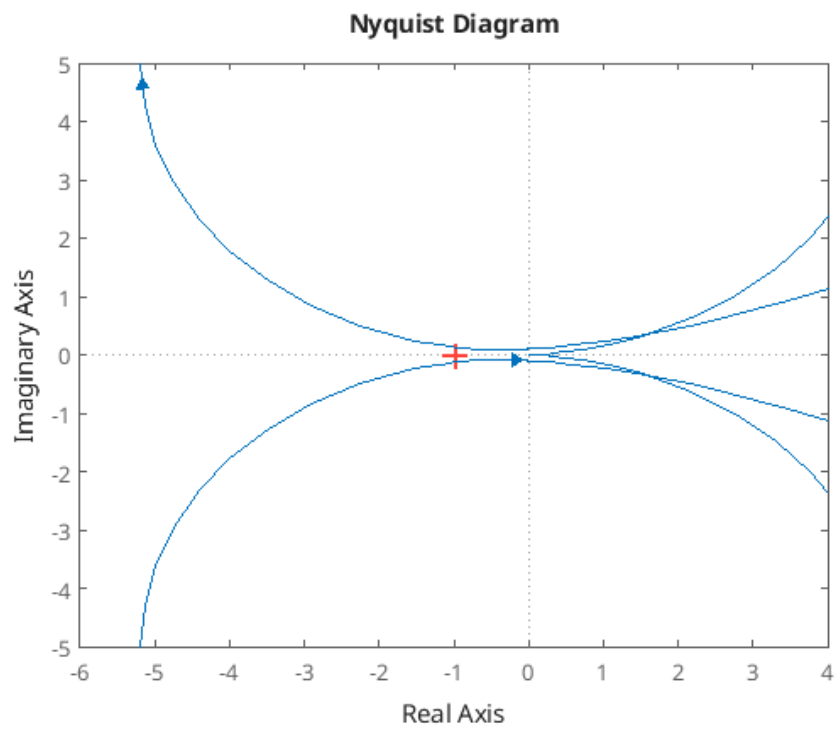
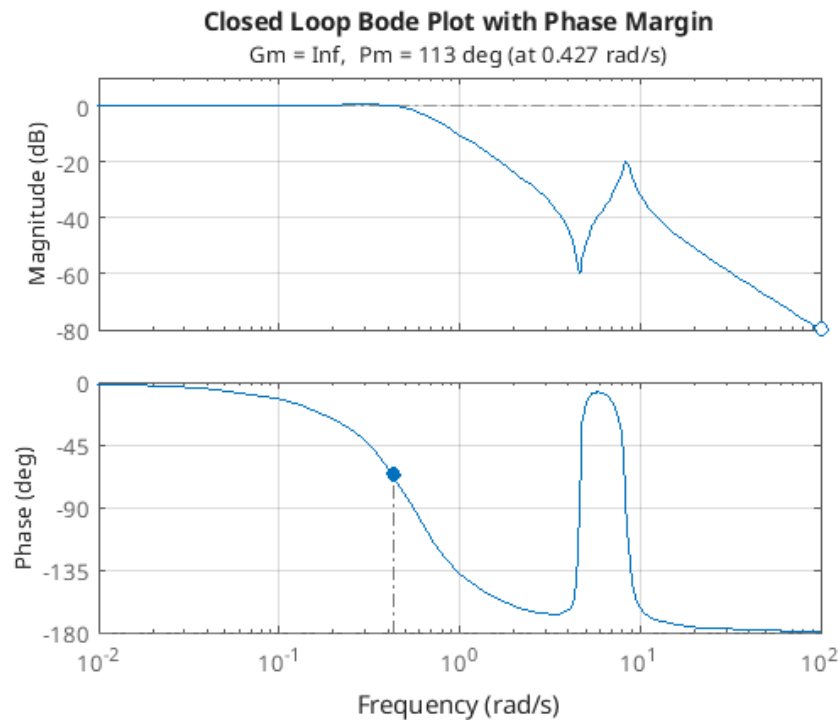


Figure 9:  $K=-100$  Nyquist Diagram

3)

[30 pts] Design a compensator  $C_1(s)$  so that the unity feedback control system with  $C(s) = KC_1(s)$  has at least 40 deg phase margin, 10 dB of gain margin, and a closed loop (-3 dB) bandwidth as close to 1 Hz as possible. Use the analytic plant model in this design. Compute and plot the closed loop tracking frequency response.

I started by plotting the unity feedback with no controller ( $G/(1+G)$ ) then decided how to compensate it to achieve the desired performance and stability.



