Homework 1

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February 3, 2025

"Due: Monday, February 3, 2025 at 11:59 pm on Canvas. Please assemble a single PDF file for submission that includes your Matlab/Simulink code/diagrams, plots, and explanations of your work and the results. Label sections to correspond with those in the assignment. Don't make it difficult to locate the text/code/plots for each section."

1.

"[10pts] Find the parameters R_M, L_M, K_τ, J_M, and K_B from the motor specification sheet, noting units. Also, find the total gear ratio N from the motor shaft to the load shaft, and estimate the load shaft moment of inertia J_L. Use these to quantify the parameters in the transfer function relating V_P to Θ_L. Also, estimate the potentiometer scale factor K_S from the data file posted on Canvas."

Stock program
Standard program
Special program (on request)

		Part Numbers										
with terminals		110181	110182	110183	110184	110185	110186	110187	110188	110189	110190	110191
with cables		353078	353079	353080	353081	329757	353082	332818	353083	353084	353085	353086
Motor Data												
Values at nominal voltage												
1 Nominal voltage	V	4.5	6	9	12	15	18	24	30	36	42	48
2 No load speed	rpm	7320	8670	6160	6780	6720	6690	5670	6090	6780	6570	6050
3 No load current	mA	78.9	77.7	30.2	26.3	20.7	17.1	9.97	8.9	8.76	7.15	5.5
4 Nominal speed	rpm	6900	8130	5000	5340	5060	5010	3940	4370	5060	4820	4280
5 Nominal torque (max. continuous torque)	mNm	4.46	5.02	11.3	13.7	15.8	15.6	15.3	15.3	15.2	15	15
6 Nominal current (max. continuous current)	A	0.84	0.84	0.84	0.84	0.766	0.627	0.391	0.336	0.31	0.254	0.204
7 Stall torque	mNm	67.3	73.5	58.8	63.5	63.6	62.1	50.3	54.2	60.2	56.4	51.4
8 Stall current	A	11.5	11.2	4.25	3.78	3.01	2.43	1.25	1.16	1.2	0.93	0.683
9 Max. efficiency	%	84	84	84	84	84	84	83	84	84	84	83
Characteristics												
10 Terminal resistance	Ω	0.39	0.536	2.12	3.17	4.99	7.41	19.2	25.8	30.1	45.1	70.2
11 Terminal inductance	mH	0.04	0.051	0.227	0.333	0.529	0.77	1.9	2.58	2.99	4.34	6.68
12 Torque constant	mNm/A	5.84	6.57	13.9	16.8	21.2	25.5	40.1	46.7	50.3	60.6	75.2
13 Speed constant	rpm/V	1640	1450	689	569	451	374	238	205	190	158	127
14 Speed/torque gradient	rpm/mNm	109	119	105	108	106	108	114	113	114	117	119
15 Mechanical time constant	ms	16.5	16	15	14.9	14.8	14.8	14.9	14.9	14.9	15	15
16 Rotor inertia	gcm²	14.4	12.9	13.6	13.2	13.3	13.1	12.5	12.6	12.5	12.2	12.1

I took the parameters from the motor specification table above using part number 110187.

$$R_M = 19.2\ \Omega \checkmark$$
$$L_M = 1.9\text{mH} = 0.0019\ \text{H} \checkmark$$
$$K_\tau = 40.1\text{mNm/A} = 0.0401\ \text{Nm/A} \checkmark$$
$$J_M = 12.5\text{gcm}^2 = 0.00000125\ \text{kgm}^2 \checkmark$$
$$K_B = 238\text{rpm/V} = 0.04\ \text{Vs/rad} \checkmark$$
Motor Gear Head?
$$N = 3.3 \checkmark$$
$$J_L = 0.018522\ \text{kgm}^2\ ?$$
$$K_S = 0.8\ \text{V/rad} \checkmark$$

I derived N by using the logic that the gear ratio measured in radii is equal to the gear ratio measured in circumferences and if the teeth on both gears are the same size then you could measure the circumference in number of teeth and take the ratio of numbers of teeth. The load shaft gear has 120 teeth and the motor shaft gear has 36, making the ratio 3.33.

I estimated J_L, using standard inertia formulas for a rod rotating about one end and a satellite (for the weight) and added them together.
Equation for rod: $J_{\text{arm}} = \frac{1}{3} m_{\text{arm}} L^2$
Equation for weight: $J_{\text{weight}} = m_{\text{weight}} L^2$

I got the mass of the arm and weight using their measurements and the average density for aluminum and brass.

