Class: Robust Multivariate Control
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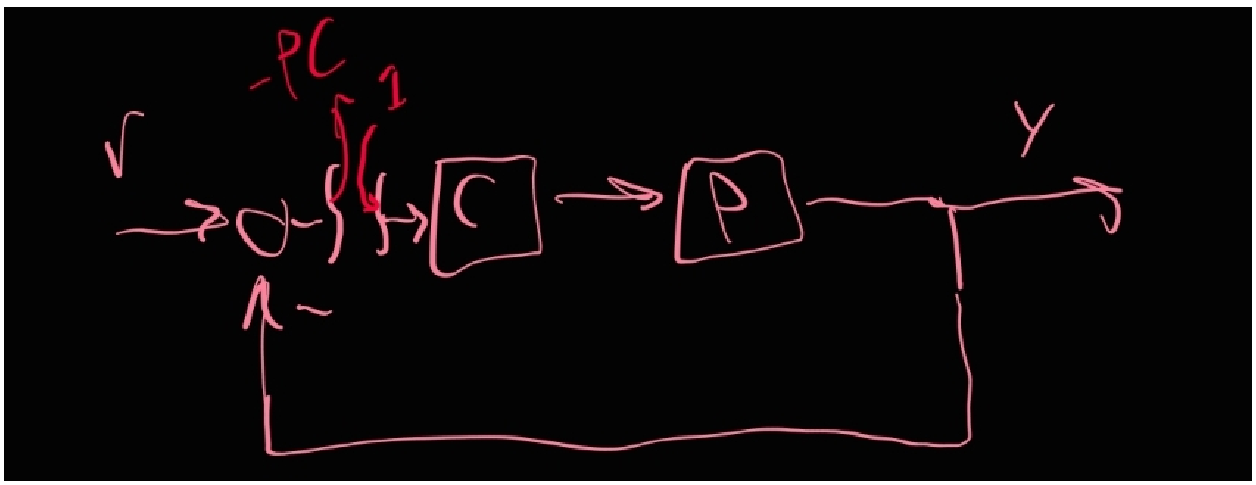
Assignment: Homework 5

1.

For the simple SISO negative feedback system with $P(s) = \frac{1}{s-1}$ and $C(s) = \frac{s-1}{s+2}$, show that at least one transfer function from exogenous signals r and d_I to the internal signals e, u and y is unstable due to the right-half plane pole/zero cancellation of $s-1$ in the loop transfer function $L = PC$.



I will start with the transfer function from r to y which for this SISO unity feedback system is:

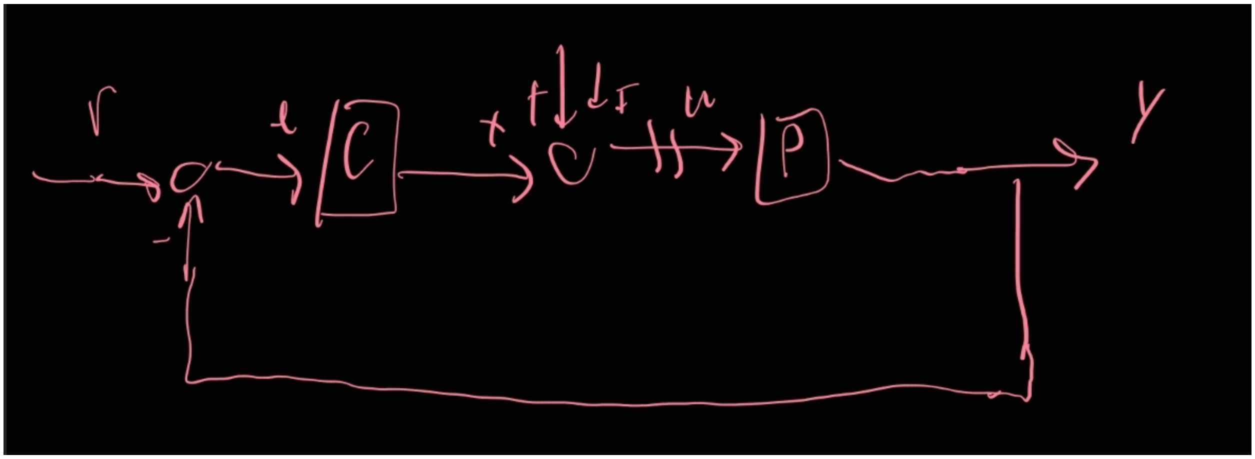
$$\frac{PC}{1+PC}$$

Or:

