Brief Papers

Adaptive High-Precision Control of Positioning Tables—Theory and Experiments

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Abstract—**Positioning tables are widely used in industries, and precise control of their motion is difficult to achieve because of uncertainty in their load and friction parameters. In this paper, we describe the development and experimentation of a new adaptive controller for precise table motion control. The global stability of the controller and global convergence of tracking errors are proven using Lyapunov stability theory under mild conditions. The adaptive controller has been experimentally implemented and extensively tested on a belt-drive high-speed table, resulting in significant accuracy improvement over conventional PID controller.**

I. Introduction

POSITIONING tables are widely used in machine tools, electronic assembly and laboratory automation. Currently, they are generally controlled by PID controllers. Even though the end-point positioning accuracy can be high under PID control, their trajectory tracking accuracy is often quite poor, since there is no direct compensation for friction and inertial forces. The low tracking accuracy can cause contour errors in path following or end-point overshoot in point-to-point motion. Use of higher PID gains can indirectly compensate the friction and inertial forces and reduce the tracking errors, but makes the closed-loop system susceptible to dangerous instability.

Direct compensation of friction forces is desirable, because there is generally significant friction in positioning tables. Direct compensation of inertial forces is useful if the desired motion involves large acceleration or deceleration, as is increasingly true with the newer machines. However, direct compensation is rarely done in current practice, because inertial and friction parameters are often difficult to determine. The reasons are that system inertia varies with load changes, and friction can vary due to various factors such as temperature, humidity and mechanical wear. Overcompensation of friction due to incorrect friction parameters can actually lead to system instability or limit cycles [1]–[2]. Friction can also vary along the length of a table due to mechanical imperfection, as was found in our laboratory table. As a result. it is interesting to explore the use of adaptive control, since parameter adaptation may reduce the uncertainty in the inertia and friction parameters.

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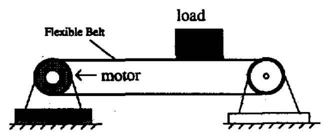

Fig. 1. A positioning table model.

The adaptive positioning table controller in this paper has been obtained by modifying the adaptive robot control method presented in [3]–[4], and similar robot controllers were developed in [5]–[7], through the incorporation of direct compensation for static and dynamic frictions and modification of inertial compensation for eliminating possible parameter drift. We point out that the original adaptive robot controllers were found to perform well in direct-drive robots, but not very well in indirect-drive robots due to transmission imperfections, particularly frictions. Friction compensation using estimates from recursive least square estimation was studied in [1], [8]–[9], leading to good accuracy for persistently exciting trajectories. However, least square estimation suffers from drawbacks such as burden of complex computation and difficulty in dealing with non-persistently exciting trajectories. Other studies on friction compensation can be found in [10]–[14].

In this paper, we examine the Lyapunov-based adaptive control of a positioning table, using a combination of theoretical analysis, computer simulation and hardware experimentation. The paper is organized as follows: In Section II, the modeling of a linear positioning table is presented; Section III discusses the design of the adaptive positioning table control system; Section IV presents the simulation and experimental results; Section V summarizes the conclusions from this study.

II. Modeling of aLinear Positioning Table

The mechanical structure of the positioning table in our lab is illustrated in Fig. 1. The load on this table is driven by a DC motor through a plastic timing belt. We now discuss mathematical modeling of the system, particularly the friction aspect.

Even though exact friction model is difficult to obtain, one can use a simplified model as an approximation. Various friction models were discussed by a number of researchers [1]–[2], [15]. Major components of friction are Coulomb friction, static friction (stiction), and viscous friction. Coulomb friction is the force between two contact surfaces with relative motion. Static friction is the friction when relative motion

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speed is zero. Viscous friction is normally thought to be linear in terms of motion velocity. According to experiments on our positioning table, its friction can be well modeled as static friction plus Coulomb friction. However, due to the imperfection of the mechanical system, friction forces are quite different in the two motion directions. Therefore, it is necessary to assign two sets of parameters for friction in the positive and negative directions. Experimental study also indicates that viscous friction component in the friction force is very small and not linear versus velocity. As a result, viscous friction term is neglected in our modeling and can be regarded as part of the disturbance to the control system.