- Arm Dimensions: $L = 30$ cm, $h = 0.7$ cm, $w = 1.1$ cm
- Volume:
$$V_{\text{arm}} = L \times h \times w = 30 \times 0.7 \times 1.1 = 23.1 \text{ cm}^3 \tag{1}$$
- Density of aluminum: $\rho_{\text{Al}} \approx 2.7 \text{ g/cm}^3$
- Mass of the arm:
$$m_{\text{arm}} = V_{\text{arm}} \times \rho_{\text{Al}} = 23.1 \times 2.7 = 62.37 \text{ g} = 0.0624 \text{ kg} \tag{2}$$
- Weight Dimensions: $3.2 \times 2.0 \times 3.4$ cm
- Volume:
$$V_{\text{brass}} = 3.2 \times 2.0 \times 3.4 = 21.76 \text{ cm}^3 \tag{3}$$
- Density of brass: $\rho_{\text{brass}} \approx 8.5 \text{ g/cm}^3$
- Mass of the brass weight:
$$m_{\text{brass}} = V_{\text{brass}} \times \rho_{\text{brass}} = 21.76 \times 8.5 = 184.96 \text{ g} = 0.185 \text{ kg} \tag{4}$$

Then I used those numbers (including length of arm for L) to calculate the moments of inertia and combine them.

$$J_{\text{arm}} = \frac{1}{3} m_{\text{arm}} L^2 \tag{5}$$

$$J_{\text{arm}} = \frac{1}{3}(0.0624)(0.30)^2 = 0.001872 \text{ kgm}^2 \tag{6}$$

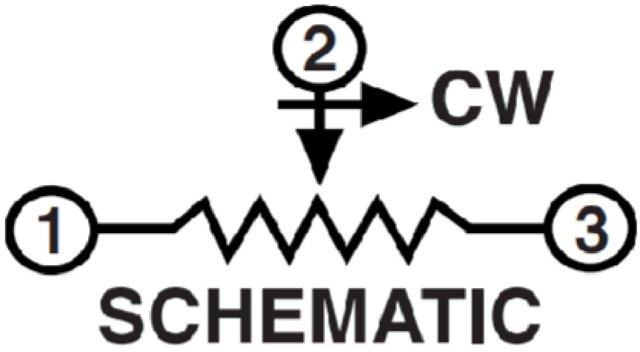
$$J_{\text{weight}} = m_{\text{weight}} L^2 \tag{7}$$

$$J_{\text{weight}} = (0.185)(0.30)^2 = 0.01665 \text{ kgm}^2 \tag{8}$$

$$J_L = J_{\text{arm}} + J_{\text{brass}} \tag{9}$$

$$J_L = 0.00187 + 0.01645 = 0.018522 \text{ kgm}^2 \tag{10}$$

MoI with respect to the potentiometer shaft



Measurement of port 2, with respect to port 1 (GND)

-90 Deg shows 1.25 V

0 Deg shows 2.488 V

+90 Deg shows 3.76 V

To derive K_S I used the readings/schematic above. I took the total voltage difference over the total angle difference (converted to radians).

$$K_S = (3.76V - 1.25V)/(\frac{\pi}{2}\text{rad} + \frac{\pi}{2}\text{rad}) \tag{11}$$

$$K_S = 0.8 \text{ V/rad} \tag{12}$$

The transfer function that relates V_P to Θ_L is:

$$\frac{-1}{\frac{1}{NK_\tau}J_{eq}L_M s^3 + \frac{1}{NK_\tau}J_{eq}R_M s^2 + NK_B s} \tag{13}$$

I derived J_{eq} using:

$$J_{eq} = J_L + N^2 J_M$$
$$J_{eq} = 0.018535612kgm^2$$

The N value considered is wrong.

Did not value in the Motor gear head ratio

Plugging in the parameter values:

$$\frac{-1}{0.000266135\ V \cdot sec^3\ s^3 + 2.6893656\ V \cdot sec^2\ s^2 + 0.132\ \frac{V \cdot sec}{rad}\ s} \tag{14}$$

2.

