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An Exponentially Stable Adaptive Friction Compensator

Teh-Lu Liao and Tsun-I Chien

***Abstract*—This note presents a novel adaptive compensation scheme for Coulomb friction in a servocontrol system. An adaptive observer for estimating the unknown Coulomb friction coefficient is also derived on the basis of the Lyapunov technique. In addition, a linearizing control law is developed to compensate for the friction force and obtain the tracking objective. The proposed adaptive compensation guarantees an exponential convergence for state errors and parameter error, and the adaptive schemes in [1] and [3] guarantee only an asymptotic (or stable) convergence. Simulation results demonstrate the effectiveness of the proposed method for a single-mass servocontrol system.**

***Index Terms*— Adaptive compensation, exponentially stable, friction, Lyapunov.**

I. Introduction

Friction is an unavoidable force and critical aspect of many physical control systems. Its impact on the performance of systems has received extensive attention [1]–[7]. These effects originating from friction may lead to steady-state tracking errors and limit cycles. Consequently, diverse friction compensation strategies have been developed in the recent decade [1]–[6], [8]. Friction compensation largely focuses on cancelling the friction by applying some control force if the accuracy friction models are available. In practice, however, friction is a natural phenomenon that is extremely difficult to model and not yet completely determined. Hence, with a lack of precise knowledge regarding friction, cancelling its effect based on the adaptive estimation of the friction in control systems is a feasible strategy [1].

A linear observer has been developed in recent years to enhance the performance of a system with Coulomb friction [8]. A pioneering work, Friedland and Park [1] constructed an adaptive nonlinear observer for estimating Coulomb function coefficient. By doing so, that investigation ensured that the error in estimation of the friction coefficient converges asymptotically to zero if the velocity is bounded away from zero. A recent work [3] has proposed a systematic method based on the Lyapunov-based technique for selecting a nonlinear function that can be used in an observer for estimating the Coulomb function coefficient. That work also released the restrictive condition on the magnitude of the velocity imposed in [1]. Moreover, asymptotic stability of the coefficient error dynamics is guaranteed if the selected nonlinear function in the observer satisfies certain conditions. The above methods, however, are limited in that the Lyapunov-like function only includes the parameter error is employed, thereby implying the asymptotic stability of the coefficient error dynamics. Moreover, the asymptotic stability of the overall control system for both tracking errors and parameters has not been thoroughly discussed.

Based on the above results in [1] and [3], this work presents an exponentially stable adaptive compensation for Coulomb friction in a simple servocontrol system. The proposed scheme provides an exponential convergence for estimating the Coulomb friction coefficient and tracking errors even without persistency of excitation.

The rest of this note is organized as follows. Section II reviews some preliminary results of [1] and [3]. Section III presents the exponentially stable adaptive friction compensation strategy. Next, Section IV provides some simulations on a single-mass servocontrol system. Merits of the proposed method are also compared with those of the method in [1] and [3]. Conclusions are finally made in Section V.

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The authors are with the Department of Engineering Science, National Cheng Kung University, Tainan, 701, Taiwan R.O.C. (e-mail: tlliao@mail.ncku.edu.tw).

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II. Review of Adaptive Friction Compensation

Consider a servocontrol system given by

$$m\ddot{x} = u - f(\dot{x}, k_c) \tag{1}$$

where

m	mass (or inertia);
x	position (or angular position);
$\dot{x}$	velocity (or angular velocity);
u	control input representing the effect of all applied forces (or torques) except the friction;
$f(\dot{x}, k_c)$	friction.

Friction models have been extensively studied [4], [6], [7], [9]. Herein, we consider the simplest Coulomb friction model, which is expressed as

$$f(\dot{x}, k_c) = k_c\ \mathrm{sgn}(\dot{x}). \tag{2}$$

The parameter k_c is the Coulomb friction coefficient.