Without considering belt flexibility, transmission backlash, and motor dynamics, our motion system can be modeled as

$$m\ddot{x} = u + f \tag{1}$$

In the above equation, x is the position of the load, u is the equivalent linear control force (converted from rotary motor torque), and f is the total friction in this system.

If we define maximum static friction of the positive and negative motion directions to be F_{s+} and F_{s-}, instant static friction to be f_{s+} and f_{s-}, and Coulomb friction of the positive and negative motion directions to be f_{c+} and f_{c-}, the total friction force f can be expressed by the following nonlinear function:

$$\begin{aligned} f = {} & [f_{c+} \cdot \eta + f_{c-} \cdot (1-\eta)] \cdot (1-\xi) \\ & + [f_{s+} \cdot \mu + f_{s-} \cdot (1-\mu)] \cdot \xi \end{aligned} \tag{2}$$

where

$$\begin{array}{llllll} \eta = 1 & (\dot{x} \geq 0) & \xi = 1 & (\dot{x} = 0) & \mu = 1 & (u \geq 0) \\ \eta = 0 & (\dot{x} < 0) & \xi = 0 & (\dot{x} \neq 0) & \mu = 0 & (u < 0) \end{array}$$

The model of friction is illustrated in Fig. 2. In order to provide plant parameters for simulation purpose, we performed some experiments and off-line estimation to identify the values of system inertia and friction for our table. A simple way of using digital computer to estimate the dynamic parameters is to start from a discretized dynamic equation. By discretizing (1), we obtain

$$\begin{aligned} x(k) = {} & 2x(k-1) - x(k-2) \\ & + \frac{T^2}{2m}[u(k-1) + u(k-2)] \\ & + \frac{T^2}{m} f_{c+} \cdot (1-\eta) \\ & + \frac{T^2}{m} f_{c-} \cdot \eta \end{aligned}$$

where T is the sampling time. The stiction parameters do not appear in this equation, because stiction only occurs at instants of zero speed. The stiction parameters can be obtained by recording the levels of control input required to move the positioning table.

This relationship is nonlinear in terms of parameters. However, if we define

$$z(k) = x(k) - 2x(k-1) + x(k-2)$$

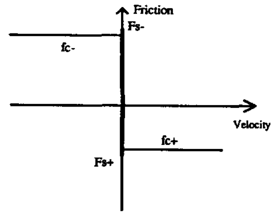

Fig. 2. Positioning table friction model.

$$a_1 = \frac{T^2}{2m} \qquad \alpha_2 = \frac{T^2}{m} f_{c+} \qquad \alpha_3 = \frac{T^2}{m} f_{c-}$$

the above equation becomes

$$\begin{aligned} z(k) = {} & \alpha_1[u(k-1) + u(k-2)] \\ & + \alpha_2 \eta + \alpha_3 (1-\eta) \end{aligned} \tag{3}$$

It is obvious that the above equation is linear in these newly-defined parameters. This is a familiar regression form in system identification theories, with α_1, α_2, α_3 being unknown parameters. In order to get *a priori* knowledge and compare with Lyapunov-based adaptation later, an off-line recursive least square estimation method is employed to get estimates of these unknown parameters, to be denoted by $\hat{a}_1$, $\hat{\alpha}_2$, $\hat{\alpha}_3$. Then, we can compute the estimated values of the corresponding physical parameters from

$$\hat{m} = \frac{T^2}{2\hat{\alpha}_1} \qquad \hat{f}_{c+} = \frac{\hat{\alpha}_2 \hat{m}}{T^2} \qquad \hat{f}_{c-} = \frac{\hat{\alpha}_3 \hat{m}}{T^2}$$