"[20pts] Simulate the system relating power amp voltage V_P to sensor voltage V_S using a single transfer function block from the Continuous library in Simulink. I suggest you compute polynomial coefficients in a separate m-file (as in the intro_sims.m example), so the simulink system can be built using simple variable names from the workspace. Use a sinusoidal power amp input voltage (1V peak, 1Hz), and plot the corresponding input and output signals, with appropriate units. Does the output follow the input closely?"

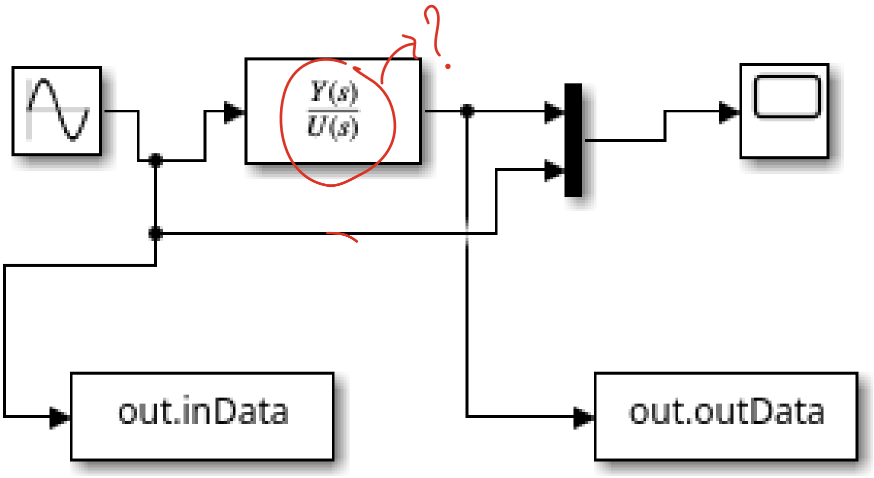
I set up the variables for the transfer function as suggested in a matlab file as shown below.

```
coeff3=0.000266135; % [Vs^3]
coeff2=2.6893656; % [Vs^2]
coeff1=0.132; % [Vs/rad]
num = [-1];
den = [coeff3, coeff2, coeff1, 0];
format longG;
disp(roots(den));
```

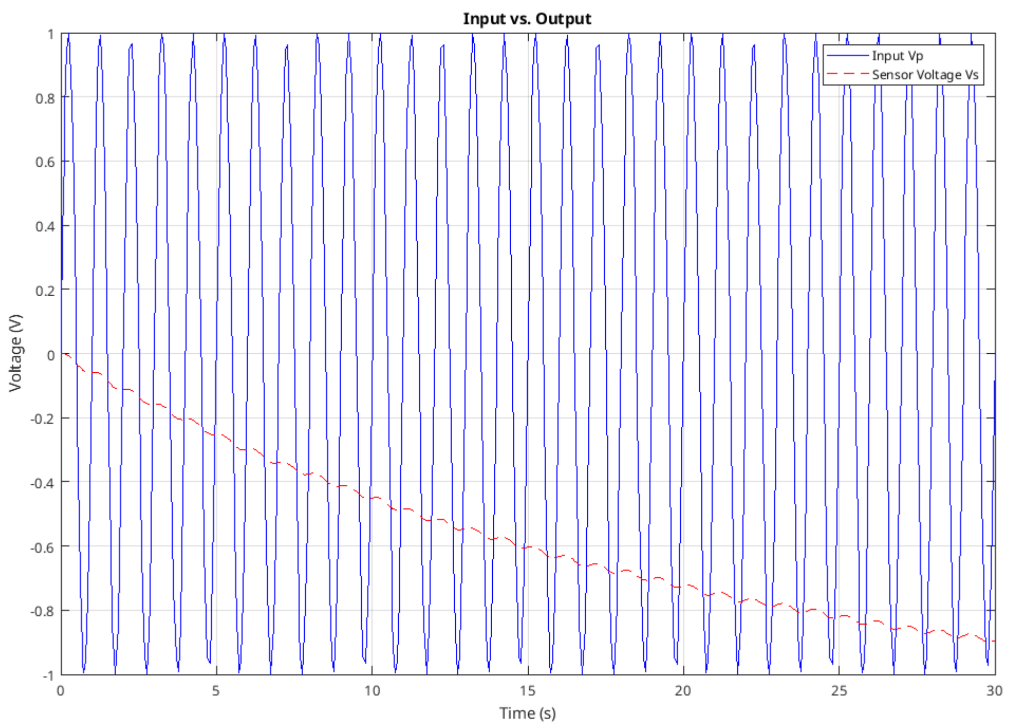
```
plot(out.inData.time, out.inData.Data, 'b-', ...
     out.outData.time, out.outData.Data, 'r--');

xlabel('Time (s)');
ylabel('Voltage (V)');
legend('Input Vp', 'Sensor Voltage Vs');
title('Input vs. Output');
grid on;
```

