Class: Robust Multivariate Control
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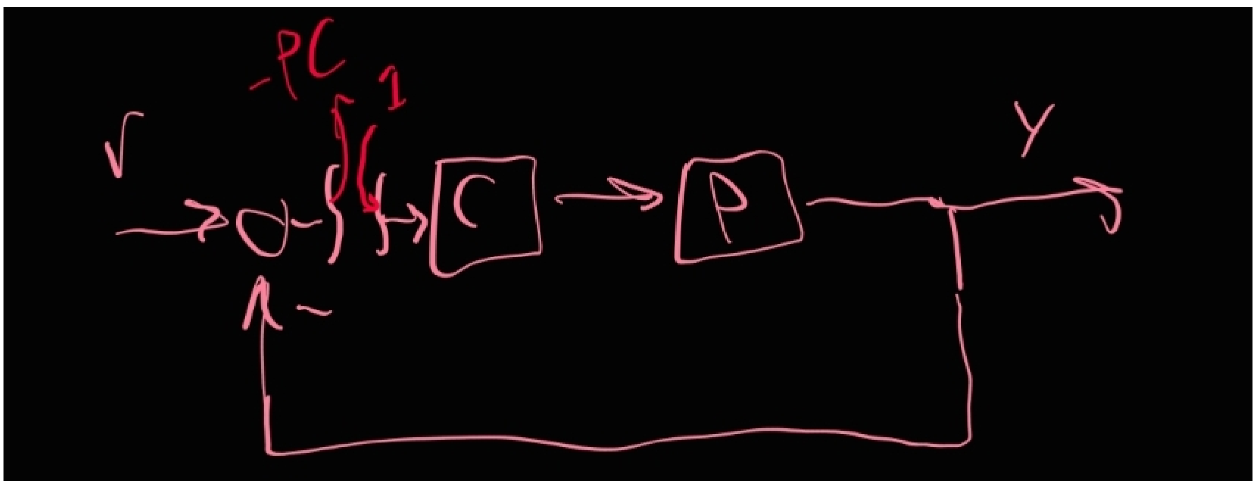
Date: March 2, 2025
Assignment: Homework 5

1.

For the simple SISO negative feedback system with $P(s) = \frac{1}{s-1}$ *and* $C(s) = \frac{s-1}{s+2}$*, show that at least one transfer function from exogenous signals* r *and* d_I *to the internal signals* e*,* u *and* y *is unstable due to the right-half plane pole/zero cancellation of* $s-1$ *in the loop transfer function* $L = PC$*.*



I will start with the transfer function from r to y which for this SISO unity feedback system is:

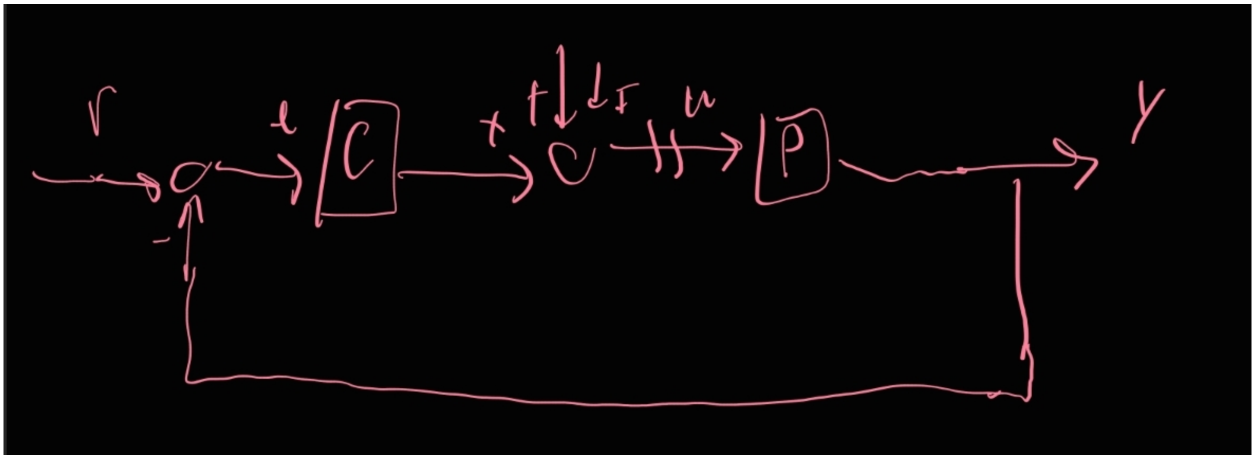
$$\frac{PC}{1+PC}$$

Or:

$$\frac{(\frac{1}{s-1})(\frac{s-1}{s+2})}{1+(\frac{1}{s-1})(\frac{s-1}{s+2})} = \frac{\frac{1}{s+2}}{1+\frac{1}{s+2}} = \frac{\frac{1}{s+2}}{\frac{s+2}{s+2}+\frac{1}{s+2}} = \frac{\frac{1}{s+2}}{\frac{s+3}{s+2}} = \frac{1}{s+3}$$

This transfer function is not only stable but it was made stable by the addition of the controller and the pole/zero cancellation. Let us see some other signals, eh.