Without loss of generality and for the sake of simplicity, assume that $m = 1$ and define state variables: $x_1 = x$ and $x_2 = \dot{x}$. Therefore, the dynamical equation (1) with (2) is written in the state space representation as

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= u - k_c\ \mathrm{sgn}(x_2).\end{aligned} \tag{3}$$

If the parameter k_c is known *a priori*, and desired position and velocity trajectories x_d and $\dot{x}_d$ are given, then a linearizing feedback control for the state tracking of this system is given by

$$u = k_c\ \mathrm{sgn}(x_2) - g_1(x_1 - x_d) - g_2(x_2 - \dot{x}_d) + \ddot{x}_d \tag{4}$$

which results in the closed-loop system in the state space

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -g_1(x_1 - x_d) - g_2(x_2 - \dot{x}_d) + \ddot{x}_d\end{aligned} \tag{5}$$

or in the error space

$$\begin{aligned}\dot{e}_1 &= e_2\\ \dot{e}_2 &= -g_1 e_1 - g_2 e_2\end{aligned} \tag{6}$$

where $e_1 = x_1 - x_d$ and $e_2 = x_2 - \dot{x}_d$ are position tracking error and velocity tracking error, respectively. If g_1 and g_2 are all positive constants and appropriately chosen such that system (6) is asymptotically stable, then the objective of state tracking is achieved. The main problem is that the coefficient k_c is generally unknown or rarely determined. Therefore, the above control law (4) is not implementable. Hence, an adaptive estimator of k_c should be designed before implementing the control law (4). A structure of the generalized estimator proposed in [2] is as follows:

$$\hat{k}_c = z - g(|x_2|) \tag{7}$$

where z denotes an intermediate variable whose dynamics is to be specified subsequently and $g(|x_2|)$ is a nonlinear function of $|x_2|$. In a pioneering work, Friedland and Park [1] proposed an adaptive estimator for the friction parameter k_c, in which the nonlinear function $g(|x_2|)$ is given by

$$g(|x_2|) = k|x_2|^{\mu}. \tag{8}$$

The estimate of k_c is then expressed as

$$\hat{k}_c = z - k|x_2|^{\mu} \tag{9}$$

and the dynamics of z is chosen as

$$\dot{z} = k\mu|x_2|^{\mu-1}(u - \hat{k}_c \operatorname{sgn}(x_2)) \operatorname{sgn}(x_2) \tag{10}$$

where $k > 0$ and $\mu > 0$ are two positive design parameters selected to achieve a proper transient response. A linearizing feedback control for this control system is then designed on the basis of the estimated parameter $\hat{k}_c$ and given by

$$u = \hat{k}_c \operatorname{sgn}(x_2) - g_1(x_1 - x_d) - g_2(x_2 - \dot{x}_d) + \ddot{x}_d. \tag{11}$$

The above estimator design, however, only results in *conditional* asymptotic stability of the error dynamics. Restated, the error dynamics is asymptotically stable, provided velocity is always bounded away from zero [1]. A recent work [3] proposed a Lyapunov-based method to systematically select the nonlinear function $g(|x_2|)$ and relax the condition imposed in [1]. In that study, this constraint may be relaxed, provided some certain criteria for selecting the nonlinear function used in the nonlinear estimator are satisfied. The proposed adaptive estimator in [3] is given by

$$\hat{k}_c = z - g(|x_2|) \tag{12}$$

and

$$\dot{z} = g'(|x_2|)(u - \hat{k}_c \operatorname{sgn}(x_2)) \operatorname{sgn}(x_2) \tag{13}$$

where $g'(|x_2|)$ denotes the derivative of $g(|x_2|)$ with respect to $|x_2|$. The nonlinear function $g(|x_2|)$ is selected on the basis of the following two-criterion [3]:

- $g(|x_2|)$ is monotonically increasing;
- $0 < g'(|x_2|) < K_{\max}$ for all $|x_2| \geq 0$.

By defining the estimation error $e_k = k_c - \hat{k}_c$, the Lyapunov function candidate is chosen as

$$V = \tfrac{1}{2}e_k^2 = \tfrac{1}{2}(k_c - \hat{k}_c)^2 \tag{14}$$

such that it can be easily shown that the time derivative of V along with (12) and (13) is a negative definite function; i.e., $\dot{V} < 0$. Therefore, the asymptotic stability of the parameter error dynamics is ensured.