I then created my model in Simulink using num and den from the m file for the transfer function, a 1V-1Hz sine wave input, and a couple 'To Workspace' blocks so I could plot the input/output.



Then I ran the simulation and plotted the results as you can see below:



As for whether the output follows the input I would say in one sense it does, the oscillations match up. When the input goes down so does the output at the same time. However, in another sense the output doesn't really follow the input, it does small oscillations but mostly just slowly converges to -1.

3.

"[10pts] Derive the closed loop transfer function between reference angle Θ_R and load shaft angle Θ_L when the controller has a proportional-plus-derivative (PD) control law, i.e. $V_P = G_P(\Theta_R - \Theta_L) + G_D\frac{d}{dt}(\Theta_R - \Theta_L)$. Determine the DC gain of the closed loop system."

I derived the closed loop transfer function using the closed loop relationship

$$\frac{\Theta_L}{\Theta_R} = \frac{PC}{1+PC}$$

Where P is the plant and C is the controller. I will use the same B/A transfer function for the plant and the proportional and derivative gains for the controller.

$$P = \frac{B}{A}$$
$$C = G_P + G_D\frac{d}{dt}$$
$$\frac{\Theta_L}{\Theta_R} = \frac{P(G_P + G_D\frac{d}{dt})}{1 + P(G_P + G_D\frac{d}{dt})}$$

The DC gain of the closed loop system can be derived by converting the transfer function to the LaPlace domain and getting the value as s goes to 0.

Limit function has to be applied infinity over infinity is not defined

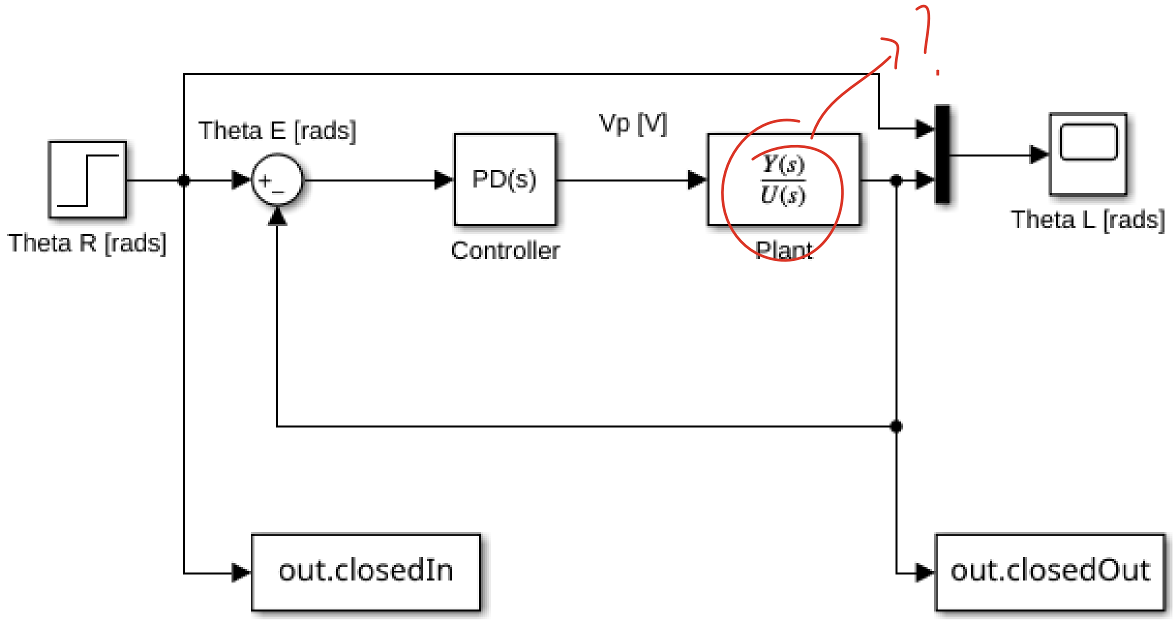
$$\frac{\Theta_L}{\Theta_R} = \frac{P(s)(G_P + G_D s)}{1 + P(s)(G_P + G_D s)}$$

The value of the plant goes to infinity as s goes to 0 since the plant is -1 over a few powers of s. This then means that the closed loop transfer function becomes $\frac{\infty \cdot G_P}{1+\infty \cdot G_P}$ effectively $\frac{\infty}{\infty}$ which is 1.

