UNIVERSITY OF COLORADO - BOULDER
Department of Mechanical Engineering

MCEN 6228 - Robust Multivariable Control

Homework #2 (Assigned: 1/24, Due: 1/30 at 5pm)

1. Find the singular value decompositions (SVDs) of the following:

   (a) $A = [\ 2\ \ -1\ \ 2\ ]$.

   (b) The $m \times n$ matrix of zeros, $\mathbf{O}_{m\times n}$.

   (c) A general $\mathbf{x} \neq 0 \in \mathbb{R}^n$ (in terms of $\mathbf{x}$ and $\|\mathbf{x}\|$)

2. The *condition number* of a matrix is defined by

$$\text{cond}(G) \quad = \quad \bar{\sigma}(G)/\underline{\sigma}(G).$$

   The condition number of a matrix (or of a transfer function of a plant plotted as a function of frequency) is a measure of the difficulty in inverting the matrix (i.e., controlling the plant). If G and K are square, invertible complex matrices, prove the following using the submultiplicative property $\bar{\sigma}(GK) \leq \bar{\sigma}(G)\bar{\sigma}(K)$ of the matrix 2-norm, $\|\cdot\|_2 = \bar{\sigma}(\cdot)$ and the identity $\bar{\sigma}(G^{-1}) = 1/\underline{\sigma}(G)$. (*Hint:* $GG^{-1} = I$).

   (a) $\bar{\sigma}(GK) \leq \bar{\sigma}(KG)\ \text{cond}(G)$

   (b) $\underline{\sigma}(GK) \geq \underline{\sigma}(KG)/\text{cond}(G)$