I played around with just increasing the gain and adding a lead compensator at first to bump the phase up but the resonance and antiresonance are situated around the 1 Hz frequency making it so that there are several gain crossover frequencies as you can see in the plot below where I just have  $K=65$  and no compensator:

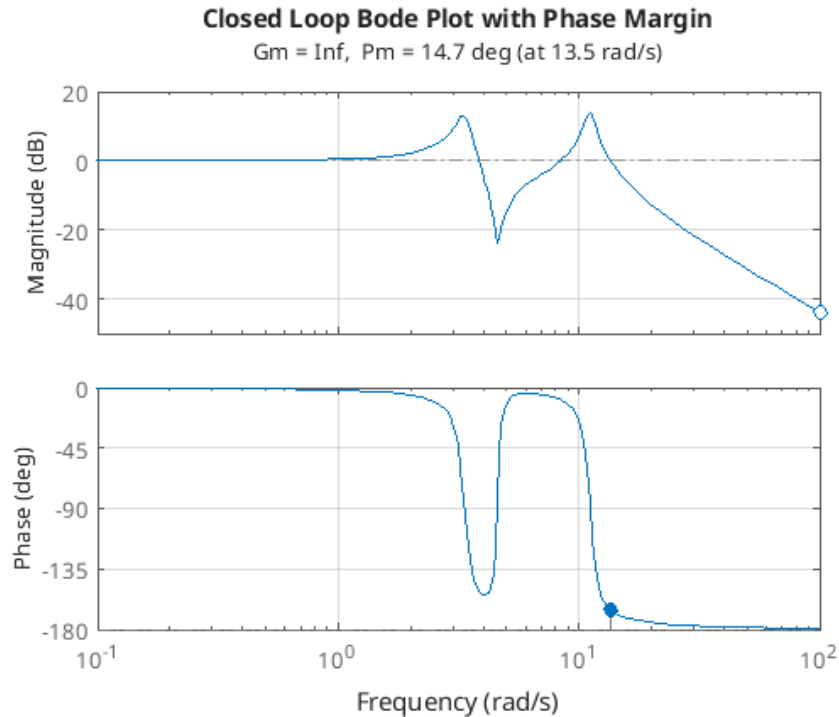


Figure 10: Closed Loop Bode Plot with Gain Only

It would not be accurate to say that the middle crossover frequency is the closed loop bandwidth because the tracking dips down significantly (way below -3dB) before that point and the purpose of the closed loop bandwidth is that the tracking is good up until that point. It wouldn't be accurate either to say that the middle crossover frequency is where you would measure the phase margin from since the phase margin is the worst case scenario, measured from the closest frequencies phase. At the beginning I had unofficially decided to myself not to just try to cancel the resonance and antiresonance because that cancellation would not work out perfectly in the real plant. However, I played around with a bit using the empirical data and decided that the cancellation would work well enough and it would be the only way to get performance that I would be satisfied with. So I added a second order compensator element to the numerator and denominator of the controller with the same values at opposite points to cancel the resonance and antiresonance and then I added the gain K and lead compensator for the extra phase like so:

```

1 K = 65;
2 a = 1;
3 b = 10;
4 leadNumC = [1 a];
5 leadDenC = [1 b];
6 numC = [1 zetaP*omegaP omegaP^2];
7 denC = [1 zetaZ*omegaZ omegaZ^2];
8 numC = conv(numC, leadNumC);
9 denC = conv(denC, leadDenC);
10 C = tf(K * numC, denC);
11 closedTF = C*estTF/(1 + C*estTF);

```

The results are below:

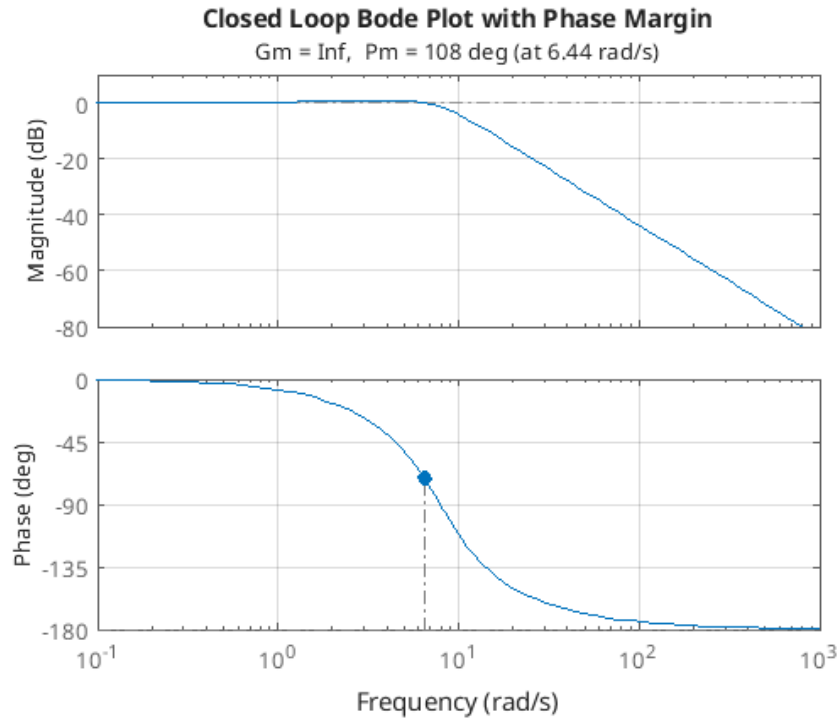


Figure 11: Closed Loop Bode Plot Full Controller

You can see the closed loop bandwidth is around 1 Hz (6.44 rad/s), the phase margin there is 108 degrees, and the gain margin is infinite since the analytical frequency response never crosses -180 degrees.