It can be proven that, even though there are three unknown parameters to be estimated, an input signal of PE order two (one pure sinusoid) can guarantee the convergence of the three parameters, due to the nonlinear nature (in terms of states) of the dynamics. To generate more informative data and obtain accurate parameter estimates in our identification, a strongly persistently exciting signal of $x_d(t) = 3\sin(8t) + 3\sin(4t)$ was actually used for generating experimental data, and a PD controller was used to stabilize the table during the tests. In this way, reasonably reliable values of the plant parameters can be obtained and later used for understanding the real-time control results. The estimated values of the three uncertain parameters were found to be

$$\hat{m} = 5.1 \text{ kg}, \qquad \hat{f}_{c+} = -12.58 \text{ N}, \qquad \hat{f}_{c-} = 25.16 \text{ N}$$

The above linear friction values correspond to rotary friction values of -0.18 Nm and 0.36 Nm, since the transmission ratio of this positioning table is 90 mm linear movement per motor revolution, i.e., 0.0143 m/rad.

III. Lyapunov-Based Adaptive Compensation

In this section, we present a direct adaptive controller for positioning table tracking, based on Lyapunov stability theory. Note that tracking error will be shown to converge to zero, whether the tracked trajectories are persistently exciting (parameters convergent) or not. Unlike conventional model-reference adaptive control, there is no need for a reference model in our framework. Instead, we require the knowledge of a desired position trajectory $x_d(t)$ which is twice differentiable. That is, the desired position $x_d(t)$, velocity $\dot{x}_d(t)$ and acceleration $\ddot{x}_d(t)$ are available to the controller. This is a very reasonable assumption, because industrial motion controllers generally have built-in planning capabilities for the generation of trajectory profiles, including $x_d(t)$, $\dot{x}_d(t)$, $\ddot{x}_d(t)$.

Assuming that the plant model we are dealing with is described by (1). The friction compensation force should be in the form of

$$\hat{f} = \left[\hat{f}_{c+} \cdot \eta + \hat{f}_{c-} \cdot (1-\eta)\right] \cdot (1-\xi) + \left[\hat{F}_{s+} \cdot \mu + \hat{F}_{s-} \cdot (1-\mu)\right] \cdot \xi \tag{4}$$

where η, ξ, μ are defined as before, and quantities with the hats represent estimated values. In on-line adaptive control, our objective is to find a good way of estimating f, such that the friction estimate $\hat{f}$ will approximate true friction f. For simplicity, the static friction coefficients will not be separately estimated. Instead, they will be assumed to be the same as the corresponding Coulomb friction coefficents. We have experimentally found this to be basically true for our table.

Therefore the entire control input can be selected as:

$$u = \hat{m}\ddot{x}_d - K_D E - \hat{f} \tag{5}$$

where

$$E = \dot{\tilde{x}} + \lambda\tilde{x}$$

is the combined error, $\tilde{x} = x - x_d$ is position tracking error, and λ and K_D are positive design parameters whose values will be chosen in experimentation. Note that the first term on the right-hand-side of (1) is inertial compensation, the second actually represents proportional plus derivative feedback, and the third term denote friction compensation. A technique of choosing the parameters λ and K_D analytically can be found in [16]. Notice that this controller is different from the controller in [3] in two ways. First, dynamic and static frictions in the transmission are directly compensated; Secondly, the desired acceleration, instead of the measurement-dependent (thus noise-contaminated) reference acceleration, is used in the compensation of the uncertain inertial force, leading to avoidance of undesirable parameter drift. As a result of the differences, the Lyapunov stability analysis has to be different, and a new Lyapunov function is given later. Note that the idea of using desired acceleration for inertial compensation was first developed and demonstrated by [7] in robotic control, but our controller here is different in that no quadratic error feedback term was used and the following Lyapunov analysis is simpler.

The Coulomb friction parameter and mass inertia can be obtained from the following adaptation law:

$$\dot{\hat{m}} = -\gamma_1 \ddot{x}_d E \tag{6-1}$$

$$\dot{\hat{f}}_{c-} = \gamma_2 E \cdot \eta \tag{6-2}$$

$$\dot{\hat{f}}_{c+} = \gamma_2 E \cdot (1-\eta) \tag{6-3}$$

where $\gamma_i (i = 1, 2)$ are positive adaptation gains. Adaptation gains can be chosen using the technique in [16] which correlate the selection of adaptation gain magnitude to the magnitude of the acceleration of the desired motion to avoid the excitation of unmodeled high-frequency dynamics. The closed-loop system dynamics can be obtained by differentiating (1) and substituting (5) and (6) into it. This leads to the following equation governing the error in the adaptively controlled system,

