UNIVERSITY OF COLORADO - BOULDER
Department of Mechanical Engineering

MCEN 6228 - Robust Multivariable Control

Homework #8 (Assigned: 4/10, Due: 4/24)
Structured Uncertainty Analysis and Robust Performance

1. In this problem you will analyze the robustness properties of a F-16 equipped with a lateral regulator. We consider the lateral dynamics (β, ϕ, p, r) augmented with aileron and rudder actuator dynamics (δ_a, δ_r) and a washout filter (x_w) in the yaw channel. The state vector of the augmented dynamics is $(\beta, \phi, p, r, \delta_a, \delta_r, x_w)$. The outputs of the model include (r_w, p, β, ϕ) where r_w is the washed out yaw rate. Assume a static controller $u = Ke$ where

$$K = \begin{bmatrix} -0.56 & -0.44 & -0.11 & -0.35 \\ -1.19 & -0.21 & -0.44 & 0.26 \end{bmatrix}.$$

An m-file with the augmented system A, B, C, D matrices and controller gains K is available for download on the course website. For the analysis, assume that the plant is subject to an unstructured inverse multiplicative output uncertainty $\tilde{G} = (I - W_o\Delta)^{-1}G$ and the desired performance metric is attenuation of measurement noise n at the output y, as described.

   (a) Draw a block diagram of the overall system including the weighting functions described below and label w, z, v, u, y_Δ and u_Δ.

   (b) Compute the generalized plant P and close the lower LFT (with $\Delta \neq 0$) to compute N. What are the corresponding tests for nominal performance and robust stability?

   (c) Assume a performance weighting function to be $W_p(s) = w_p(s)I$ where $w_p(s) = (s/M + \omega_B^*)/(s + \omega_B^* A)$, with $A = 4$; $M = 10$; $\omega_B^* = 3$. Does the system satisfy this nominal performance criterion you derived in part (b)?

   (d) Assume an uncertainty weighting function to be $W_o(s) = w_o(s)I$ where $w_o(s) = (\tau s + r_0)/(\tau s/r_\infty + 1)$, with $\tau = 0.02$, $r_0 = 0.05$, $r_\infty = 0.4$, and full (unstructured) uncertainty. Is the system robustly stable using the criterion you derived in part (b)?

   Hint: to compute the state space model for the controller, assume $D = K$ (static gain) and the A, B, C matrices are all zeros with appropriate dimensions.

2. Consider a model of the longitudinal dynamics of a missile

$$\begin{aligned} \frac{d}{dt}\begin{bmatrix}\alpha \\ q\end{bmatrix} &= \begin{bmatrix} Z_\alpha & Z_q \\ M_\alpha & 0\end{bmatrix}\begin{bmatrix}\alpha \\ q\end{bmatrix} + \begin{bmatrix} Z_{\delta_u} \\ M_{\delta_u}\end{bmatrix}\delta_u \\ n_z &= \begin{bmatrix} Z_\alpha & 0\end{bmatrix}\begin{bmatrix}\alpha \\ q\end{bmatrix} + Z_{\delta_u}\delta_u \end{aligned}$$

where the states are the angle of attack α, the pitch rate q, n_z is the output (normal acceleration), δ_u is the tail deflection input. Assume the following variations in stability derivatives: $M_\alpha = M_\alpha^0 + \delta_\alpha$ and $Z_q = Z_q^0 + \delta_q$. Generate the analytical state space model as discussed in lecture for the extended plant N by "pulling out the deltas" for this case of structured uncertainty. Assume $w = \delta_u$, $z = n_z$ and $\Delta = \text{diag}\{\delta_\alpha,\delta_q\}$.

3. Consider the following lateral flight dynamics model for the A-4D aircraft:

$$\frac{d}{dt}\begin{bmatrix}\beta \\ r\end{bmatrix} = \begin{bmatrix} Y_\beta/u_0 & -(1 - Y_r/u_0) \\ N_\beta & N_r\end{bmatrix}\begin{bmatrix}\beta \\ r\end{bmatrix} + \begin{bmatrix} 0 & Y_{\delta_r}/u_0 \\ N_{\delta_a} & N_{\delta_r}\end{bmatrix}\begin{bmatrix}\delta_a \\ \delta_r\end{bmatrix}.$$

Assume the sea level reference flight condition (Condition 1) from the attached data sheet, and Mach 1 at sea level is 1126 ft/s (use this to compute u_0). Build the state space A and B matrices using the regular stability and control derivatives ($Y_\beta, N_\beta, N_r, Y_{\delta_r}, N_{\delta_a}, N_{\delta_r}$), not the 'primed' derivatives, and assume $Y_r = 0$. Also assume the C matrix is the identity so that the outputs are the two states β and r.

(a) Using the nominal plant, synthesize a H_∞ controller using `hinfsyn` with a disturbance rejection performance weighting function $W_p(s) = w_p(s)I$ where $w_p(s) = (s/M + \omega_B^*)/(s + \omega_B^* A)$, with $A = 0.005$; $M = 2$; $\omega_B^* = 1$. Does the closed loop system satisfy nominal performance? Verify with a plot of $\bar{\sigma}[S(j\omega)]$ and $1/|w_p(j\omega)|$.

(b) Now allow the parameter Y_β to have 20% real uncertainty and compute $\mu(N_{11})$. Use the MATLAB command `mussv` and generate the N and Δ blocks using the commands `lft` and `lftdata` as was done in lecture. Does the controller you designed provide robust stability for the perturbation in Y_β?

(c) Now compute the robust performance test $\mu(N)$ using the appropriate unstructured Δ_p and structured Δ blocks as was discussed in lecture. Does the controller you designed provide robust performance for the perturbation in Y_β?

(d) Now add an additional perturbation of 30% real uncertainty of the parameter N_β and check to see if your controller provides robust stability and robust performance for this uncertainty case.