4.

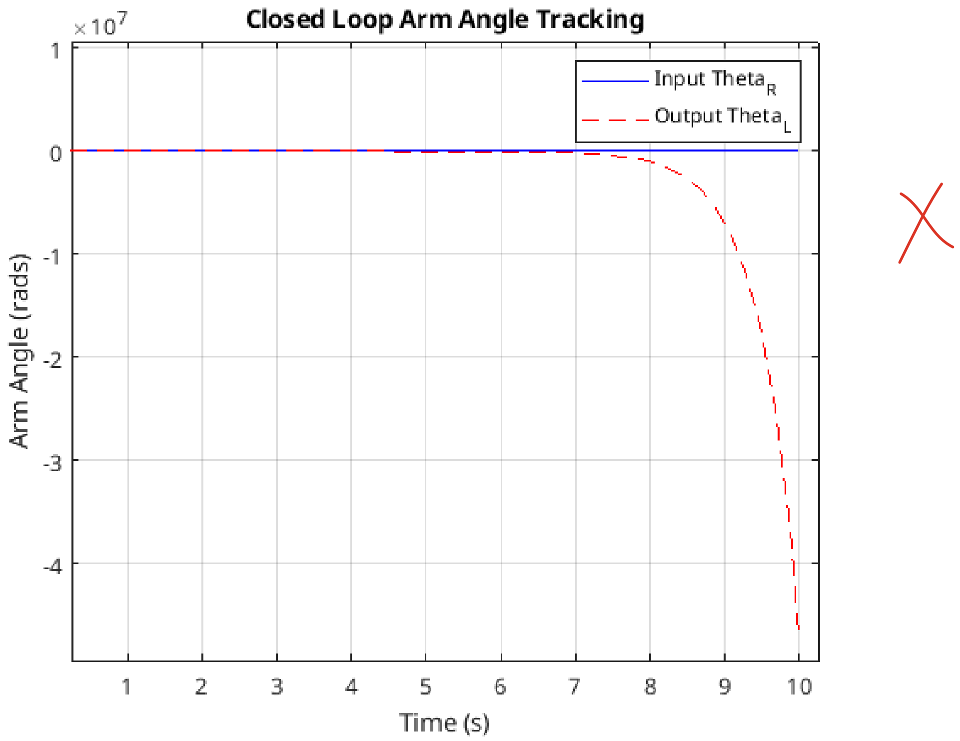
"[20pts] Construct a Simulink model of the control system including both proportional and derivative control. DO NOT create one block that represents the closed loop transfer function derived in Part 3. Instead, create a second block for the control law itself and then connect it in a unity-feedback control system with the Plant block from part 2. Label this block diagram with signal names and units"

I constructed the closed loop model using the same plant from the open loop, the PID Controller Block which I just set to PD and set the gains to 10 and 0.1 and left the filter coefficient at 100 because I don't really understand it but I believe it is intended to stabilize the signal at high frequencies, and then I put in a step source instead of the sine wave source.



5.

