$$\|f\|_2 = \sqrt{\int_a^b |f(t)|^2 \, dt}$$

1.) a) $f(t) = e^{-3t} \quad [0,\infty)$

$$\|f(t)\|_2 = \sqrt{\int_0^\infty e^{-6t} \, dt}$$

$$= \sqrt{-\frac{1}{6} e^{-6t} \Big|_0^\infty}$$

$$= \sqrt{0 + \frac{1}{6}}$$

$$= \sqrt{\frac{1}{6}}$$

b) $f(t) = \sin t \quad [0,\infty)$

$$\|f(t)\|_2 = \sqrt{\int_0^\infty \sin^2 t \, dt}$$

$$= \sqrt{\frac{t}{2} - \frac{1}{4}\sin(2t) \Big|_0^\infty}$$

$\sin(\infty)$ does not converge so 2-norm doesn't exist

c) $f(t) = \begin{bmatrix} e^{-t} \\ 1 \end{bmatrix}$ $[0, T)$

$$\|f(t)\|_2 = \begin{bmatrix} \sqrt{\int_0^T e^{-2t}\,dt} \\ \sqrt{\int_0^T 1\,dt} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{-\frac{1}{2} e^{-2t} \Big|_0^T} \\ \sqrt{t \Big|_0^T} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{-\frac{1}{2} e^{-2T} - \frac{1}{2}} \\ \sqrt{\ } \end{bmatrix}$$

$$= \sqrt{\frac{1}{2} - \frac{1}{2} e^{-2T} + T}$$

$$\|f\|_2 = \sqrt{\frac{1}{2\pi} \int_{-\infty}^{\infty} |\hat{f}(j\omega)|^2 \, d\omega}$$

2.) a.) $\hat{f}(j\omega) = \frac{1}{j\omega + a}, \quad a > 0$

$$\|\hat{f}(j\omega)\|_2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{\omega^2 + a^2} \, d\omega$$

$$= \frac{1}{2\pi} \left( \frac{1}{a} \arctan\left(\frac{\omega}{a}\right) \Big|_{-\infty}^{\infty} \right)$$

$$= \frac{1}{2\pi} \left( \frac{\pi}{a\,2} + \frac{\pi}{2a} \right)$$

$$= \sqrt{\frac{1}{2a}}$$

b) $\hat{f}(s) = \frac{1}{(s+a)^2}$, $a > 0$

$$\|\hat{f}(j\omega)\|_2 = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\frac{1}{(j\omega+a)^2}\right|^2 d\omega}$$

$$= \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{(\omega^2+a^2)^2}\, d\omega}$$

$$= \sqrt{\frac{1}{2\pi}\,\frac{\pi}{2a^3}}$$

$$= \sqrt{\frac{1}{4a^3}}$$

c) $\hat{f}(s) = \begin{bmatrix} \frac{1}{s+a} \\ \frac{1}{s+b} \end{bmatrix}$ for $a > 0$, $b > 0$

$$\|\hat{f}(j\omega)\|_2 = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\left|\frac{1}{j\omega+a}\right|^2 + \left|\frac{1}{j\omega+b}\right|^2\right) d\omega}$$

$$= \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{1}{\omega^2+a^2} + \frac{1}{\omega^2+b^2}\right) d\omega}$$

$$= \sqrt{\frac{1}{2\pi}\left(\frac{\pi}{a}+\frac{\pi}{b}\right)}$$

$$= \sqrt{\frac{1}{2a}+\frac{1}{2b}}$$

3.)

$$\|g\|_1 = \int_0^\infty |g(t)| dt$$

$$\leq \int_0^\infty c e^{-at} dt$$

$$\leq -\frac{c}{a} e^{-at} \Big|_0^\infty$$

$$\leq 0 + \frac{c}{a}$$

$$\leq \frac{c}{a}$$

C

$\frac{}{q}$ finite for any positive $c$ and $q$

$\|g\|_1 < \infty$, can be bounded