$$m\dddot{\tilde{x}} + K_D\ddot{\tilde{x}} + (\lambda K_D + \gamma_1 \ddot{x}_d^2 + \gamma_2)\dot{\tilde{x}} + (\gamma_1 \ddot{x}_d^2 + \gamma_2)\lambda\tilde{x} = \tilde{m}\dddot{x}_d \tag{7}$$

where $\tilde{m} = \hat{m} - m$. Typically, the control system error varies much faster than the desired acceleration of the motion. This implies that the coefficients in the above equation are quasi-static constants. Note that these coefficients determine the response speed, i.e., the bandwidth, of control system. Since the mechanical structure and other components in the system have inherent unmodeled high-frequency dynamics which should not be excited, these coefficients should be chosen below certain levels. Therefore, smaller adaptation gain γ_1 should be used for trajectories with higher acceleration $\ddot{x}_d(t)$ in order to keep the bandwidth of the closed-loop system, and thus the excitation of the unmodeled dynamics, at the same level. If the acceleration of the desired motion increases n times, the parameter γ_1 should be decreased by roughly n^2 times. The effectiveness of this idea has been confirmed in a number of experiments.

In order to prove stability, the Lyapunov function can be chosen as:

$$V = \frac{1}{2}mE^2 + \frac{1}{2\gamma_1}\tilde{m}^2 + \frac{\lambda}{2}(2K_D - m\lambda)\tilde{x}^2 + \frac{1}{2\gamma_2}\tilde{f}_{c-}^2 + \frac{1}{2\gamma_2}\tilde{f}_{c+}^2 \tag{8}$$

where $\tilde{m} = \hat{m} - m$, $\tilde{f}_{c-} = \hat{f}_{c-} - f_{c-}$ and $\tilde{f}_{c+} = \hat{f}_{c+} - f_{c+}$.

By differentiating the function and substituting (5) and (6) into the derivative, we obtain

$$\dot{V} = -K_D\lambda^2\tilde{x}^2 - (K_D - m\lambda)\dot{\tilde{x}}^2. \tag{9}$$

If the design parameters K_D and λ are chosen such that $K_D > m\lambda$ is satisfied, $\dot{V}$ is a globally negative definite function. Therefore, the tracking errors $\tilde{x}$ and $\dot{\tilde{x}}$ will converge to zero asymptotically, based on La Salle's theorem or Barbalat's lemma [17]–[18]. Note that the convergence of tracking errors does not imply the convergence of parameter errors. The estimated parameters from the adaptation law will converge to the true parameters if the desired trajectories are persistently exciting.

Normally, a *priori* knowledge can be obtained before the implementation of real time control. Hence we may easily make use of the known upper bound of the system inertia to guarantee global stability. Specifically, if we know the range of the system inertia

$$M_{\min} \le m \le M_{\max}.$$

Then, we may simply choose

$$K_D \ge M_{\max}\lambda.$$

In practice, the inertia values of the motor shaft and the mechanical transmission are well known and the only uncertainty is on the inertia of the load mass. In such cases, a similar condition can be easily obtained, i.e.,

$$K_D \geq (m_0 + M_{\text{load-max}})\lambda$$

where m_0 is the combined inertia of the motor shaft and transmission, and $M_{\text{load-max}}$ is the known maximum bound of the inertia of the handled loads.

The above adaptive control design can be easily extended to include adaptive compensation of viscous friction. A similar Lyapunov analysis will lead to the same stability and convergence result as before. In our experimental system, we found that viscous friction is relatively small, so its compensation is not examined in detail.