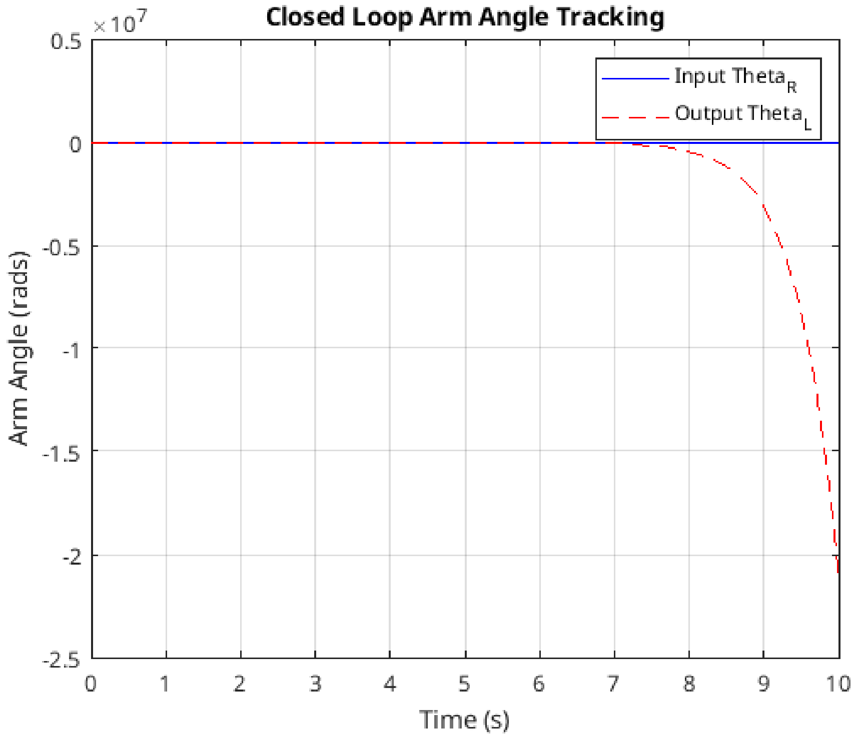
"[10pts] Plot the simulated step response (0.4 radian amplitude) of this closed loop control system using a proportional gain of 10 V/rad and a derivative gain of 0.1 V/rad/s . Compare the steady state tracking behavior to that predicted by the DC gain calculated in part 3."

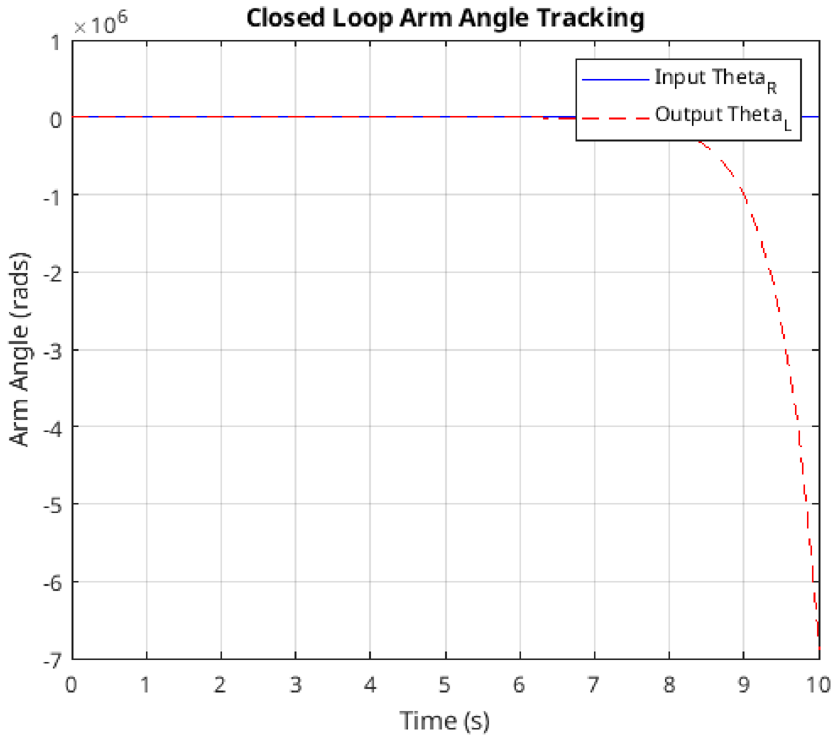


This does not look good, definitely not converging on the reference value. Exploding to negative infinity. I think something is wrong thought the final value theorem for the plant would indicate negative infinity, the final value theorem also only applies if a system is stable.

6.

"[20pts] Repeat part 5, but use sinusoid reference signals at 0.4 rad amplitude, with frequencies of 0.2 Hz and 2 Hz. Comment on the relative tracking ability of the system at these two frequencies."





More explosions. Yep!!

The feedback PID controller takes in the values of Gp and Gd in a negative fashion

so Gp = -10
and Gd = -0.1

Okay, I think I figured out the issue. I wrote out the closed loop transfer function so I could derive those roots and see what the behavior should look like.