Full system diagram with d_I:



Let's try r→u:
Direct path: C Return path: $1 + PC$

$$\frac{C}{1+PC}$$

$$\frac{\frac{s-1}{s+2}}{1+\frac{1}{s-1}\frac{s-1}{s+2}} =$$

$$\frac{\frac{s-1}{s+2}}{1+\frac{1}{s+2}} = \frac{\frac{s-1}{s+2}}{\frac{s+3}{s+2}} = \frac{s-1}{s+3}$$

$$= \frac{s-1}{s+3}$$

Again the pole is at -3 and the transfer function is stable. Let's try $d_I \rightarrow y$:

$$\frac{P}{1+CP}$$

$$\frac{\frac{1}{s-1}}{\frac{s+3}{s+2}}$$

$$= \frac{s+2}{(s-1)(s+3)}$$

There we go, a pole at $s = 1$, the transfer function from d_I to y is unstable.

2.

For the following longitudinal model for an F-4 Phantom with 4 perturbation states (pitch rate Δq, forward speed Δu, angle of attack $\Delta\alpha$, pitch angle $\Delta\theta$), 2 inputs (elevator deflection $\Delta\delta_e$, flaperon deflection $\Delta\delta_f$) and 2 outputs (Δq and $\Delta\alpha$). Note that the D matrix entries are zeros for this output selection.

$$A = \begin{pmatrix} -0.8 & -0.0006 & -12 & 0 \\ 0 & -0.014 & -16.64 & -32.2 \\ 1 & -0.0001 & -1.5 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} -19 & -2.5 \\ -0.66 & -0.5 \\ -0.16 & -0.6 \\ 0 & 0 \end{pmatrix}.$$

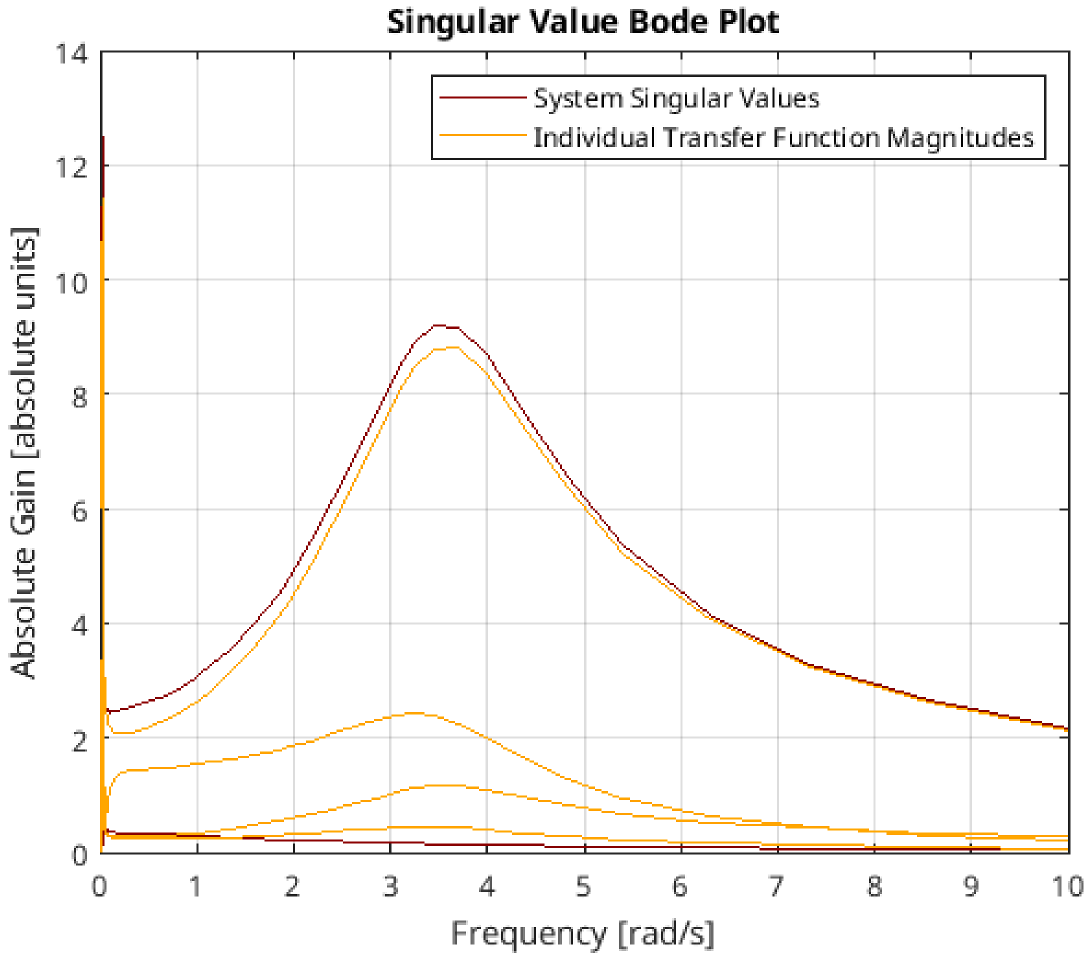
(a)

Generate a MATLAB plot that contains the singular value bode plots in red combined with the bode magnitude plots of the 4 individual transfer functions from the 2 inputs to 2 outputs. Does the maximum singular value plot envelope the individual transfer function plots?

