Homework 1

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February 3, 2025

"Due: Monday, February 3, 2025 at 11:59 pm on Canvas. Please assemble a single PDF file for submission that includes your Matlab/Simulink code/diagrams, plots, and explanations of your work and the results. Label sections to correspond with those in the assignment. Don't make it difficult to locate the text/code/plots for each section."

1.

"[10pts] Find the parameters R_M, L_M, K_τ, J_M, and K_B from the motor specification sheet, noting units. Also, find the total gear ratio N from the motor shaft to the load shaft, and estimate the load shaft moment of inertia J_L. Use these to quantify the parameters in the transfer function relating V_P to Θ_L. Also, estimate the potentiometer scale factor K_S from the data file posted on Canvas."

Stock program
Standard program
Special program (on request)

		Part Numbers										
with terminals		110181	110182	110183	110184	110185	110186	110187	110188	110189	110190	110191
with cables		353078	353079	353080	353081	329757	353082	332818	353083	353084	353085	353086
Motor Data												
Values at nominal voltage												
1 Nominal voltage	V	4.5	6	9	12	15	18	24	30	36	42	48
2 No load speed	rpm	7320	8670	6160	6780	6720	6690	5670	6090	6780	6570	6050
3 No load current	mA	78.9	77.7	30.2	26.3	20.7	17.1	9.97	8.9	8.76	7.15	5.5
4 Nominal speed	rpm	6900	8130	5000	5340	5060	5010	3940	4370	5060	4820	4280
5 Nominal torque (max. continuous torque)	mNm	4.46	5.02	11.3	13.7	15.8	15.6	15.3	15.3	15.2	15	15
6 Nominal current (max. continuous current)	A	0.84	0.84	0.84	0.84	0.766	0.627	0.391	0.336	0.31	0.254	0.204
7 Stall torque	mNm	67.3	73.5	58.8	63.5	63.6	62.1	50.3	54.2	60.2	56.4	51.4
8 Stall current	A	11.5	11.2	4.25	3.78	3.01	2.43	1.25	1.16	1.2	0.93	0.683
9 Max. efficiency	%	84	84	84	84	84	84	83	84	84	84	83
Characteristics												
10 Terminal resistance	Ω	0.39	0.536	2.12	3.17	4.99	7.41	19.2	25.8	30.1	45.1	70.2
11 Terminal inductance	mH	0.04	0.051	0.227	0.333	0.529	0.77	1.9	2.58	2.99	4.34	6.68
12 Torque constant	mNm/A	5.84	6.57	13.9	16.8	21.2	25.5	40.1	46.7	50.3	60.6	75.2
13 Speed constant	rpm/V	1640	1450	689	569	451	374	238	205	190	158	127
14 Speed/torque gradient	rpm/mNm	109	119	105	108	106	108	114	113	114	117	119
15 Mechanical time constant	ms	16.5	16	15	14.9	14.8	14.8	14.9	14.9	14.9	15	15
16 Rotor inertia	gcm²	14.4	12.9	13.6	13.2	13.3	13.1	12.5	12.6	12.5	12.2	12.1

I took the parameters from the motor specification table above using part number 110187.

$$
\begin{aligned}
R_M &= 19.2\ \Omega \\
L_M &= 1.9\text{mH} = 0.0019\ \text{H} \\
K_\tau &= 40.1\text{mNm/A} = 0.0401\ \text{Nm/A} \\
J_M &= 12.5\text{gcm}^2 = 0.00000125\ \text{kgm}^2 \\
K_B &= 238\text{rpm/V} = 0.04\ \text{Vs/rad} \\
N &= 3.3 \\
J_L &= 0.018522\ \text{kgm}^2 \\
K_S &= 0.8\ \text{V/rad}
\end{aligned}
$$

I derived N by using the logic that the gear ratio measured in radii is equal to the gear ratio measured in circumferences and if the teeth on both gears are the same size then you could measure the circumference in number of teeth and take the ratio of numbers of teeth. The load shaft gear has 120 teeth and the motor shaft gear has 36, making the ratio 3.33.

I estimated J_L, using standard inertia formulas for a rod rotating about one end and a satellite (for the weight) and added them together.
Equation for rod: $J_{\text{arm}} = \frac{1}{3} m_{\text{arm}} L^2$
Equation for weight: $J_{\text{weight}} = m_{\text{weight}} L^2$

I got the mass of the arm and weight using their measurements and the average density for aluminum and brass.