Starting from the transfer function I derived above:

$$\frac{\theta_L(s)}{\theta_R(s)} = \frac{P(s) \cdot (G_P + G_D s)}{1 + P(s) \cdot (G_P + G_D s)}$$

Substitute $P(s)$ with the plant's transfer function:

$$P(s) = \frac{-1}{0.000266135s^3 + 2.6893656s^2 + 0.132s}$$

Therefore, the closed-loop transfer function becomes:

$$\frac{\theta_L(s)}{\theta_R(s)} = \frac{\left(\frac{-1}{0.000266135s^3+2.6893656s^2+0.132s}\right) \cdot (G_P + G_D s)}{1 + \left(\frac{-1}{0.000266135s^3+2.6893656s^2+0.132s}\right) \cdot (G_P + G_D s)}$$

Simplify the expression:

$$\frac{\theta_L(s)}{\theta_R(s)} = \frac{-(G_P + G_D s)}{0.000266135s^3 + 2.6893656s^2 + 0.132s - G_P - G_D s}$$

Combine like terms in the denominator:

$$\frac{\theta_L(s)}{\theta_R(s)} = \frac{-(G_P + G_D s)}{0.000266135s^3 + 2.6893656s^2 + (0.132 - G_D)s - G_P}$$

The final closed-loop transfer function is:

$$\frac{\theta_L(s)}{\theta_R(s)} = \frac{-G_D s - G_P}{0.000266135s^3 + 2.6893656s^2 + (0.132 - G_D)s - G_P}$$

I used the built-in roots function to get the roots of this transfer function in Matlab.

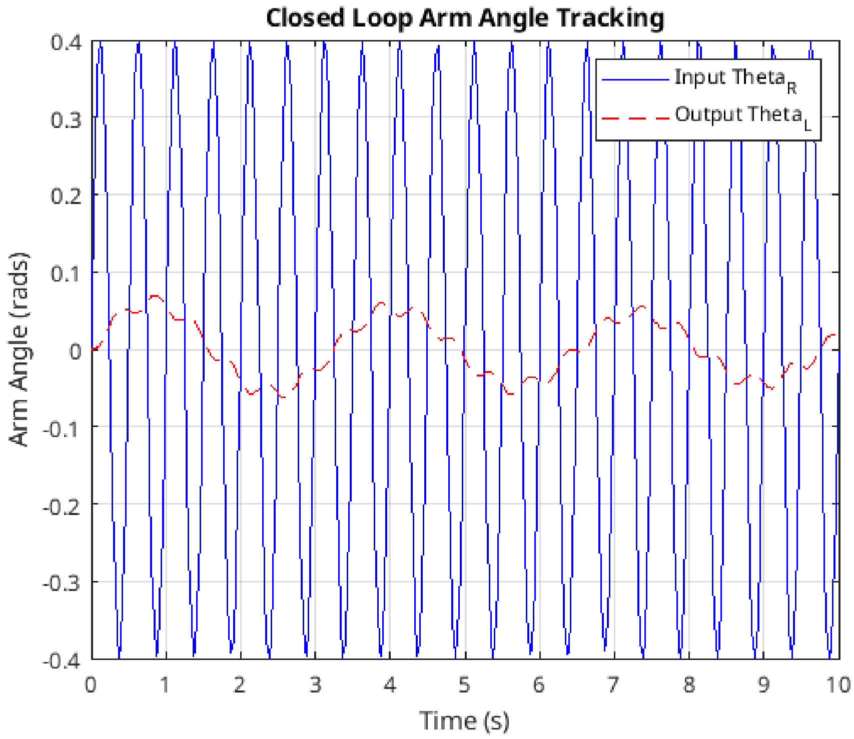
```
Gp = 10;
Gd = 0.1;
numCL = [-Gd -Gp];
denCL = [0.000266135, 2.6893656, 0.132 - Gd, -Gp];
disp(roots(denCL));
```

Results: -10105.2561121366 -1.9344463849097 1.92217969120724

Aha! A positive root. Not good.

I changed the gains to be negative and look:

Nice!



That's the 2Hz sine with -10 proportional gain and -0.1 derivative gain. Not exactly tracking but stable. I suspect this was either a trick question type situation or my calculations missed a sign somewhere tragically. More investigation required.

We expect a good tracking for a 0.2Hz signal and
for the 2Hz we need to observe an amplitude attenuation and phase offset significantly.