Remark 1: A possible function of $g(|x_2|)$ has been pointed out in [3] and given by $g(|x_2|) = \ln(1/1 + \exp(-|x_2|))$, which is a monotonically increasing function in $|x_2|$ with its derivative $g'(|x_2|) = (\exp(-|x_2|)/1 + \exp(-|x_2|))$ bounded by 0.5. The function $g'(|x_2|)$, however, converges to zero as $|x_2|$ approaches infinity, thereby implying that the time derivative of V is only a negative semidefinite function. Consequently, the parameter error dynamics is stable, but not asymptotically stable.

III. An Exponentially Stable Adaptive Friction Compensator

In this section, we derive a novel adaptive compensator for Coulomb friction with the property of exponential convergence of both the tracking errors and the parameter error. Before developing the adaptive compensator, define the following notation and matrices: $e^T = [e_1\ e_2]$ representing the tracking errors vector, and matrices

$$A = \begin{bmatrix} 0 & 1 \\ -g_1 & -g_2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 \\ -1 \end{bmatrix}.$$

The tracking control law employed for the servocontrol system in (1) is designed as (11). Meanwhile, the adaptive estimator for the friction parameter k_c is given by

$$\hat{k}_c = z - g(|x_2|) \tag{15}$$

and

$$\dot{z} = g'(|x_2|)(u - \hat{k}_c \operatorname{sgn}(x_2)) \operatorname{sgn}(x_2) + 2e^T PB \operatorname{sgn}(x_2) \tag{16}$$

where P is a symmetric and positive-definite matrix; i.e., $P = P^T > 0$, and is a solution of the following Lyapunov equation:

$$A^T P + PA = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{17}$$

Moreover, the nonlinear function $g(|x_2|)$ satisfies the following condition:

$$0 < K_{\min} \leq g'(|x_2|) \leq K_{\max} \quad \text{for all } |x_2| \geq 0. \tag{18}$$

Remark 2: Many functions $g(|x_2|)$ satisfy the condition (18). One example is $g(|x_2|) = a|x_2| + \ln(1/1 + \exp(-|x_2|))$ and $a > 0$, for which the constants in (18) can be easily verified that $K_{\min} = a$ and $K_{\max} = a + 0.5$.

According to (15) and (16), the dynamics the estimated parameter $\hat{k}_c$ can be obtained as

$$\begin{aligned} \dot{\hat{k}}_c &= \dot{z} - g'(|x_2|)\dot{x}_2 \operatorname{sgn}(x_2) \\ &= g'(|x_2|)(u - \hat{k}_c \operatorname{sgn})) \operatorname{sgn}(x_2) \\ &\quad - g'(|x_2|)\dot{x}_2 \operatorname{sgn}(x_2) + 2e^T PB \operatorname{sgn}(x_2) \\ &= g'(|x_2|)(k_c - \hat{k}_c) + 2e^T PB \operatorname{sgn}(x_2). \end{aligned} \tag{19}$$

Remark 3: The second term on the right-hand side of the final equality in (19) is an integral part of the adaptive control for linear or nonlinear systems in which unknown parameters appear linearly. A control law along with an updated algorithm with $2e^T PB\operatorname{sgn}(x_2)$ can cancel the nonlinear term expressed by linear-in-k_c in system (1).

Main Theorem: Consider the servocontrol system in (1), and give the desired position trajectory x_d and the desired velocity trajectory $\dot{x}_d$ that are all bounded. If the control law given by (11) and the adaptive estimator of (15) and (16) with the property of (18) are employed, then the tracking errors and parameter error exponentially converge to zero.

Proof: To prove stability, the Lyapunov stability theorem is adopted herein. If a Lyapunov function candidate is chosen as

$$V = e^T Pe + \tfrac{1}{2}(k_c - \hat{k}_c)^2 \tag{20}$$

then the time derivative of V along the trajectory of the closed-loop system leads to

$$\begin{aligned} \dot{V} &= -e^T e + 2e^T PB \operatorname{sgn}(x_2)(k_c - \hat{k}_c) - (k_c - \hat{k}_c)\dot{\hat{k}}_c \\ &= -e^T e + 2e^T PB \operatorname{sgn}(x_2)(k_c - \hat{k}_c) \\ &\quad - (k_c - \hat{k}_c)g'(|x_2|)(k_c - \hat{k}_c) \\ &\quad - 2e^T PB \operatorname{sgn}(x_2)(k_c - \hat{k}_c) \\ &= -e^T e - g'(|x_2|)(k_c - \hat{k}_c)^2 \\ &\leq -\|e\|^2 - K_{\min}(k_c - \hat{k}_c)^2. \end{aligned} \tag{21}$$