$$\frac{(\frac{1}{s-1})(\frac{s-1}{s+2})}{1+(\frac{1}{s-1})(\frac{s-1}{s+2})} = \frac{\frac{1}{s+2}}{1+\frac{1}{s+2}} = \frac{\frac{1}{s+2}}{\frac{s+2}{s+2}+\frac{1}{s+2}} = \frac{\frac{1}{s+2}}{\frac{s+3}{s+2}} = \frac{1}{s+3}$$

This transfer function is not only stable but it was made stable by the addition of the controller and the pole/zero cancellation. Let us see some other signals, eh.

Full system diagram with d_I:



Let's try r→u:
Direct path: C Return path: $1 + PC$

$$\frac{C}{1+PC}$$

$$\frac{\frac{s-1}{s+2}}{1+\frac{1}{s-1}\frac{s-1}{s+2}} =$$

$$\frac{\frac{s-1}{s+2}}{1+\frac{1}{s+2}} = \frac{\frac{s-1}{s+2}}{\frac{s+3}{s+2}} = \frac{s-1}{s+3}$$

$$= \frac{s-1}{s+3}$$

Again the pole is at -3 and the transfer function is stable. Let's try $d_I \to y$:

$$\frac{P}{1+CP}$$

$$\frac{\frac{1}{s-1}}{\frac{s+3}{s+2}}$$

$$= \frac{s+2}{(s-1)(s+3)}$$

There we go, a pole at $s = 1$, the transfer function from d_I to y is unstable.

2.

For the following longitudinal model for an F-4 Phantom with 4 perturbation states (pitch rate Δq, forward speed Δu, angle of attack $\Delta\alpha$, pitch angle $\Delta\theta$), 2 inputs (elevator deflection $\Delta\delta_e$, flaperon deflection $\Delta\delta_f$) and 2 outputs (Δq and $\Delta\alpha$). Note that the D matrix entries are zeros for this output selection.

$$A = \begin{pmatrix} -0.8 & -0.0006 & -12 & 0 \\ 0 & -0.014 & -16.64 & -32.2 \\ 1 & -0.0001 & -1.5 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} -19 & -2.5 \\ -0.66 & -0.5 \\ -0.16 & -0.6 \\ 0 & 0 \end{pmatrix}.$$