#### 4)

[10 pts] Repeat part 4 using the empirical plant frequency response. Adjust the control design as necessary to try to maintain the stability and performance objectives. Comment on the effects of the unmodeled dynamics at high frequency.

I applied the same controller to the empirical data using a frequency response data ('frd') Matlab object for the empirical data as below:

```
1 G = frd(10.^(pResponse(:,1)/20) .* exp(1j * deg2rad(pResponse(:,2))), pResponse(:,3));
2 empiricalClosedTF = G * C/(1 + G * C);
```

As you can see in the plot below the controller does not work as well on the empirical data although the basic shape is maintained.

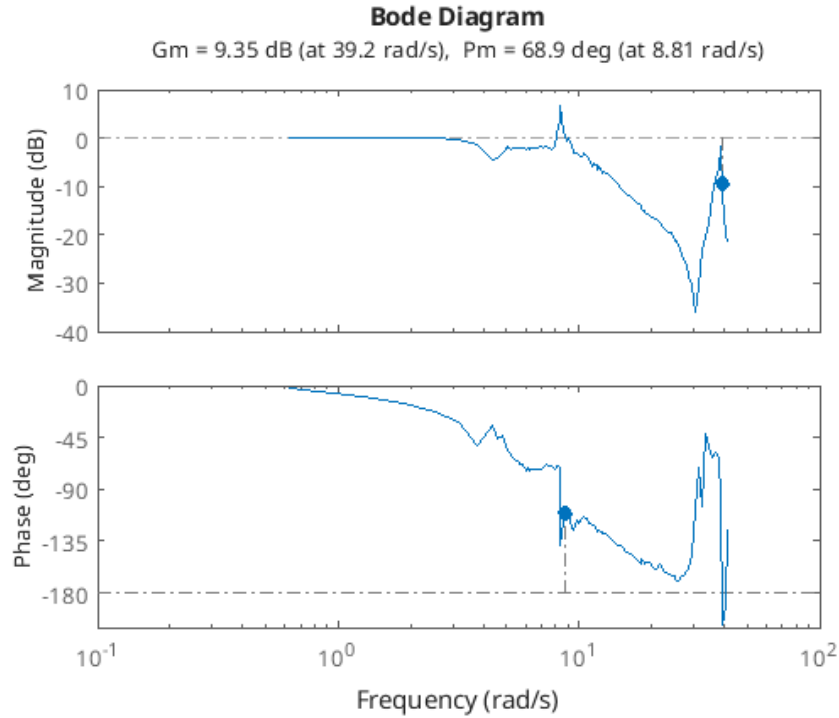


Figure 12: Empirical Closed Loop Bode Plot Full Controller

The closed loop bandwidth is around 8.81 rad/s because of the not-completely-cancelled resonances, the worst case phase margin is still 68.9 degrees which meets the stability requirements, the gain margin is 9.35 dB at 39.2 rad/s which is pretty close to the requirement. It's hard to get the same performance with the empirical data because of the uncanceled dynamics and the high frequency unmodelled dynamics. You can see the unmodelled dynamics in the plot above the resonance goes all the way up almost to 0 db indicating that there is some high frequency where there is good tracking. You can also see how the unmodelled dynamics create phase dip at high frequencies which cross over -180 degrees which indicates that there is some high frequency where the system could be unstable.

Below I show the effects of adding some compensation to cancel the high frequency resonances and you can see how that improves the gain margin a bit.