As demonstrated earlier, $\dot{V}$ is negative definite. This process subsequently implies that $\dot{V} \leq -\lambda_1\|X\|^2 \leq 0$, where

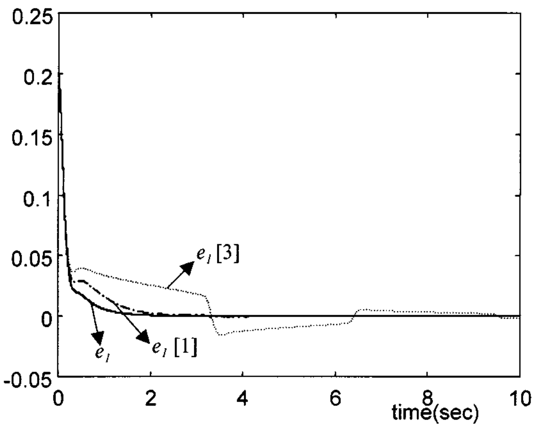

Fig. 1. Time responses of position tracking error by the proposed method (e_1), the method of [1] (e_1 [1]), and the method of [3] (e_1 [3]) for k_c = 50, respectively.

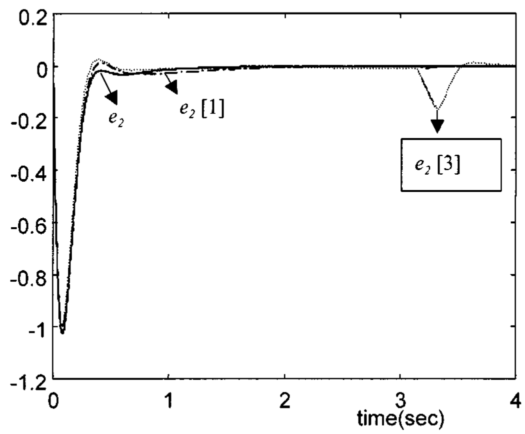

Fig. 2. Time responses of velocity tracking error by the proposed method (e_2), the method of [1] (e_2 [1]), and the method of [3] (e_2 [3]) for k_c = 50, respectively.

$0 < \lambda_1 \equiv \min(K_{\min}, 1)$ and $X^T = [e^T \;\; k_c - \hat{k}_c]$. Furthermore, according to (20) we have $\lambda_2\|X\|^2 \leq V \leq \lambda_3\|X\|^2$, where $0 < \lambda_2 \min \equiv \{\lambda_{\min}(P), \frac{1}{2}\}$, $0 < \lambda_3 \equiv \max\{\lambda_{\max}(P), \frac{1}{2}\}$, and $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ denote the minimum and maximum eigenvalue of matrix P, respectively. Therefore, the global asymptotic stability of $X = 0$ is ensured. Next, using (21) yields $\dot{V} \leq -(\lambda_1/\lambda_3)V$, and, hence, $V(t) \leq V(0)\exp[-(\lambda_1/\lambda_3)t]$, where $V(0)$ denotes the initial Lyapunov function, which is bounded. Therefore, X converges to zero at least exponentially [10]. This completes the proof.

