Class: Robust Multivariate Control
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Assignment: Homework 7

1.

In this problem you will analyze the robustness properties of a F-16 equipped with a lateral regulator. We consider the lateral dynamics (β, ϕ, p, r) *augmented with aileron and rudder actuator dynamics* (δ_a, δ_r) *and a washout filter* (x_w) *in the yaw channel. The state vector of the augmented dynamics is* $(\beta, \phi, p, r, \delta_a, \delta_r, x_w)$*. The outputs of the model include* (r_w, p, β, ϕ) *where* r_w *is the washed out yaw rate. Assume a static controller* $u = Ke$ *where*

$$K = \begin{bmatrix} -0.56 & -0.44 & -0.11 & -0.35 \\ -1.19 & -0.21 & -0.44 & 0.26 \end{bmatrix}.$$

An m-file with the augmented system A, B, C, D *matrices and controller gains* K *is available for download on the course website. For the analysis, assume that the plant is subject to an unstructured inverse multiplicative output uncertainty* $\bar{G} = (I - W_o\Delta)^{-1}G$ *and the desired performance metric is attenuation of measurement noise* n *at the output* y*, as described.*

(a)

Draw a block diagram of the overall system including the weighting functions described below and label w, z, v, u, y_Δ *and* u_Δ*.*

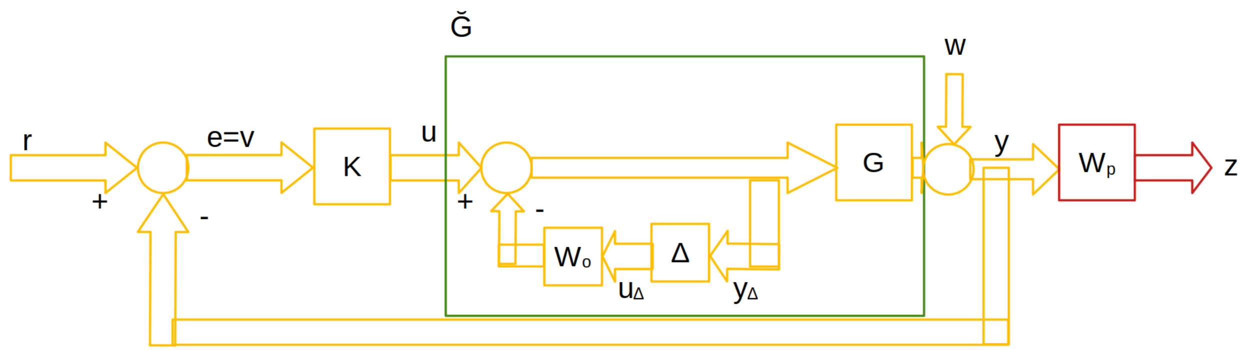

Figure 1: Block diagram of the overall system with labeled components.

(b)

Compute the generalized plant P and close the lower LFT (with $\Delta \neq 0$) to compute N. What are the corresponding tests for nominal performance and robust stability?

Writing y_Δ, v, z in terms of u_Δ, w, u:

$$\begin{aligned} y_\Delta &= u - W_o u_\Delta \\ z &= W_p(w + G(u - u_\Delta)) \\ v &= -y \\ &= -w - G(u - u_\Delta) \end{aligned}$$

$$\begin{bmatrix} y_\Delta \\ z \\ v \end{bmatrix} = \begin{bmatrix} -W_o & 0 & I \\ -W_pG & W_p & W_pG \\ G & -I & -G \end{bmatrix} \begin{bmatrix} u_\Delta \\ w \\ u \end{bmatrix}$$

Closing lower LFT:

$$\begin{aligned} N = F_l(P, K) &= P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21} \\ &= \begin{bmatrix} -W_o & 0 \\ -W_pG & W_p \end{bmatrix} + \begin{bmatrix} I \\ W_pG \end{bmatrix} K(I - GK)^{-1} \begin{bmatrix} G & -I \end{bmatrix} \\ &= \begin{bmatrix} -W_o + K(I - GK)^{-1}G & -K(I - GK)^{-1} \\ -W_pG + W_pGK(I - GK)^{-1}G & W_p - W_pGK(I - GK)^{-1} \end{bmatrix} \\ &= \begin{bmatrix} -W_o + T_I & -KS_O \\ W_pGS_O & W_pS_O \end{bmatrix} \end{aligned}$$

The test for robust stability is comes from the N_{11} block so $-W_o + T_I$. The test for nominal performance comes from the N_{22} block so W_pS_O. Need those H_∞ norms to be less than 1.

(c)

Assume a performance weighting function to be $W_p(s) = w_p(s)I$ where $w_p(s) = (s/M + \omega_B^*)/(s + \omega_B^* A)$, with $A = 4$; $M = 10$; $\omega_B^* = 3$. Does the system satisfy this nominal performance criterion you derived in part (b)?

(d)

Assume an uncertainty weighting function to be $W_o(s) = w_o(s)I$ where $w_o(s) = (\tau s + r_0)/(\tau s/r_\infty + 1)$, with $\tau = 0.02$, $r_0 = 0.05$, $r_\infty = 0.4$, and full (unstructured) uncertainty. Is the system robustly stable using the criterion you derived in part (b)?

Hint: to compute the state space model for the controller, assume $D = K$ (static gain) and the A, B, C matrices are all zeros with appropriate dimensions.