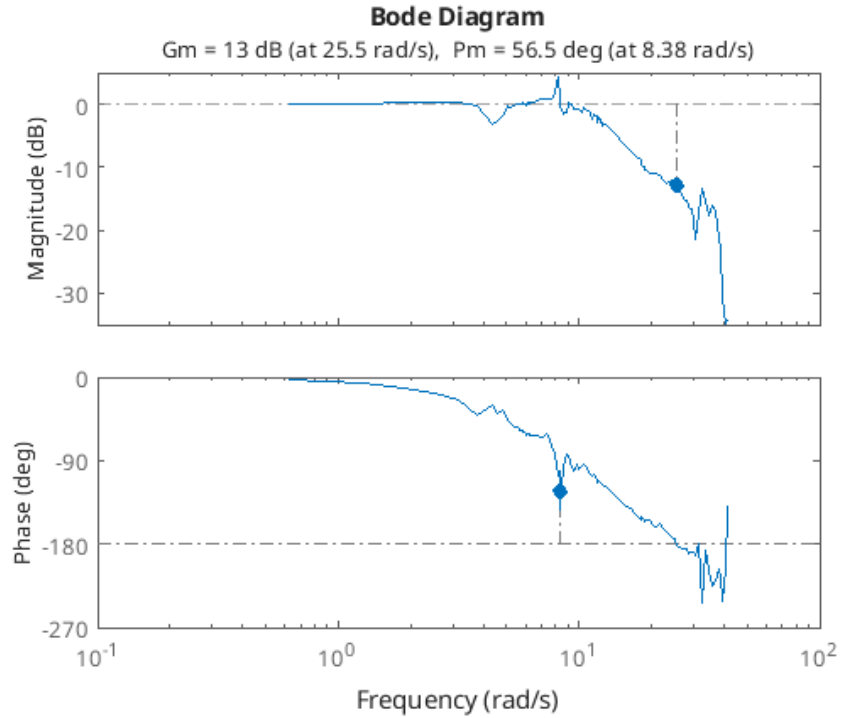


Figure 13: Empirical Closed Loop Bode Plot with Cancelled Unmodelled Dynamics

5)

[20 pts] Simulate the analytic model in a unity feedback control loop in Simulink using the control design from part 4. Plot the responses to a 0.5 rad step input, and to single-sinusoid inputs at 0.5 Hz and 2.0 Hz, each with amplitude 0.5 rad. Also plot the torque input signal, along with its limiting values. Compare these responses with expectations from loop gain analysis for closed loop frequency response, DC tracking accuracy, and stability margins.

I created the model in Simulink using two continuous transfer function blocks, a step/sine generator, a sum block, and some scope blocks. I put the transfer function variables directly into the blocks from the workspace and you can see the model below:

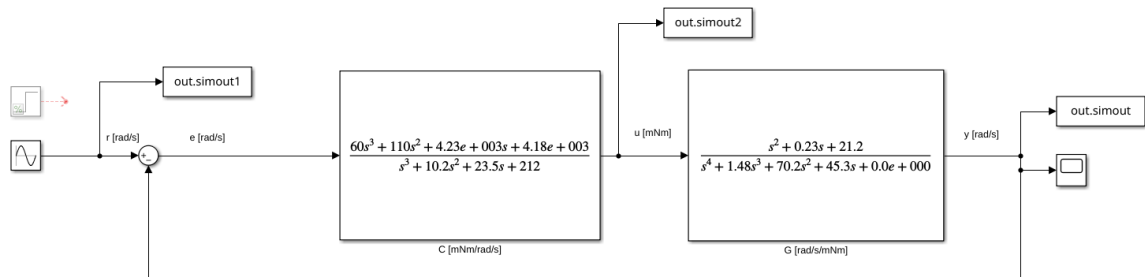
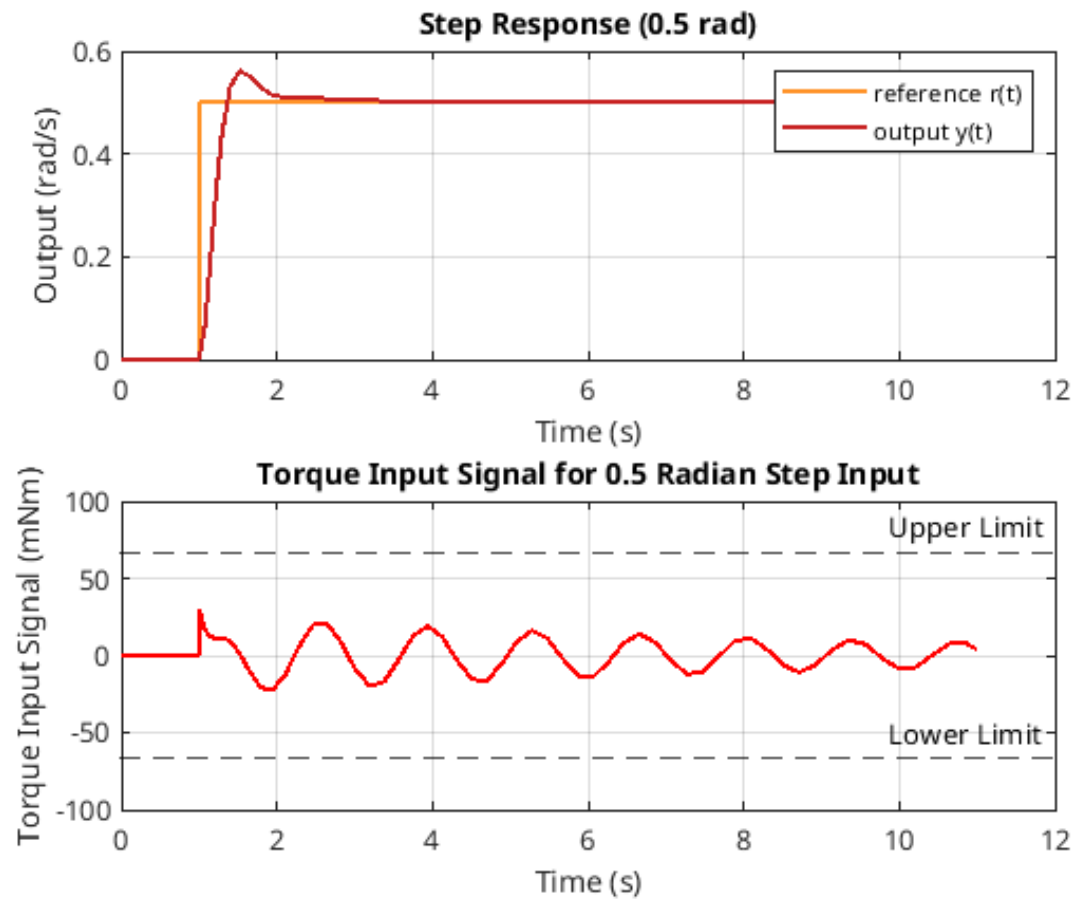