- Arm Dimensions: $L = 30$ cm, $h = 0.7$ cm, $w = 1.1$ cm
- Volume:
$$V_{\text{arm}} = L \times h \times w = 30 \times 0.7 \times 1.1 = 23.1 \text{ cm}^3 \tag{1}$$
- Density of aluminum: $\rho_{\text{Al}} \approx 2.7$ g/cm$^{\mathbf{3}}$
- Mass of the arm:
$$m_{\text{arm}} = V_{\text{arm}} \times \rho_{\text{Al}} = 23.1 \times 2.7 = 62.37 \text{ g} = 0.0624 \text{ kg} \tag{2}$$
- Weight Dimensions: $3.2 \times 2.0 \times 3.4$ cm
- Volume:
$$V_{\text{brass}} = 3.2 \times 2.0 \times 3.4 = 21.76 \text{ cm}^3 \tag{3}$$
- Density of brass: $\rho_{\text{brass}} \approx 8.5$ g/cm$^{\mathbf{3}}$
- Mass of the brass weight:
$$m_{\text{brass}} = V_{\text{brass}} \times \rho_{\text{brass}} = 21.76 \times 8.5 = 184.96 \text{ g} = 0.185 \text{ kg} \tag{4}$$

Then I used those numbers (including length of arm for L) to calculate the moments of inertia and combine them.

$$J_{\text{arm}} = \frac{1}{3} m_{\text{arm}} L^2 \tag{5}$$

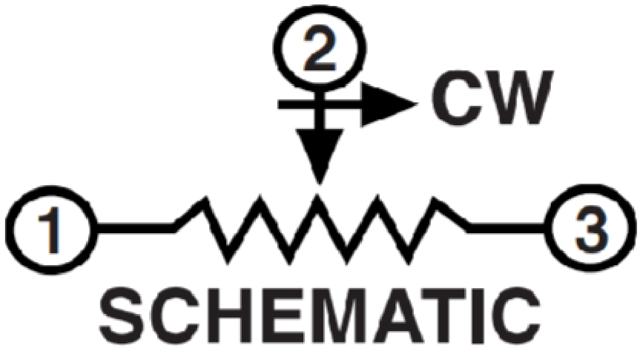
$$J_{\text{arm}} = \frac{1}{3}(0.0624)(0.30)^2 = 0.001872 \text{ kgm}^2 \tag{6}$$

$$J_{\text{weight}} = m_{\text{weight}} L^2 \tag{7}$$

$$J_{\text{weight}} = (0.185)(0.30)^2 = 0.01665 \text{ kgm}^2 \tag{8}$$

$$J_L = J_{\text{arm}} + J_{\text{brass}} \tag{9}$$

$$J_L = 0.00187 + 0.01645 = 0.018522 \text{ kgm}^2 \tag{10}$$



Measurement of port 2, with respect to port 1 (GND)

-90 Deg shows 1.25 V

0 Deg shows 2.488 V

+90 Deg shows 3.76 V

To derive K_S I used the readings/schematic above. I took the total voltage difference over the total angle difference (converted to radians).

$$K_S = (3.76V - 1.25V)/(\frac{\pi}{2}\text{rad} + \frac{\pi}{2}\text{rad}) \tag{11}$$

$$K_S = 0.8 \text{ V/rad} \tag{12}$$

The transfer function that relates V_P to Θ_L is:

$$\frac{-1}{\frac{1}{NK_\tau} J_{eq} L_M s^3 + \frac{1}{NK_\tau} J_{eq} R_M s^2 + NK_B s} \tag{13}$$

I derived J_{eq} using:

$$J_{eq} = J_L + N^2 J_M$$
$$J_{eq} = 0.018535612 kgm^2$$

Plugging in the parameter values:

$$\frac{-1}{0.000266135\ V \cdot sec^3\ s^3 + 2.6893656\ V \cdot sec^2\ s^2 + 0.132\ \frac{V \cdot sec}{rad}\ s} \tag{14}$$

2.

"[20pts] Simulate the system relating power amp voltage V_P to sensor voltage V_S using a single transfer function block from the Continuous library in Simulink. I suggest you compute polynomial coefficients in a separate m-file (as in the intro_sims.m example), so the simulink system can be built using simple variable names from the workspace. Use a sinusoidal power amp input voltage (1V peak, 1Hz), and plot the corresponding input and output signals, with appropriate units. Does the output follow the input closely?"

