ASEN 5014 Linear Control Design

Midterm Exam 1 2024

Instructions: Do not communicate with any other person concerning the exam, except the instructor. Sign the Honor Code Pledge at the end of this exam and scan this to attach to your soultions. Any violation will receive a 0 for the exam. Be sure to verify any numerical answer, and attach any code used including explanatory comments. Answers without explanations will earn no credit. Explanations using the terminology and results from class are expected. The exam is due at 11:59 pm on October 16, 2024 on Canvas (in a single PDF file).

1. (50 Pts) For each of the following statements, either show why it is true, or fix it to make it true and provide a corresponding explanation why.
   a. $span\left\{\begin{bmatrix}1\\0\\1\end{bmatrix},\begin{bmatrix}0\\1\\0\end{bmatrix},\begin{bmatrix}1\\1\\1\end{bmatrix}\right\} = 2.$
   b. A basis for $\mathbb{R}^3$ is $\left\{\begin{bmatrix}1\\1\end{bmatrix},\begin{bmatrix}0\\2\end{bmatrix},\begin{bmatrix}-1\\0\end{bmatrix}\right\}$.
   c. The set $\left\{\begin{bmatrix}1\\0\\0\\1\end{bmatrix},\begin{bmatrix}0\\0\\1\\0\end{bmatrix},\begin{bmatrix}1\\0\\1\\0\end{bmatrix},\begin{bmatrix}1\\0\\1\\1\end{bmatrix}\right\}$ cannot be a spanning set because it is linearly dependent.
   d. For any vector space V, there can be only one minimal spanning set.
   e. A linear equation $y = Mx$ has unique solutions iff $y \in CS(M)$.
   f. A linear equation $y = Mx$ fails to have a solution iff $RN(M)$ is non-trivial.
   g. $span\left\{\begin{bmatrix}1\\-1\\3\\0\\5\end{bmatrix},\begin{bmatrix}2\\1\\2\\4\\6\end{bmatrix},\begin{bmatrix}1\\2\\-1\\4\\1\end{bmatrix}\right\} > span\left\{\begin{bmatrix}1\\1\end{bmatrix},\begin{bmatrix}0\\2\end{bmatrix},\begin{bmatrix}-1\\0\end{bmatrix}\right\}$.
   h. $span\left\{\begin{bmatrix}1\\0\\0\\1\end{bmatrix},\begin{bmatrix}0\\0\\1\\0\end{bmatrix},\begin{bmatrix}1\\1\\1\\0\end{bmatrix}\right\} = span\left\{\begin{bmatrix}1\\0\\1\end{bmatrix},\begin{bmatrix}0\\1\\0\end{bmatrix},\begin{bmatrix}1\\1\\1\end{bmatrix}\right\}$.
   i. The $RS(M)$ is always non-trivial.
   j. If a mapping is onto, then M has linearly independent columns

2. (15 pts) Given the basis sets B and C below
   a. find the coordinate representation of $\underline{z}^T = [18 \quad 2.6 \quad 3.1 \quad 3.0 \quad 34 \quad 7.1]$ in each basis.

$$B = \left\{ \begin{bmatrix} 3 \\ 1 \\ -1 \\ 2 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \\ -1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 3 \\ -2 \\ 4 \\ 1 \end{bmatrix} \right\}, \quad C = \left\{ \begin{bmatrix} 5 \\ 1 \\ 2 \\ 0 \\ 8 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -3 \\ 3 \\ 6 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \\ -8 \\ 5 \end{bmatrix} \right\}$$

b. Determine if w given below lies in span(B).

$$\underline{w}^T = \begin{bmatrix} 3 & 24 & 3 & -12 & 1 & 6 \end{bmatrix}$$

c. Find an orthonormal basis for span(B). Is this also an orthonormal basis for span(C)?

3. (15 pts) For the following differential equation

$$\dddot{y}(t) - 3\ddot{y}(t) + 2y(t) = \ddot{u}(t) - 3\dot{u}(t)$$

a. Find the transfer function relating $u(s)$ and $y(s)$.
b. Find a state space model in the controllable canonical form.
c. Draw a simulation diagram and label the state variables on the diagram.

4. (20 pts) Show whether or not the set of positive real numbers $\mathbb{R}^+$ is a vector space over $\mathbb{R}$, i.e. V = (G,F) = ($\mathbb{R}^+$, $\mathbb{R}$) under the following operations: group operator " $+$ " defined by

$$x\ "+"\ y = x * y$$

where $x, y \in \mathbb{R}^+$ and the product on the right is the usual product of real numbers, and scalar multiplication " $\cdot$ " is defined by $c\ "\cdot"\ x = x^c$ for $x \in \mathbb{R}^+$ and $c \in \mathbb{R}$, and the exponentiation on the right is as usual for real numbers.

Honor Code Pledge

On my honor, as a University of Colorado Boulder student, the solutions submitted are entirely my own, and I have not communicated with any other person (except the instructor) concerning the exam. Also, I have not utilized any materials except my own notes, the posted materials for this class on Canvas, and the textbook for the class.

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