Figure 14: Simulink Closed Loop Model

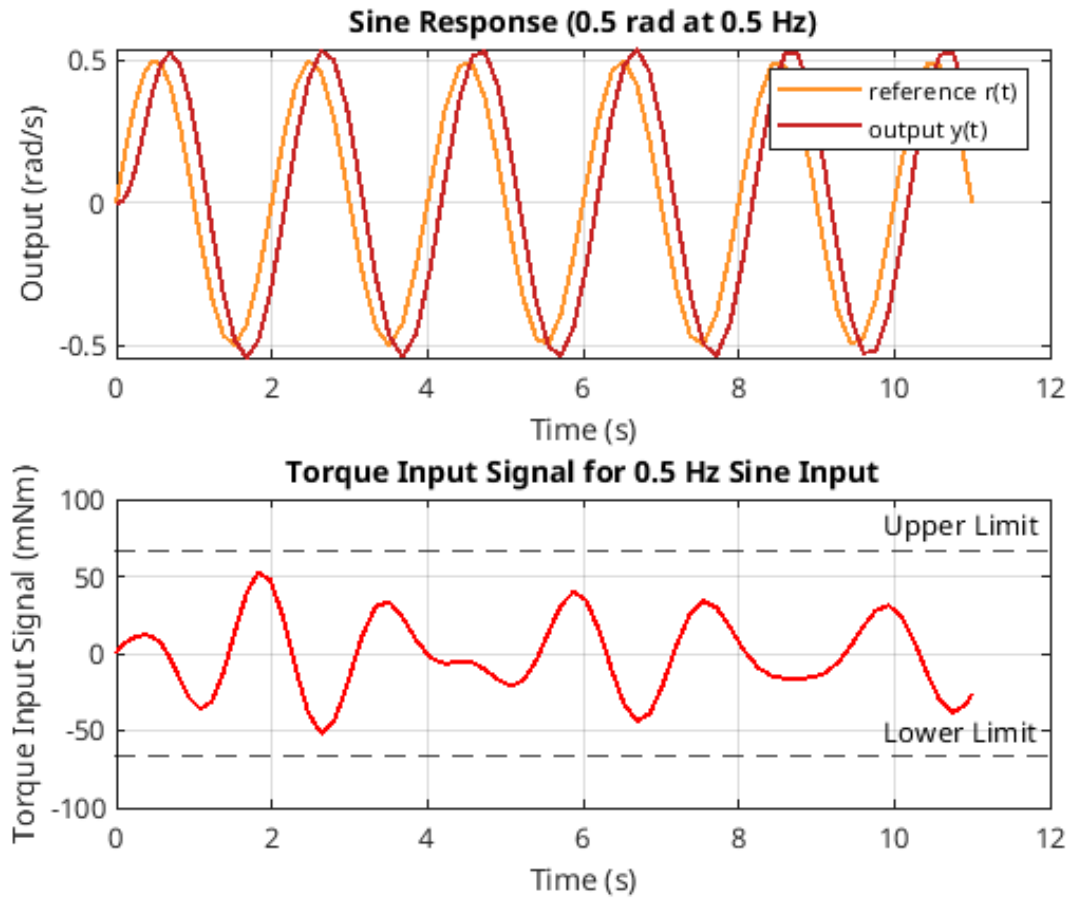
I plotted the torque input signal using a scope block on the output of the controller and setting the limits to  $\pm 67$  mNm as per motor torque limit specs.

