UNIVERSITY OF COLORADO - BOULDER
Department of Mechanical Engineering

MCEN 6228 - Robust Multivariable Control

Homework #3 (Assigned: 2/7, Due: 2/18 at 5pm)

Note: A screenshot of the output of the functions `feasp`, `mincx` or `gevp` *is sufficient to turn in for credit, along with all your individually written MATLAB m-files and functions and any required conclusions based on your results.*

1. ($\mathbb{D}$-*stability of a System*) In lecture it was shown that the LMI to verify stability of the system $\dot{x} = Ax$ is given by

$$\begin{cases} P > 0 \\ A^T P + PA < 0, \end{cases}$$

which verifies that the eigenvalues of A are in the left half plane. Modifications can be made to this LMI that provide conditions to verify the eigenvalues of A are in a subset $\mathbb{D}$ of the left half plane, which is called $\mathbb{D}$-stability. The simplest case is to verify $\lambda_i(A)$ are in region that lies to the left of some value of α in the left half plane. In this case the resulting LMI conditions are

$$\begin{cases} P > 0 \\ A^T P + PA + 2\alpha P < 0. \end{cases}$$

Code up this new LMI as a `feasp` problem using the MATLAB LMI Toolbox and use it to estimate the largest region of $\mathbb{D}$-stability for the following systems $\dot{x} = Ax$ where

$$A = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix}, \qquad A = \begin{pmatrix} -1.5 & 1 & 0.1 \\ -4 & -1 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

2. (*Spectral Norm of a Matrix*) In lecture we derived the following LMI representing the inequality $\|A\|_2 = \bar{\sigma}(A) < \gamma$,

$$\begin{pmatrix} \gamma^2 & A^T \\ A & I \end{pmatrix} > 0.$$

Code up this LMI as a `mincx` problem using the MATLAB LMI Toolbox. Assume the minimization variable is $\rho = \gamma^2$:

$$\begin{cases} \min \rho \\ \begin{pmatrix} \rho & A^T \\ A & I \end{pmatrix} > 0. \end{cases}$$

For the following matrices, use the code above to estimate $\bar{\sigma}(A)$. Remember to back solve for $\gamma = \sqrt{\rho}$.

$$A_1 = \begin{pmatrix} 2 & -1 & 2 \end{pmatrix}, \quad A_2 = \begin{pmatrix} -1 & 0 \\ 1 & -1 \\ 0 & 1 \end{pmatrix}.$$

3. (H_∞ *Norm of a System*) In this problem you will use an alternate LMI form for the Bounded Real Lemma and implement it as a generalized eigenvalue problem (`gevp`) using the MATLAB LMI Toolbox. If we start with part (c) from the Bounded Real Lemma theorem from lecture and move the γ^2 term to the right side, we have

$$\begin{pmatrix} A^T P + PA + C^T C & PB + C^T D \\ B^T P + D^T C & D^T D \end{pmatrix} < \gamma^2 \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

In this form with P as the matrix variable, it can be cast as a generalized eigenvalue problem

$$\begin{cases} \min \lambda \\ A(x) < \lambda B(x) \\ B(x) > 0 \\ C(x) > 0, \end{cases}$$

where $\lambda = \gamma^2$, $A(x)$ is the left hand side of the above expression, $C(x) = P$, and

$$B(x) = \begin{pmatrix} \varepsilon & 0 \\ 0 & 1 \end{pmatrix}.$$

Here we need to add a small number $\varepsilon \approx 0.00001$ to the matrix so that it is positive definite for numerical stability.

(a) Implement the above LMI as a function that accepts matrices A, B, C, and D from a general state space representation $\dot{x} = Ax + Bu$, $y = Cx + Du$ of a system G and uses the function `gevp` to compute the infinity norm for the system, $\|G\|_\infty$. Be sure to back solve for $\gamma = \sqrt{\lambda}$ in your function.

(b) Use your code from (a) to compute $\|G\|_\infty$ for the following stable (from Problem 1) dynamical systems. You can check your results using the MATLAB function `hinfnorm(sys)` where `sys` is a state space object constructed using the command `ss`.

$$\begin{aligned} \dot{x} &= \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u \\ y &= \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} x \end{aligned}, \qquad \begin{aligned} \dot{x} &= \begin{pmatrix} -1.5 & 1 & 0.1 \\ -4 & -1 & 0 \\ 0 & 1 & 0 \end{pmatrix} x + \begin{pmatrix} -0.2 \\ -1.8 \\ 0 \end{pmatrix} u \\ y &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u \end{aligned}.$$