IV. Simulation and Experimental Results

In this section we present some simulation and experimental results based on our positioning table shown in Fig. 1. Performance of the adaptive control system, which is in the form of PD plus adaptive feedforward, will be compared with that of a PD controller. The PD controller is considered instead of a complete PID controller, because it is much more convenient to compare PD due to the similarity of its structure to the adaptive controller, and PD performance is quite representative of those of PID controllers in tracking problems. The integral term in a PID controller is mainly used for eliminating the steady-state error in point-to-point control tasks and it is not much useful in tracking tasks, as has been verified in a number of our experiments. The motor for driving the table is a brushless servo motor powered by an analog servo drive which can operate in both velocity and torque modes. In our experiments, torque mode is selected because of its simplicity.

It was pointed out by several authors that control system performance may be quite different in low speed and high speed motions. Therefore, we carry out our simulation and experiments for two sinusoidal trajectories, one being of low velocity and the other being high velocity. The low-speed position profile is

$$x_d(t) = 0.143\cos(t) \quad (m)$$

The high speed profile is

$$x_d(t) = 0.143\cos(8t) \quad (m)$$

These sinusoidal trajectories can be thought of representing the motion of one motor axis of a x-y positioning table in circular contour motion.

A. Computer Simulations

For all the following simulations and experiments, the gains of the PD part ($K_D E$ term) in the adaptive controller were chosen to be

$$K_D = 400 \qquad (N/m/s),$$
$$K_p = K_d\lambda = 9300 \qquad (N/m).$$

The identical gains were used by a PD controller (same level of stability) for fairness of comparison. Actually, the above gains have been experimentally determined so that the PD controller have a fast and well-damped response. Thus the comparison is more in the PD controller's favor, since the adaptive controller relies mostly on direct cancellation of inertial and friction forces to reduce tracking error, instead of PD gains. In experiments, we have found that the robustness of the adaptive controller with respect to high-frequency unmodeled dynamics is basically the same as that of the PD controller, i.e., they go unstable when K_p and K_D reach the same level of high values.

The responses of the PD and adaptive controllers are shown in Fig. 3. Only tracking errors are shown due to space limitation. The tracking errors in low speed and high speed tasks for the PD controller are shown in Figs. 3(a) and 3(c). As expected, the tracking errors do not converge to zero, and the tracking error in the high speed task is much larger than in the low speed task. Figs. 3(b) and 3(d) show the results of the adaptive controller in the low speed and high speed tasks. The responses of the adaptive parameters are not shown, but they are quite similar to the experimental results shown later in Figs. 5 and 6. For the low-speed motion, the adaptation gains are chosen to be $\gamma_1 = 3 \times 10^{-3}$, $\gamma_2 = 1$. In this test, the initial values of the estimated parameters are chosen to be zero (no a priori knowledge) to highlight the effect of the adaptation. As a result, the adaptive controller is the same as the PD controller in the initial period. As adaptation goes on, the adaptive feedforward part of the adaptive controller has more and more obvious effect. The tracking error is seen to converge to zero after a short time. In this task, the inertial and friction parameters converge to the true plant parameters, and the adaptive controller converges to a PD plus feedforward controller. It is seen that the tracking errors converge asymptotically to zero. In the high velocity case, adaptation gains are chosen to be $\gamma_1 = 5 \times 10^{-5}$, $\gamma_2 = 1$. The inertia adaptation gain is chosen smaller than in the low speed case because the acceleration in this task is higher, as discussed in Section III. Fig. 3(d) show the tracking results of the fast motion tracking. It is seen that the simulated performance of the adaptive controller is consistently good for both low speed and high speed. In the following, we show that the experimental results indeed confirm the above simulation results.

B. Hardware Implementation

For the experiment setup, we used a personal computer for implementing the adaptive controller. The computer was a PC486 made by Gateway 2000, Inc. An encoder attached to the back of the motor was used to obtain position information for feedback. The encoder has 1000 lines/resolution, and, after quadrature decoder, provides an accuracy of about 0.09 deg/count. The table has a transmission ratio of 90 mm per motor revolution. Therefore, the equivalent linear resolution of the sensor is 0.0225 mm, or 22.5 μm. We also point out that the timing belt has a backlash of about 4 encoder counts (0.1 mm), which provides one limitation of the table motion accuracy. The 1.1 m long plastic belt has some obvious sagging, leading to another source of hardware-induced motion error. A decoder board from Keithley Metrabyte is used to