Then I ran simulations with the step input and the two different sine inputs and the results are below:

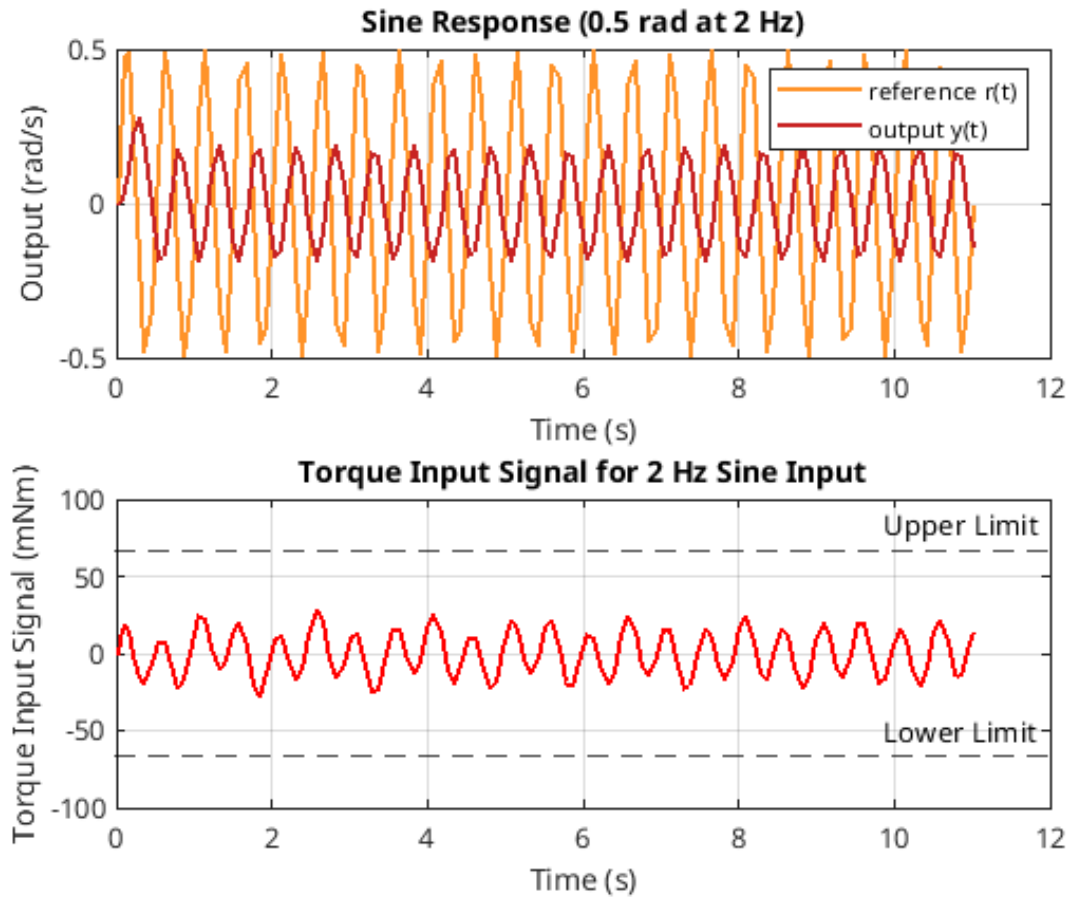


Firstly from the step response you can see that the response is lightly damped and closely tracks the reference indicating good phase margin and tracking which aligns with the loop gain analysis. You can see the control effort below that and how it spikes up to 30 mNm initially and then slowly settles.





Next on the 0.5 Hz sine response you can see how the DC tracking and phase is pretty close which you would expect for any frequency below the closed loop bandwidth (1 Hz). The control effort here is also the highest of the three simulations here and spikes up to  $\pm 50$  mNm at one point.



Lastly in the 2 Hz sine response you can see the attenuation that you would expect for frequencies above 1 Hz and you can see the increased phase lagging effect you would expect from the frequency response analysis. The lack of instability in the responses also speaks to the stability margins being sufficient as expected.

6)

[20 pts – graduate credit only] Suppose the spacecraft had a model where the first resonance occurred before the anti-resonance in frequency. Sketch this Bode plot, and determine if your controller designed in Part 4 would produce a stable system. Comment on the difficulty of achieving the above closed loop stability and tracking bandwidth performance with this “non-colocated” plant.

So I started by plotting the same analytical transfer function with the location of the resonance and antiresonance flipped so the resonance occurs first. You can see immediately that flipping the order creates a big phase dip instead of the phase bump we had before which is obviously going to create stability issues, in the plot the phase dips down to  $-360^\circ$  at the first resonance instead of staying above  $-180^\circ$  like it was before.

