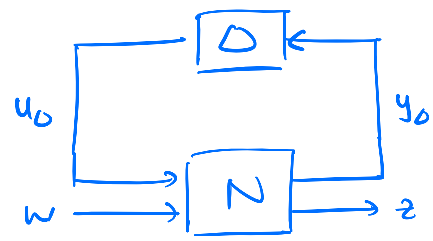
MODELS FOR STRUCTURED UNCERTAINTY

"PULL OUT THE DELTAS": GIVEN A SET OF REAL UNCERTAINTIES, HOW DO WE PUT THE SYSTEM INTO $N-\Delta$ FORM?

$$\begin{pmatrix} y_\Delta \\ z \end{pmatrix} = \left(\begin{array}{c|c} N_{11} & N_{12} \\ \hline N_{21} & N_{22} \end{array}\right) \begin{pmatrix} u_\Delta \\ w \end{pmatrix}$$

$$= \left(\begin{array}{c|c} u_\Delta \to y_\Delta & w \to y_\Delta \\ \hline u_\Delta \to z & w \to z \end{array}\right) \begin{pmatrix} u_\Delta \\ w \end{pmatrix}$$

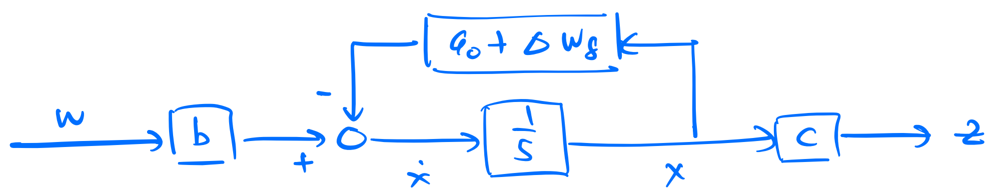


FIRST ORDER PLANT

$$\dot{x} = -ax + bu \qquad G(s) = \frac{cb}{s+a}$$
$$y = cx$$

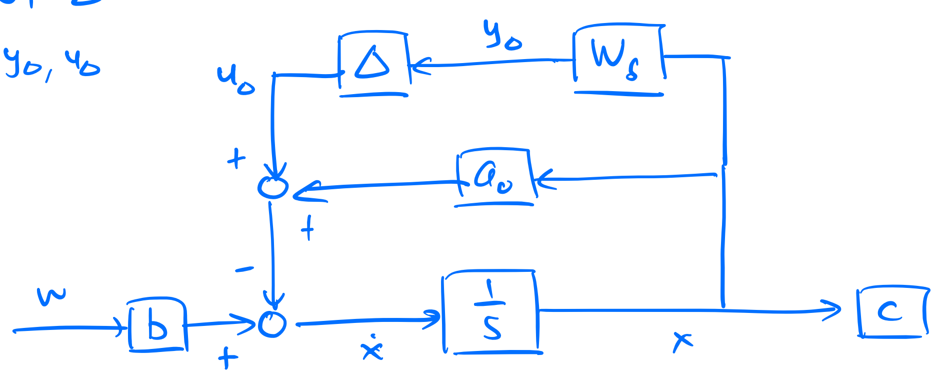
ASSUME $a = a_0 + \Delta W_\delta$ $\quad$ Δ, W_δ SCALAR, REAL, $|\Delta| \le 1$

