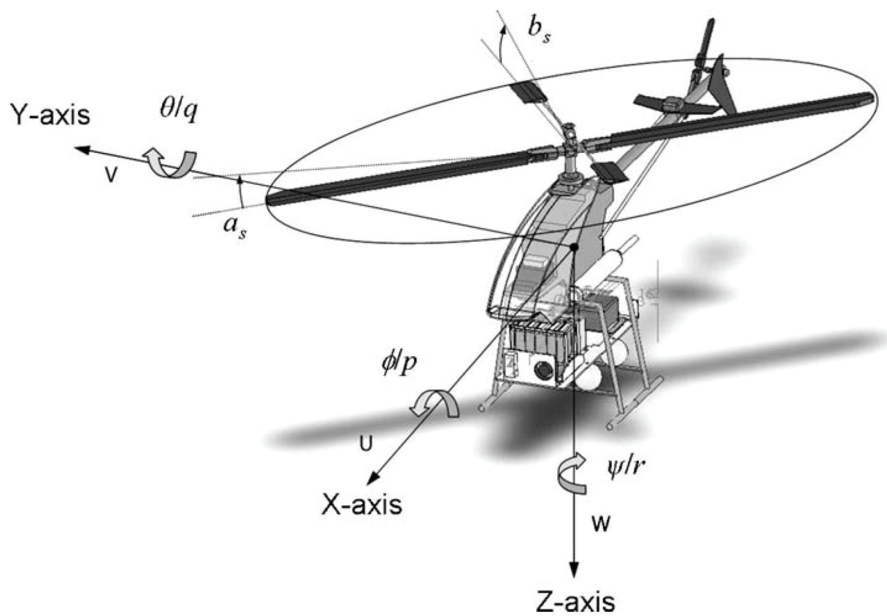
UNIVERSITY OF COLORADO - BOULDER
Department of Mechanical Engineering

MCEN 6228 - Robust Multivariable Control

Final Project (Due: 5/6 at 5pm)
Robust Inner Loop Control of a 6-DOF Micro-Helicopter



For this project you will synthesize a series of inner loop controllers using H_∞ and μ-synthesis techniques for an 11-state rotary wing micro-helicopter and analyze their robustness properties. The state vector is given by $x = [u \;\; v \;\; p \;\; q \;\; \phi \;\; \theta \;\; a_s \;\; b_s \;\; w \;\; r \;\; r_{fb}]^T$. The variables u, v, w are the vehicle translational velocities along the vehicle's longitudinal, lateral and vertical body axis, p, q, r are the vehicle roll, pitch and yaw rate respectively, ϕ and θ are the roll and pitch attitude, a_s and b_s are the rotor dynamics states (flap of the rotor relative to the body) and r_{fb} is the washed out yaw rate, a filtering state associated with on-board yaw rate feedback. The control input vector is $u = [\delta_{long} \; \delta_{lat} \; \delta_{ped}]^T$, corresponding to the longitudinal cyclic, the lateral cyclic and the tail rotor inputs. The outputs available for feedback are the pitch and roll attitude θ and ϕ, and the washed out yaw rate r_{fb}. The equations of motion linearized about hover are given in the MATLAB m-file on the Canvas website.

The objective of the project is to design a controller to ensure robust command tracking of $\theta_r \rightarrow \theta$ and $\phi_r \rightarrow \phi$ for the reference $r = [\theta_r \; \phi_r]^T$ in the presence of a structured multiplicative input uncertainty.

(a) Construct the corresponding block diagram and formulate the generalized plant with the relevant uncertainty and performance weighting functions to achieve the desired command tracking behavior. Note that the output as prescribed has three states; you will include the r_{fb} state in the reference input as well so that the G and K matrix transfer functions are square. However from the design perspective we are only interested in the performance in the $\theta_r \rightarrow \theta$ and $\phi_r \rightarrow \phi$ channels.

(b) Synthesize a H_∞ controller and analyze its nominal performance via singular value plots of the open loop transfer functions G, GK and closed loop transfer functions S_o, T_o and performance weight W_p. Provide plots of step responses in $\theta_r \rightarrow \theta$ and $\phi_r \rightarrow \phi$ and the output disturbance quantities $d_\theta \rightarrow \theta$ and $d_\phi \rightarrow \phi$ for a disturbance vector $d_o = [d_\theta \; d_\phi]^T$. Comment on the performance of your design in terms of its bandwidth, tracking error and disturbance rejection capabilities.

(c) Update the generalized plant to include a *diagonal* structured complex uncertainty with a relative weight of $w_i(s) = (s+0.2)/(0.5s+1)$ in each channel. Assume $\|\Delta\|_\infty \leq 1$, and compute the upper and lower bounds on the structured singular value for the controller obtained above, and determine the margins ($1/\mu_{peak}$) on robust stability and robust performance. Provide an iteration on the performance weighting function to obtain improved margins. Note that if you are using only θ, ϕ and r_{fb} as outputs, I don't expect you to achieve robust performance for the given uncertainty profiles, but rather *use these tools to assess how much uncertainty your design can tolerate.* Again provide plots of step responses in θ and ϕ and the disturbance channels and comment on the performance of your design.

(d) Use D-K iteration to redesign your controller to achieve improved robust performance. Re-generate all the appropriate plots and comment on your final design, in particular the tradeoff between high bandwidth and robust performance in the presence of uncertainty.