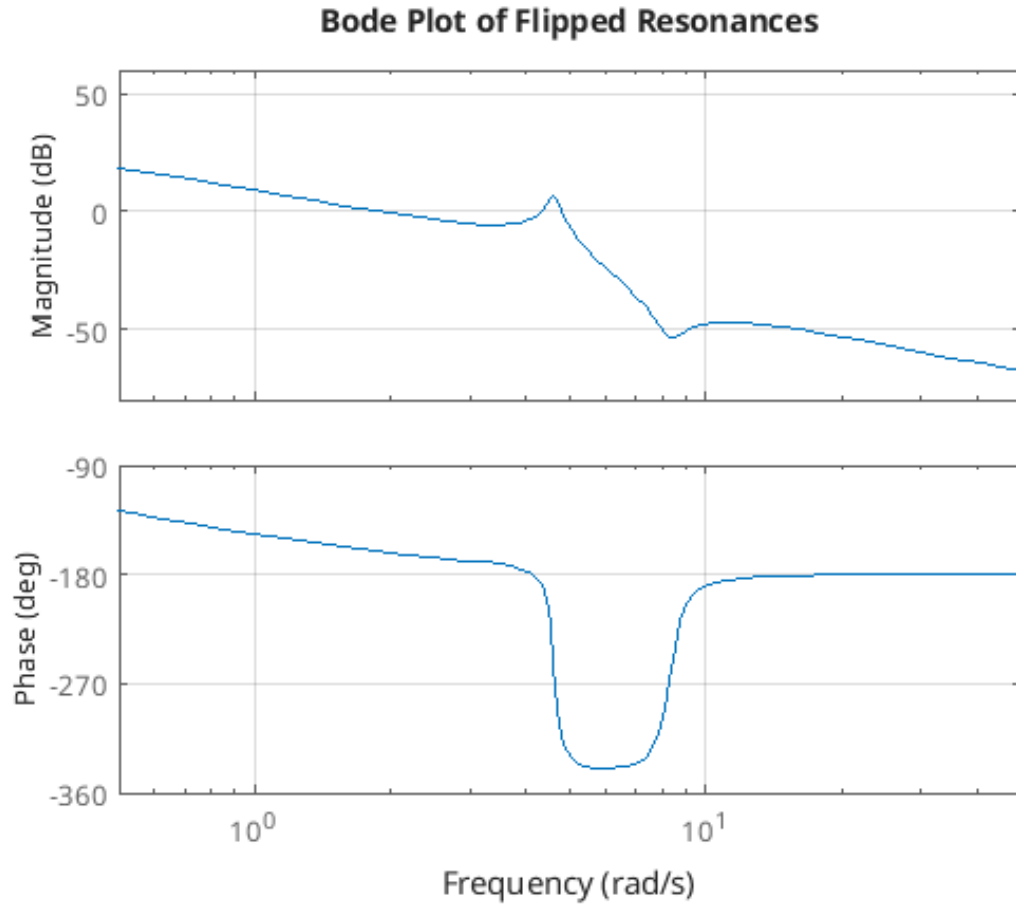


Figure 15: Bode Plot with Flipped Resonance

The system cannot be stabilized with exactly the same controller as above because the controller elements that cancelled the resonances before would amplify the opposite resonances. However, the same technique could be used to cancel the corresponding resonances with a similar effect except that the imperfect cancellation problem would be more of a problem in this case. The system overall has much more phase lagging behavior and so more of the uncanceled dynamics would be phase lagging and therefore would likely reduce your phase margin and create more concerns with stability. There is more likely to be some unmodelled, uncanceled dynamics that dip below -180 degrees and create an unstable system. It does seem that getting good tracking bandwidth performance would be easier however because as you can see in the plot above there are less points where you would get multiple gain crossover frequencies, although maybe the phase margin difficulties would make it harder to place the gain crossover frequency where you want it.