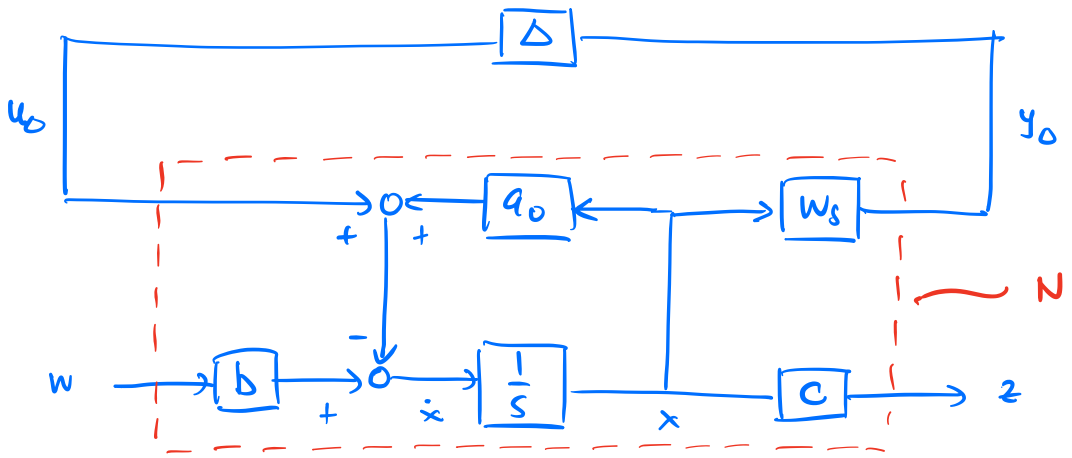
LET $w = u$, $z = y$



PULL OUT Δ

LABEL y_Δ, u_Δ





EXTENDED PLANT:

$$\dot{x} = -a_0 x - u_\Delta + b w \quad \Big] \text{ STATE EQUATION}$$

$$\left.\begin{aligned} y_\Delta &= W_s x \\ z &= c x \end{aligned}\right] \text{ OUTPUT EQUATION}$$

STATE SPACE MODEL:

$$\dot{x} = \underbrace{-a_0}_{A} x + \underbrace{\begin{pmatrix} -1 & b \end{pmatrix}}_{B} \begin{pmatrix} u_\Delta \\ w \end{pmatrix}$$

$$\begin{pmatrix} y_\Delta \\ z \end{pmatrix} = \underbrace{\begin{pmatrix} W_s \\ c \end{pmatrix}}_{C} x + \underbrace{\begin{pmatrix} 0 \\ 0 \end{pmatrix}}_{D} \begin{pmatrix} u_\Delta \\ w \end{pmatrix}$$

TRANSFER FUNCTION:

$$N = C(sI - A)^{-1} B = \begin{pmatrix} W_s \\ c \end{pmatrix} \left(\frac{1}{s+a_0} \right) \begin{pmatrix} -1 & b \end{pmatrix}$$

$$N = \left(\begin{array}{c|c} -\frac{W_s}{s+a_0} & \frac{b W_s}{s+a_0} \\ \hline -\frac{c}{s+a_0} & \frac{c b}{s+a_0} \end{array} \right) = \left(\begin{array}{c|c} N_{11} & N_{12} \\ \hline N_{21} & N_{22} \end{array} \right)$$

ROBUST STABILITY TEST: $M = N_{11} = \frac{-W_s}{s+a_0}$, $\|M\|_\infty \leq 1$

NOMINAL PERFORMANCE ($w \to z$ TF): $N_{22} = \frac{cb}{s+a_0}$

SECOND ORDER PLANT

UNCERTAINTIES:

$$m\ddot{x} + c\dot{x} + kx = f(t)$$

$m = m_0 + \delta_m$

$c = c_0 + \delta_c$

$k = k_0 + \delta_k$

CAN MAKE GENERAL

$\delta = \Delta W$

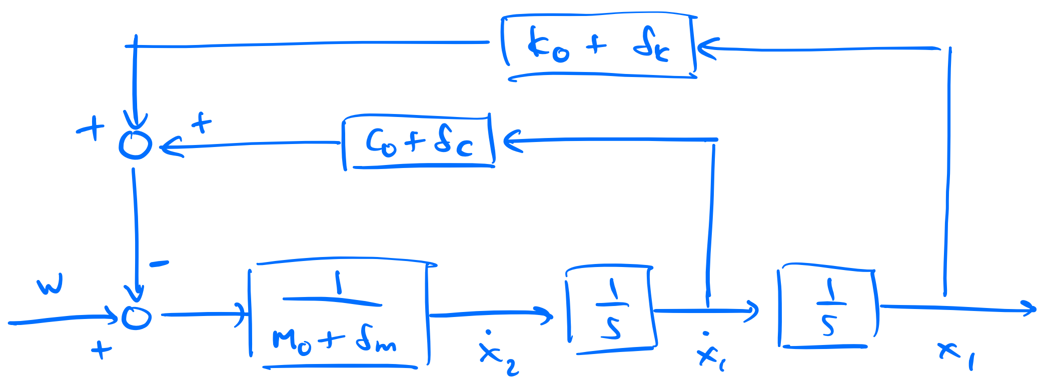
$x_1 = x \qquad w = f(t)$

$x_2 = \dot{x} \qquad z = x_1 = x$

$$\dot{x}_1 = x_2$$

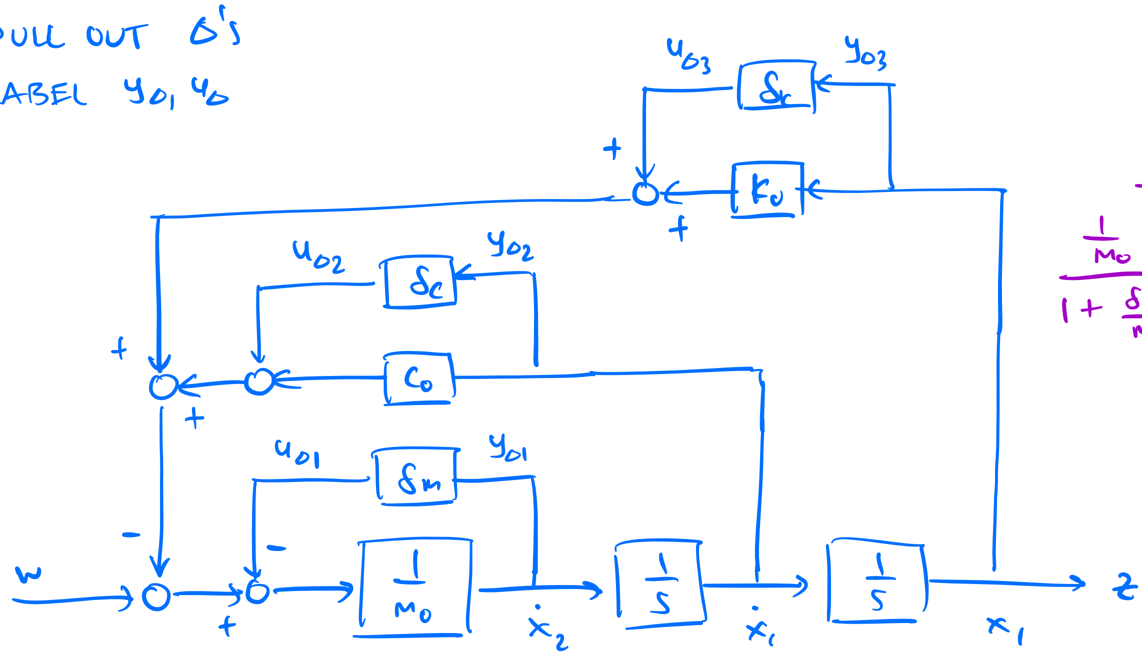
$$\dot{x}_2 = \frac{1}{m_0+\delta_m}\left[-(k_0+\delta_k)x_1 - (c_0+\delta_c)x_2 + w\right]$$

$$z = x_1$$



PULL OUT δ's

LABEL y_δ, u_δ



NOTE:

$$\frac{\frac{1}{m_0}}{1+\frac{\delta_m}{m_0}} = \frac{1}{m_0+\delta_m}$$

EXTENDED PLANT:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{1}{m_o}\left[-u_{o1} - (u_{o2} + c_o x_2) - (u_{o3} + k_o x_1) + w\right]$$

