1.1 $\frac{y^{58}}{y^4 \cdot y^{12}} = \frac{y^{58}}{y^{16}} = y^{42}$

1.2 $8^2 \cdot 2^x = 2^9$

$2^6 \cdot 2^x = 2^9$

$6 + x = 9$

$x = 3$

1.3 $\frac{x}{y} = 3$

$x^{-2}y^2 = ? \rightarrow \frac{y^2}{x^2} = \frac{x^{-2}}{y^2}$

$\left(\frac{x}{y}\right)^{-2} = 3^{-2}$

$x^{-2}y^2 = \frac{1}{9}$

1.4 $\frac{\sqrt{2^{13}}}{\sqrt{8^3}} = \frac{\sqrt{2^{13}}}{\sqrt{2^9}} = \frac{2^{6,5}}{2^{4,5}} = 2^2 = 4$

1.5

a) $x + y = y + x$ True

b) $x(y+z) = xy + xz$ True

c) $x^{y+z} = x^y x^z$ True

d) $\frac{x^y}{x^z} = x^{y-z}$ True

1,6 $\frac{x^2 - 25}{x-5} = 3$

$\frac{(x+5)(x-5)}{x-5} = 3$ $\quad x \neq 0 \Rightarrow x \neq 5$

$x + 5 = 3$

$x = -2$

2.1 $y = ax + b$

$0°K = -460°f$

$1000°K = 1340°f$

$K = a + bf$

$0 = a + b(-460)$ (1)

$1000 = a + b1340$ (2)

$1000 - 0 = a + b \cdot 1340 - (a + b \cdot (-460))$

$1000 = 1800b$

$b = \frac{5}{9}$

(1) $0 = a + \frac{5}{9} \cdot (-460)$

$-a = -\frac{5}{9} \cdot (-460)$

$a = 255{,}56$

$K = 255{,}56 + \frac{5}{9}F$

$\frac{5}{9}F = K - 255{,}56$

$F = \frac{9}{5}K - 460$

$K = \frac{5}{9}F$

$F = \frac{9}{5}F - 460$

$\frac{4}{5}F = 460$

$F = 460 \cdot \frac{5}{4} = \underline{\underline{575}}$

22 $f(x) = 2x + 3$

$f(y) = 17$

$f(y) = 2x + 3 = 17$

$2x = 14$

$x = \underline{\underline{7}}$

2.3 $3^{2x^2-4x+3} = 27$

$3^{2x^2-4x+3} = 3^3$

$2x^2 - 4x + 3 = 3$

$2x^2 - 4x = 0$

$2x(x-2) = 0$

$2x = 0$ or $x - 2 = 0$

$x = 0$ $\quad x = 2$

2.4 $(1,01)^x \cdot GDP = 2\,GDP$

$(1,01)^x = 2 \Rightarrow (1,01)^{70} = 2 \Rightarrow$ 70 years

2.5 $\ln\left(\frac{e^2}{e^3}\right) = \ln(e^2) - \ln(e^3) = 2-3 = -1$

3.1 $\sum_{i=0}^{\infty}\left(\frac{1}{6^i} + 0{,}25^i\right) = \sum_{i=0}^{\infty}\left(\frac{1}{6^i} + \frac{1}{4^i}\right) = \sum_{i=0}^{\infty}\left(\frac{1}{6^i}\right) + \sum_{i=0}^{\infty}\left(\frac{1}{4^i}\right) =$

$= 1+1 = 2$

3.2 $\lim_{x\to\infty} \frac{x^2-9}{x-3} = \lim_{x\to\infty} \frac{(x+3)(x-3)}{x-3} = \lim_{x\to\infty} x+3 = \infty$

3.3 $f(x) = x^3 - 4$

$f'(x) = 3x^2$

$f'(-1) = 3\cdot(-1)^2 = 3$

$f'(-5) = 3\cdot(-5)^2 = 75$

$\frac{y_2-y_1}{x_2-x_1} = \frac{75-3}{-5-(-1)} = \frac{72}{-4} = -18$

3.4 $f(x) = \left(\frac{x^2+3}{x+2}\right)$

$f'(x) = \frac{2x(x+2) - 1(x^2+3)}{(x+2)^2} = \frac{2x^2+4x-x^2-3}{x^2+2x+4} = \frac{x^2+4x-3}{x^2+2x+4}$