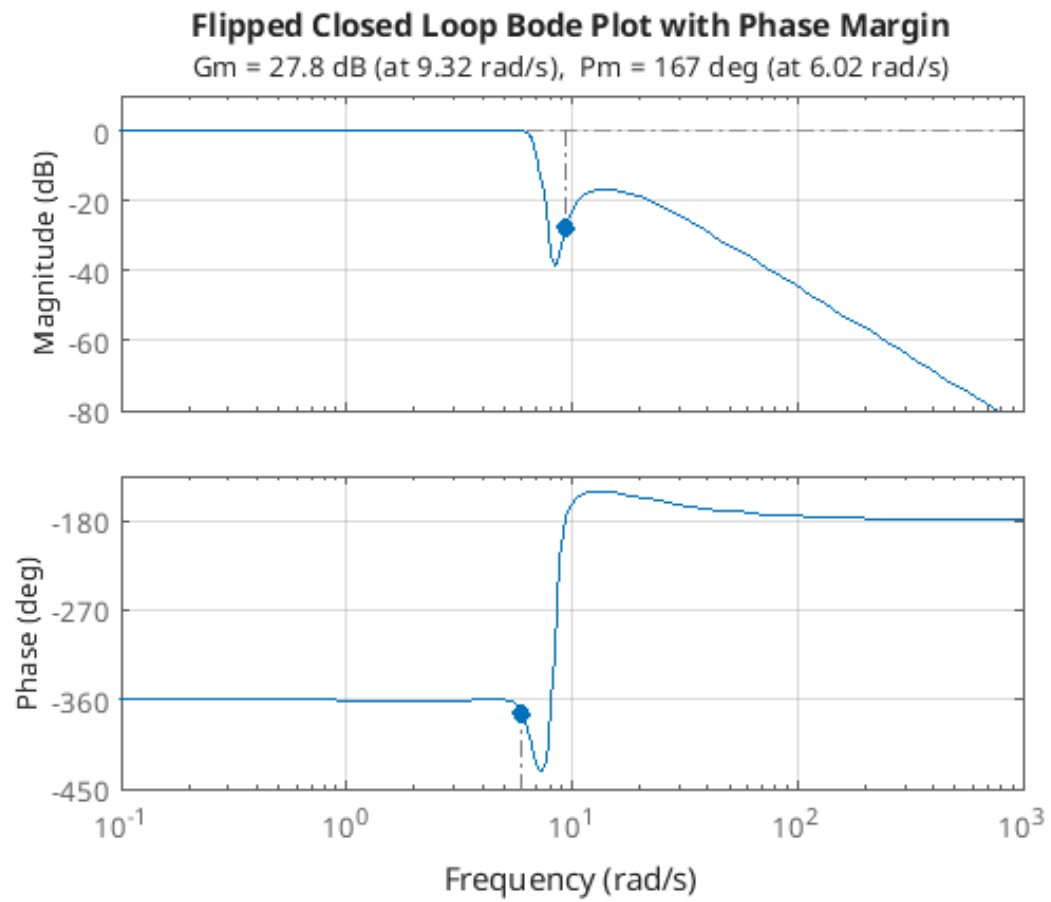


Figure 16: Flipped Resonance Bode Plot with Same Controller

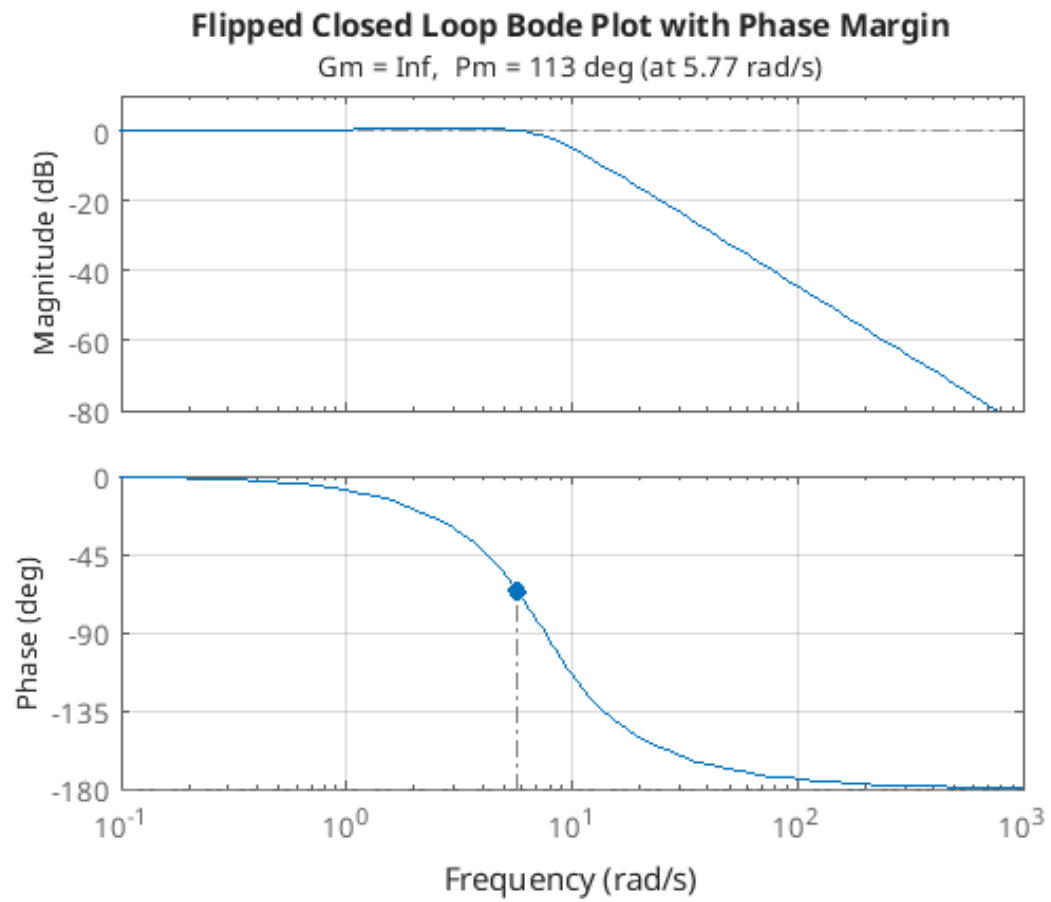


Figure 17: Flipped Resonance Bode Plot with Cancellation Controller