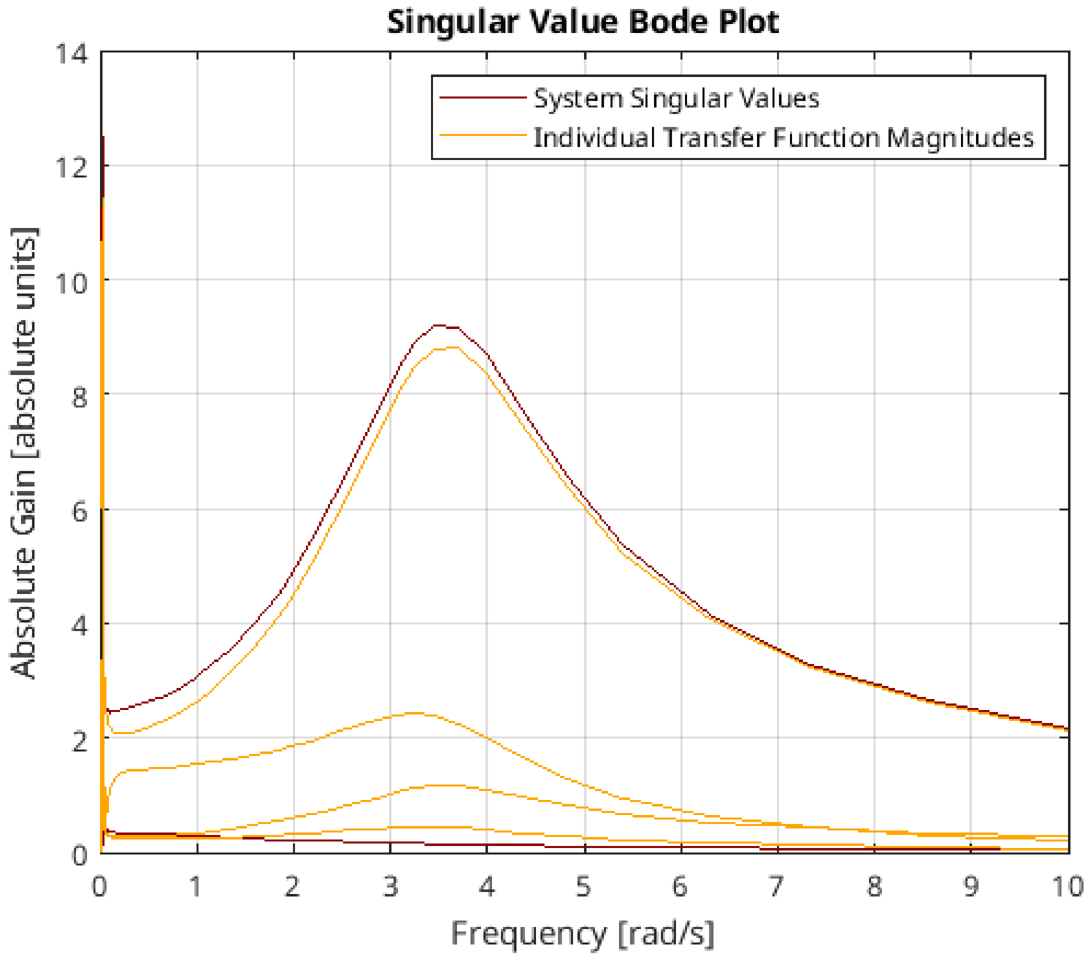
(a)

Generate a MATLAB plot that contains the singular value bode plots in red combined with the bode magnitude plots of the 4 individual transfer functions from the 2 inputs to 2 outputs. Does the maximum singular value plot envelope the individual transfer function plots?

I generated the system, transfer function, and plots using the 'ss', 'ss2tf', and 'bode' Matlab functions. You can see my implementation below:

```
A = [ -0.8 -0.0006 -12 0; 0 -0.014 -16.64 -32.2; 1 -0.0001 -1.5 0; 1 0 0 0];
B = [ -19 -2.5; -0.66 -0.5; -0.16 -0.6; 0 0];
C = [1 0 0 0; 0 0 1 0];
D = [0 0; 0 0];

phantomSS = ss(A,B,C,D);
[sv,wout] = sigma(phantomSS, {0,10});

figure;
plot(wout, sv, 'Color', [0.545, 0, 0]);
grid on;
title('Singular Value Bode Plot');
xlabel('Frequency [rad/s]');
ylabel('Absolute Gain [absolute units]');
hold on;

[num, den] = ss2tf(A,B,C,D,1);
tf11 = tf(num(1,:), den);
tf12 = tf(num(2,:), den);
[num, den] = ss2tf(A,B,C,D,2);
tf21 = tf(num(1,:), den);
tf22 = tf(num(2,:), den);

[mag11, phase11, wout11] = bode(tf11, {0,10});
[mag12, phase12, wout12] = bode(tf12, {0,10});
[mag21, phase21, wout21] = bode(tf21, {0,10});
[mag22, phase22, wout22] = bode(tf22, {0,10});
plot(wout11, squeeze(mag11), 'Color', [1,0.65,0]);
plot(wout12, squeeze(mag12), 'Color', [1,0.65,0]);
plot(wout21, squeeze(mag21), 'Color', [1,0.65,0]);
plot(wout22, squeeze(mag22), 'Color', [1,0.65,0]);
legend('System Singular Values', 'Individual Transfer Function Magnitudes');
```

The plot is below and you can see that yes in fact thee maximum singular value plot (in red) envelopes the individual transfer functions (in orange):


Singular Value Bode Plot
System Singular Values
Individual Transfer Function Magnitudes
Absolute Gain [absolute units]
Frequency [rad/s]
0
2
4
6
8
10
12
14
0
1
2
3
4
5
6
7
8
9
10