IV. Comparative Numerical Simulations

In this section, the proposed method for friction compensation is applied to a simple single-mass servocontrol system. To demonstrate the superiority of the proposed adaptive compensation scheme over the methods proposed by [1] and [3], we consider the tracking control of a single-mass servocontrol system discussed in Section II. The desired trajectory of position is chosen to be $x_d(t) = -1 + \cos(t)$, and its corresponding velocity is $\dot{x}_d(t) = -\sin(t)$ such that the desired position and desired velocity at time $t = 0$ are zero. The value of Coulomb friction coefficient k_c is selected as 50, and the control feedback law is given as (11) with gains g_1 = 200 and $g_2 = 20$. In the proposed scheme, the adaptive observer for estimating k_c is designed as shown in (15) and (16) along with $g(|x_2|)$ given in Remark 2, in which the parameter a is chosen as two. Meanwhile, the adaptive observer of [1] is given by (9) and (10) with $k = 1$ and $\mu = 2$, and the adaptive observer of [3] is shown as (12) and (13) associated with $g(|x_2|)$, which is given in Remark 1. The initial conditions in all simulations are chosen as $x_1(0) = 0.2$, $x_2(0) = 0.1$, and $\hat{k}_c(0) = 40$. Figs. 1–3 depict the simulation results when the above-mentioned compensation schemes are implemented. In Fig. 2, the time responses of velocity tracking error between $0 \leq t \leq 4$ are shown so that the differences between the methods become more apparent. According to Figs. 1 and 2, after the transients die out, position and velocity tracking can be achieved, and an apparent delay in the response by the method of [3] exists. Moreover, the tracking performance of the proposed method is better than that of [1]. According to Fig. 3, the estimated parameter $\hat{k}_c$ in the compensation method of [3] does not yet converge to its true parameter. Moreover, the parameter convergence of our proposed method is faster than that of [1].

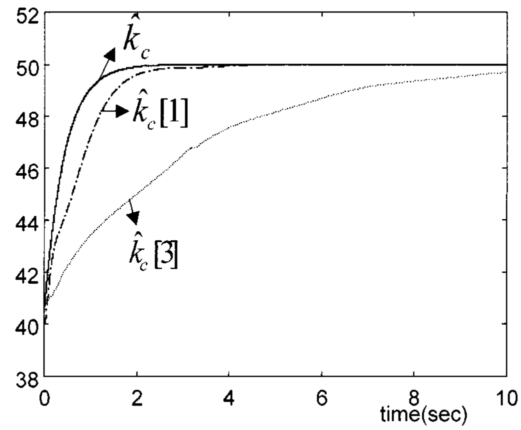

Fig. 3. Time responses of the estimated parameter $\hat{k}_c$ by the proposed method $(\hat{k}_c)$, the method of [1] ($\hat{k}_c$ [1]), and the method of [3] ($\hat{k}_c$ [3]) for k_c = 50, respectively.

The above simulations confirm that the superiority of the proposed scheme over the method of [1] and that of [3].

V. Conclusions

This paper presents a novel, exponentially stable adaptive compensation scheme for Coulomb friction in a servocontrol system. An adaptive observer is also derived for estimating the unknown Coulomb friction coefficient whose nonlinear function satisfies certain criteria. In addition, a linearizing control law is developed to compensate for the friction force and attain the tracking objective. Moreover, the exponential stability of state errors and the parameter error is ensured using the Lyapunov stability technique, and the adaptive schemes in [1] and [2] guarantee only an asymptotic (or stable) convergence. Simulations results demonstrate the superiority of the proposed method over the scheme in [3] for a single-mass servocontrol system.

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Stochastic Adaptive One-Step-Ahead Optimal Controllers Based on Input Matching

Kueiming Lo and Dachun Zhang

***Abstract*—Optimal adaptive controller based on the ELS algorithm is established using the input matching technique. The control signal is reduced to a constant weighted sum of the measurable information-state vector components using a one-step-ahead quadratic cost function to govern the behavior of the stochastic linear systems. The control effort can be estimated globally. The algorithm also predicts the convergence rate. With no excitation condition, the closed-loop system is globally stable and the input converges to the one-step-ahead optimal input.**

***Index Terms*—ELS algorithm, global stability, input matching, multivariable stochastic systems, optimal adaptive control.**

I. Introduction

The investigation of optimal adaptive controllers has been motivated by a desire to control incompletely specified plant operations via formation of an appropriate input signal and by tracking a reference signal while simultaneously minimizing the control effort. Recent advances in this area have provided a variety of approaches to the control of complex and unknown plant operations (see [1]–[5]). The common assumptions in most of these works are that the system has a stable inverse and some persistent excitation (PE) conditions must be required. Many global convergence results are invalid for adaptive control algorithms applied to systems that do not satisfy these assumptions. Furthermore, these types of optimal adaptive controllers are difficult to implement because they rely on approximations to compute a Riccati equation and require a great deal of computational time.