STATE EQUATION

$$y_{o1} = \dot{x}_2 \qquad z = x_1$$

$$y_{o2} = \dot{x}_1$$

$$y_{o3} = x_1$$

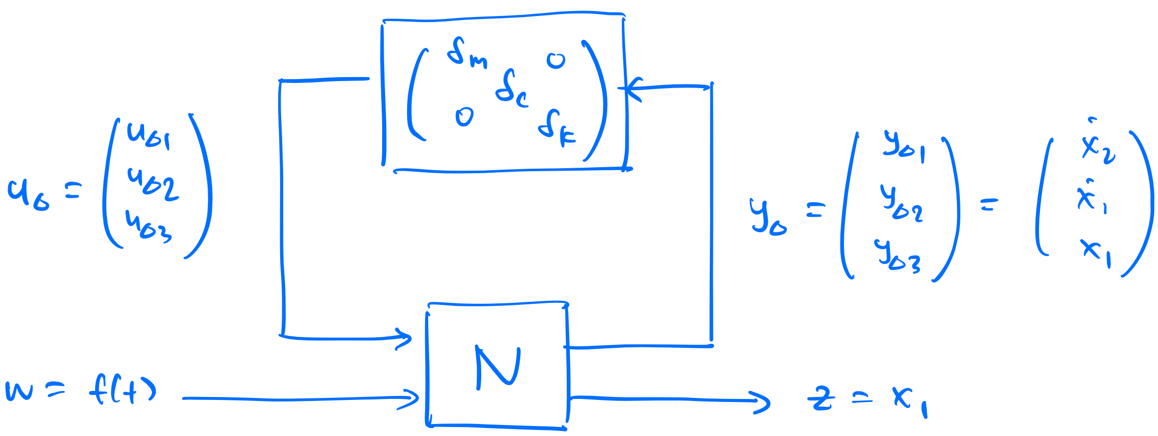
OUTPUT EQUATION

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \overbrace{\begin{pmatrix} 0 & 1 \\ -\frac{k_o}{m_o} & -\frac{c_o}{m_o} \end{pmatrix}}^{A} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \overbrace{\begin{pmatrix} 0 & 0 & 0 & 0 \\ -\frac{1}{m_o} & -\frac{1}{m_o} & -\frac{1}{m_o} & \frac{1}{m_o} \end{pmatrix}}^{B} \begin{pmatrix} u_{o1} \\ u_{o2} \\ u_{o3} \\ w \end{pmatrix}$$

$$\begin{pmatrix} y_{o1} \\ y_{o2} \\ y_{o3} \\ z \end{pmatrix} = \underbrace{\begin{pmatrix} -\frac{k_o}{m_o} & -\frac{c_o}{m_o} \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{pmatrix}}_{C} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \underbrace{\begin{pmatrix} -\frac{1}{m_o} & -\frac{1}{m_o} & -\frac{1}{m_o} & \frac{1}{m_o} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}}_{D} \begin{pmatrix} u_{o1} \\ u_{o2} \\ u_{o3} \\ w \end{pmatrix}$$

STATE SPACE MODEL FOR $N = \left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right)$

$N = C(sI - A)^{-1}B + D \longleftarrow$ YIELDS N_{11}, N_{22} ETC..



UNCERTAIN MODELS IN MATLAB

(1) USE 'ureal' TO DEFINE AN UNCERTAIN PARAMETER:

IF $m\ddot{x} + c\dot{x} + kx = f(t)$

m = ureal('m', 3, 'PERCENTAGE', [-5, 5])

$\underbrace{3}_{\text{NOMINAL}}$ $\underbrace{[-5, 5]}_{\text{DEVIATION +/-}}$ (5 %)

c = ureal('c', 1, 'RANGE', [0.8 1.2])

$\underbrace{1}_{\text{NOMINAL}}$ [0.8 1.2]: INTERVAL CONTAINING NOMINAL

k = ureal('k', 2, 'PLUSMINUS', [-0.2 0.2])

$\underbrace{2}_{\text{NOMINAL}}$ [-0.2 0.2]: DEVIATION FROM NOMINAL

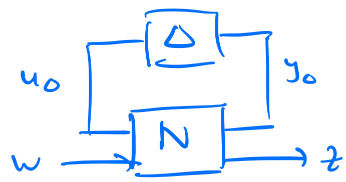
(2) CAN NOW USE THESE PARAMETERS (m, c, k)

TO BUILD TF, SS MODELS

(3) IN PLOTTING UNCERTAIN TF, SS MODELS, USE 'gridureal' TO GET EVEN SPACING.

(4) PULLING OUT DELTAS FOR STATES > 2 : HASSLE!
'lftdata' DOES THIS AUTOMATICALLY, AND COMPUTES THE EXTENDED PLANT:

[N, DELTA, BLKSTRUCT] = lftdata(sys)



N: EXTENDED PLANT (N_{11}, N_{22} ETC)

DELTA: A UMAT OBJECT

BLKSTRUCT: CONVERTS DELTA FOR MUSSV (LATER)