UNIVERSITY OF COLORADO - BOULDER
Department of Mechanical Engineering

MCEN 6228 - Robust Multivariable Control

Homework #6 (Assigned: 4/2, Due: 4/10, 5pm)
Robust Stability for Unstructured Uncertainty

1. For the following MIMO uncertainty models, sketch the block diagram and transform the standard G and K feedback system into the $M\Delta$ structure and determine M, and apply the Small Gain Theorem to find the associated robust stability test:
   (a) $\tilde{G} = (I + \Delta W_o)G$ (Multiplicative Output)
   (b) $\tilde{G} = G(I - W_i\Delta)^{-1}$ (Inverse Multiplicative Input)

2. For the following MIMO plant and controller,

$$G(s) = \begin{pmatrix} \frac{2}{s+1} & \frac{1}{(s+1)(s+2)} \\ \frac{1}{(s+1)(s+2)} & \frac{2}{s+2} \end{pmatrix}, \quad K(s) = \begin{pmatrix} \frac{2}{s+2} & 0 \\ 0 & \frac{1}{s+3} \end{pmatrix},$$

   (a) Generate singular value bode plots for G, T_o, and S_o ($= S_I$ in this case) and label the min and max singular values. The MATLAB command `sigma` will be useful here.
   (b) What percentage of tracking error would we expect for signals up to 0.1 rad/s for a unity negative feedback system?
   (c) For 10 rad/s and above, what level of noise attenuation would we expect this plant to provide at the output for a unity negative feedback system? Recall a gain of 0.1 corresponds to a 10X reduction.
   (d) Determine the minimum perturbation level $\|\Delta\|_\infty \leq \gamma$ and frequency that destabilizes the system for multiplicative output uncertainty and inverse multiplicative input uncertainty (assume the weights are identity).

3. Consider the MIMO controller and plant:

$$G(s) = \frac{1}{\tau s + 1}\begin{pmatrix} -87.8 & 1.4 \\ -108.2 & -1.4 \end{pmatrix}, \quad K(s) = \frac{\tau s + 1}{s}\begin{pmatrix} -0.0015 & 0 \\ 0 & -0.075 \end{pmatrix}$$

   Assume a time constant $\tau = 50$.

   (a) Assume the uncertainty in the two channels can be represented using a multiplicative output uncertainty model. The first channel (weight $W_1(s)$) has a 10% error at low frequency, increases to 100% at 10 rad/s, and reaches 120% (1.2X) at the high frequency range. The corresponding MATLAB command to generate the first order weight for this channel is `W1 = makeweight(0.1,10,1.2)`. Similarly, the second channel (weight $W_2(s)$) has a 5% error at low frequency, increases to 100% at 50 rad/s, and reaches 150% (1.5X) at the high frequency range. Generate magnitude plots of the two weighting functions (in absolute units, not dB) and label them on the same graph (using `bodeplot` or `sigmaplot` with appropriate plot options specified).

(b) Assume these weights can be represented by the block diagonal weighting matrix

$$W(s) = \begin{bmatrix} W_1(s) & 0 \\ 0 & W_2(s) \end{bmatrix}$$

and check to determine whether or not the system is robust with respect to (a) multiplicative output uncertainty, and (b) inverse multiplicative input uncertainty. Be sure to generate the singular value plots in absolute units as a function of frequency (using the MATLAB command `sigmaplot`). Also compute the robustness margin β for each case.