After analyzing one-step-ahead controllers, a new input matching technique was suggested by [6] and [7]. Both the tracking error and the control effort are bounded by this direct adaptive scheme. It combines the global convergence estimate of model matching and the robustness of optimally based input specification. The advantages of this method are that there is no Riccati equation in the optimal adaptive control design and the minimum phase restriction can be relaxed. The optimal adaptive controller based on input matching was developed from the SG algorithm in [7].

The characteristics of the LS algorithm, which is normally used in adaptive control, are superior to those of the SG algorithm (see [8]–[12]). In practice, much more rapid convergence can be achieved with recursive least squares-type matrix gain sequence. However, because of this intrinsic characteristic of the LS algorithm the problems of analyzing convergence and closed-loop stable become truly formidable. Some excitation conditions or modified algorithm had to be imposed in previous works. The one-step-ahead optimal adaptive controller for a single-input/single-output (SISO) system based on a modified LS algorithm was proposed in [8] and [13]. This algorithm, as pointed out in [8] and [13], differed from the standard LS algorithm. Therefore, the question of whether the one-step-ahead optimal adaptive control and the closed-loop stability are based on the full ELS algorithm is still unanswered if any PE conditions are not brought to bear on a SISO stochastic system with colored noise.

The multivariable stochastic system with colored noise interferences is analyzed in this paper. A full one-step-ahead ELS algorithm is established by the technique of input matching. The full ELS algorithm is used to design a one-step-ahead adaptive controller. The parameter estimation convergence rate is obtained. Without any PE conditions, it is shown that multi-input/multi-output (MIMO) stochastic systems with colored noise interference are closed-loop globally stable and the adaptive control converges to the one-step-ahead optimal control.

II. ELS Algorithm and Adaptive Controller

Consider a time series $\{y_n\}$ that can be described by an ARMAX model of the following form:

$$\left.\begin{array}{ll} A(q^{-1})y_n = q^{-1}B(q^{-1})u_n + C(q^{-1})w_n, & n > 0 \\ y_n = u_n = w_n = 0, & n \leq 0 \end{array}\right\} \tag{1}$$

where y_n, u_n, and w_n denote the output, input, and disturbance, respectively. $A(q^{-1}), B(q^{-1})$, and $C(q^{-1})$ are matrix polynomials in the backward operator q^{-1}

$$A(q^{-1}) = I + A_1 q^{-1} + \cdots + A_p q^{-p}$$
$$B(q^{-1}) = B_0 + B_1 q^{-1} + \cdots + B_m q^{-m}$$
$$C(q^{-1}) = I + C_1 q^{-1} + \cdots + C_r q^{-r}$$

with unknown coefficients and with known upper bounds p, m, and r for order and $\det B_0 \neq 0$. Let $\{y_n^*\}$ be an a.s. bounded desired output sequence with y_{n+1}^* being $\mathcal{F}_n$-measurable, where $\mathcal{F}_n$ denotes the increasing sigma algebra generated by the observations up to and including time n. The objective in controlling the system via formation of an appropriate input signal is to provide reasonable tracking of the reference signal $\{y_n^*\}$ while simultaneously maintaining a bounded control effort. The one-step-ahead cost function

$$J(u_n) = E\{\|y_{n+1} - y_{n+1}^*\|^2 \mid \mathcal{F}_n\} + \lambda\|u_n\|^2, \qquad \lambda \geq 0 \tag{2}$$

is considered. Similar to the proof in [6] and [13], the optimal single control minimizing (2) is inferred to satisfy

$$B_0^\tau(y_{n+1} - w_{n+1} - y_{n+1}^*) + \lambda u_n = 0. \tag{3}$$

Let θ be the vector of the system parameters and φ_n^0 be the regression vector

$$\theta^\tau = (-A_1 \cdots - A_p \quad B_1 \cdots B_m \quad C_1 \cdots C_r)$$
$$\varphi_n^0 = \left(y_n^\tau \cdots y_{n-p+1}^\tau \quad u_{n-1}^\tau \cdots u_{n-m+1}^\tau \quad w_n^\tau \cdots w_{n-r+1}^\tau\right)^\tau.$$

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K. Lo is with the Department of Mathematical Sciences, Tsinghua University, Beijing 100084, China.

D. Zhang is with the Department of Mathematics, Changsha Institute of Electric Power Engineering, Hunan 410077, China.

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