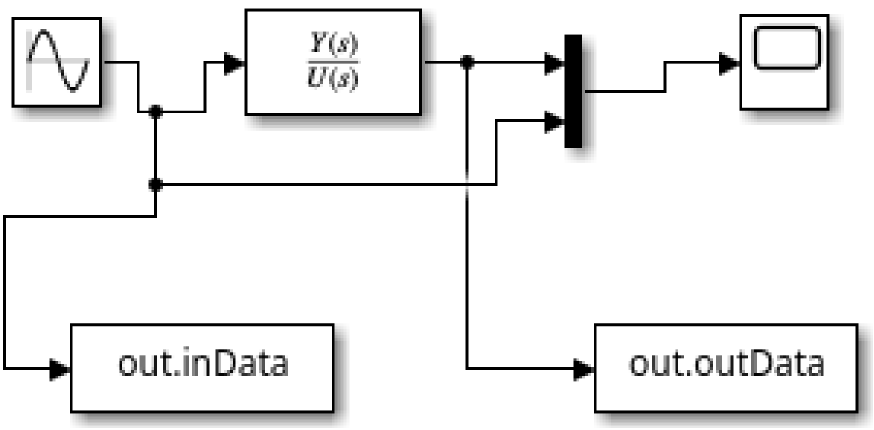
I set up the variables for the transfer function as suggested in a matlab file as shown below.

```
coeff3=0.000266135; % [Vs^3]
coeff2=2.6893656; % [Vs^2]
coeff1=0.132; % [Vs/rad]
num = [-1];
den = [coeff3, coeff2, coeff1, 0];
format longG;
disp(roots(den));
```

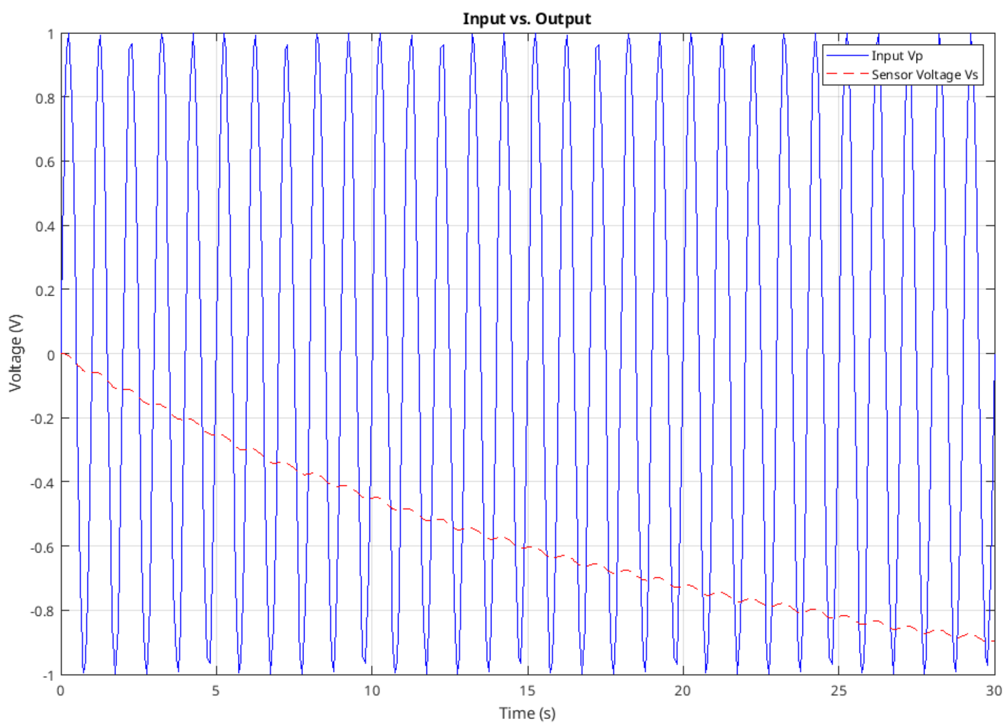
```
plot(out.inData.time, out.inData.Data, 'b-', ...
     out.outData.time, out.outData.Data, 'r--');

xlabel('Time (s)');
ylabel('Voltage (V)');
legend('Input Vp', 'Sensor Voltage Vs');
title('Input vs. Output');
grid on;
```

I then created my model in Simulink using num and den from the m file for the transfer function, a 1V-1Hz sine wave input, and a couple 'To Workspace' blocks so I could plot the input/output.



Then I ran the simulation and plotted the results as you can see below:



As for whether the output follows the input I would say in one sense it does, the oscillations match up. When the input goes down so does the output at the same time. However, in another sense the output doesn't really follow the input, it does small oscillations but mostly just slowly converges to -1.

3.

"[10pts] Derive the closed loop transfer function between reference angle Θ_R *and load shaft angle* Θ_L *when the controller has a proportional-plus-derivative (PD) control law, i.e.* $V_P = G_P(\Theta_R - \Theta_L) + G_D \frac{d}{dt}(\Theta_R - \Theta_L)$*. Determine the DC gain of the closed loop system."*