3.5 $f(x) = x^7 + 4x^2$

$f'(x) = 7x^6 + 8x$

$f''(x) = 42x^5 + 8$

3.6 $f(x) = \frac{x^4+4x}{\ln(x)}$

$f'(x) = \frac{(4x^3+4)\cdot\ln(x) - \frac{1}{x}\cdot(x^4+4x)}{(\ln(x))^2}$

3.7 $f(x) = 3x^3 - 9x$

$f'(x) = 9x^2 - 9 = 0$

$9x^2 = 9$

$x^2 = 1 \rightarrow x_1 = 1;\ x_2 = -1$

$f''(x) = 18x$

$f''(x_1) = 18 \rightarrow$ max

$f''(x_2) = -18 \rightarrow$ min

3.8 $f(xy) = x^2 + 2y^3$

$f(2,3) = 2^2 + 2 \cdot 3 = 4 + 54 = 58$

3.9 $f(xy) = \ln(2x - y) > 0$

$2x - y > 0$

$2x > y$

$x > \frac{y}{2}$

3.10 $f(xy) = x^5 e^y + x^2 y^3$

$f_x(xy) = 5x^4 \cdot e^y + 2x \cdot y^3$

$f_y(xy) = x^5 \cdot e^y + 3y^2 \cdot x^2$

3.11 $f(xy) = \sqrt{xy} - 0.7x - 0.7y = (xy)^{0.5} - 0.7x - 0.7y$

$f_x'(xy) = 0,5x^{-0.5} \cdot y^{0,5} - 0,7 = 0$

$f_y'(xy) = 0,5y^{-0,5} \cdot x^{0,5} - 0,7 = 0$

$0,5x^{-0,5} \cdot y^{0,5} - 0.7 = 0.5y^{-0,5} \cdot x^{0,5} - 0,7 \;/2$

$x^{-0.5} \cdot y^{0,5} = y^{-0,5} \cdot x^{0,5}$

$\frac{\sqrt{y}}{\sqrt{x}} = \frac{\sqrt{x}}{\sqrt{y}}$

$\sqrt{y} = \frac{x}{\sqrt{y}}$

$y = x \rightarrow 0,5\sqrt{\frac{x}{x}} - 0,7 = 0$

$,5 - 0.7 = 0 \rightarrow 0$

(4)

inflection point

4.1 $A = \begin{pmatrix} 2 & 3 \\ 4 & 1 \\ 1 & 2 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 4 & 1 \\ 2 & 1 & 2 \end{pmatrix}$

AB	1	4	1
	2	1	2
2 3	8	11	8
4 1	6	17	6
1 2	5	6	5

4.2

BA	2	3
	4	1
	1	2
1 4 1	19	9
2 1 2	10	11

4.3 $A = \begin{pmatrix} 3.3 & 5.1 \\ 6.1 & 1.23 \\ 45.76 & 0 \end{pmatrix}$ $A^T = \begin{pmatrix} 3.3 & 6.1 & 45.76 \\ 5.1 & 1.23 & 0 \end{pmatrix}$

4.4 $A = \begin{pmatrix} 2 & 3 & 0 \\ 4 & 5 & 2 \\ 2 & 5 & 3 \end{pmatrix}$ $\det(A) = 2 \cdot 5 \cdot 3 + 3 \cdot 2 \cdot 2 + 0 \cdot 4 \cdot 5 - (0 \cdot 5 \cdot 2) - (3 \cdot 4 \cdot 3) - (2 \cdot 2 \cdot 5) = 42 - 56 = -14$

5.1 $\Omega = \{H, T\}$

5.2 $30 = n$ competitors

1, 2, 3 place $= n-1, n-2, n-3$ $\Rightarrow 30 \cdot 29 \cdot 28 = 24360$

5.3 $n = 6^2 = 36$, $k = 3^2 = 9$ $\Rightarrow P(A) = \frac{9}{36} = \frac{1}{4}$