I generated the system, transfer function, and plots using the 'ss', 'ss2tf', and 'bode' Matlab functions. You can see my implementation below:

```
A = [ -0.8 -0.0006 -12 0; 0 -0.014 -16.64 -32.2; 1 -0.0001 -1.5 0; 1 0 0 0];
B = [ -19 -2.5; -0.66 -0.5; -0.16 -0.6; 0 0];
C = [1 0 0 0; 0 0 1 0];
D = [0 0; 0 0];

phantomSS = ss(A,B,C,D);
[sv,wout] = sigma(phantomSS, {0,10});

figure;
plot(wout, sv, 'Color', [0.545, 0, 0]);
grid on;
title('Singular Value Bode Plot');
xlabel('Frequency [rad/s]');
ylabel('Absolute Gain [absolute units]');
hold on;

[num, den] = ss2tf(A,B,C,D,1);
tf11 = tf(num(1,:), den);
tf12 = tf(num(2,:), den);
[num, den] = ss2tf(A,B,C,D,2);
tf21 = tf(num(1,:), den);
tf22 = tf(num(2,:), den);

[mag11, phase11, wout11] = bode(tf11, {0,10});
[mag12, phase12, wout12] = bode(tf12, {0,10});
[mag21, phase21, wout21] = bode(tf21, {0,10});
[mag22, phase22, wout22] = bode(tf22, {0,10});
plot(wout11, squeeze(mag11), 'Color', [1,0.65,0]);
plot(wout12, squeeze(mag12), 'Color', [1,0.65,0]);
plot(wout21, squeeze(mag21), 'Color', [1,0.65,0]);
plot(wout22, squeeze(mag22), 'Color', [1,0.65,0]);
legend('System Singular Values', 'Individual Transfer Function Magnitudes');
```

The plot is below and you can see that yes in fact thee maximum singular value plot (in red) envelopes the individual transfer functions (in orange):



(b)

From the plot in (a), estimate the worst case output 2-norm (in absolute units, not dB) for any $L_2[0, \infty)$ input signal with 2-norm equal to 1.

There's a peak near 0 rad/s at about 12.5 if we take this as the max singular value then that would indicate the highest possible amplification of the input so the worst case for the output 2-norm for an input with 2-norm of 1 would be 12.5.

(c)

For the frequency $\omega = 10$ rad/s, what input direction is amplified most? (the MATLAB command evalfr is useful here)

```
disp(evalfr(phantomSS, 1i*10));
```

I used the 'evalfr' function with 10 rad/s as shown above and the result was:

$$\begin{bmatrix} -0.2559 + 2.1211i & -0.1085 + 0.2593i \\ 0.2013 + 0.0718i & 0.0150 + 0.0731i \end{bmatrix}$$

Indicating that the 1st input to the 1st output has the strongest response at 10 rad/s.

3.

In this problem you will design a feedback control for the thrust vector actuator for the RL10-B2 liquid rocket engine which is on the second stage of the Delta IV launch vehicle. The gimbal actuator takes an input voltage $v(t)$ and rotates the nozzle an angle $\phi(t)$, whose dynamics are described by

$$P(s) = \frac{\phi(s)}{v(s)} = \frac{40(s+1)}{(s^2+s+4)(s+6)}$$

The desired closed loop system should have decent tracking behavior so that it can follow the reference command from the rocket's guidance system. The system has a closed loop bandwidth constraint due to the large inertia of the nozzle and the limits of the gimbal actuator to rotate it. Assume the following specifications for your initial controller design:

- *Maximum closed loop bandwidth of 2 Hz (12.56 rad/s)*
- *Tracking error of less than 10% from 0 to 0.5 Hz (3.14 rad/s)*
- *Closed loop step response with a maximum overshoot of 20%*

(a)

Convert the closed loop performance requirements above into open loop constraints. Design a dynamic compensator $C(s)$ and generate a bode plot of $L(s) = P(s)C(s)$ that includes all the constraints so that you can visually verify that your design meets all performance requirements.

For the maximum closed loop bandwidth of 12.56 rad/s we want the crossover point (the frequency where the open loop gain drops below 0dB) to be equal to the bandwidth frequency. For the tracking error we want the open loop gain to be greater than 2 or 6dB, this is derived from the tracking error being equal to the magnitude of the sensitivity function which is the reciprocol of the open loop function. For the overshoot we would like the phase margin to be 138.0213461 at the crossover point defined above, this is derived from the damping ratio formula $\zeta = (\frac{[ln(M_p)]^2}{\pi^2+[ln(M_p)]^2})^{\frac{1}{2}}$ where M_p is the max overshoot and the approximation that the phase margin is $100 \cdot \zeta$ in degrees.

The plan is to begin by plotting the plant bode plot and then using my knowledge of the effects of compensators to adjust the system to our needs.