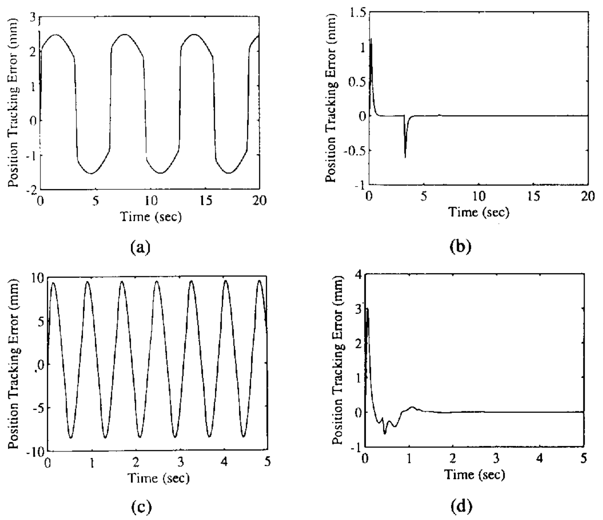

Fig. 3. Tracking errors from computer simulations (a) PD control of slow motion; (b) adaptive control of slow motion; (c) PD control of fast motion; (d) adaptive control of fast motion.

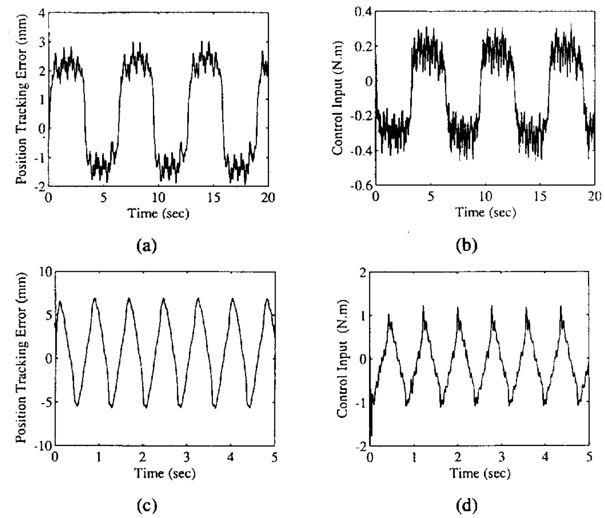

Fig. 4. PD control experiments (a) tracking error (slow motion); (b) control torque (slow motion); (c) tracking error (fast motion); (d) control torque (fast motion).

interface the incremental encoders to the controller in the PC. The relatively old data acquisition board DT 2801-A from Data Translation Inc. was used to output control commands to the motor drive, limiting the sampling rate to 250 Hz. In order to obtain velocity measurement, numerical differentiation of position measurement was used. The obtained velocity information was then filtered by a second-order digital low-pass filter to alleviate the effects of noise. Without the filter, the response of the table looked quite oscillatory, due to the noise introduced by numerical differentiation. However, after filtering the velocity information through the digital filter, the table motion became much smoother. Since the differentiation noise has the same frequency as the sampling rate, the bandwidth of the filter is chosen to be 10 times smaller, which leads to a noise reduction of about 100 times.

The experimental results using PD are shown in Fig. 4, with (a) and (b) denoting the tracking error and motor torque in the low velocity motion and (c) and (d) in the high velocity motion. In the low velocity case, maximum position error is about 3 mm. Most of this error is due to the friction force, and only a small fraction is due to inertia force because the acceleration is small in this motion. In the high speed case, the maximum position error is 7 mm. The tracking error in the high speed case is mostly due to three sources: the friction force, the inertial force, and the computer time delay. In this case, the inertia force and computer time delay are major causes of tracking error.

The experimental result for adaptive compensation are shown in Figs. 5 and 6, starting with zero initial estimates. In the case of low speed motion, the adaptive compensation allowed us to obtain a maximum tracking error of less than 1 mm, representing a three times reduction in the tracking error. In the case of high speed motion, adaptive controller accuracy also provides a more than 100% decrease in error, compared with the PD controller. We point out that the inertial and friction parameters obtained by the adaptive controller are

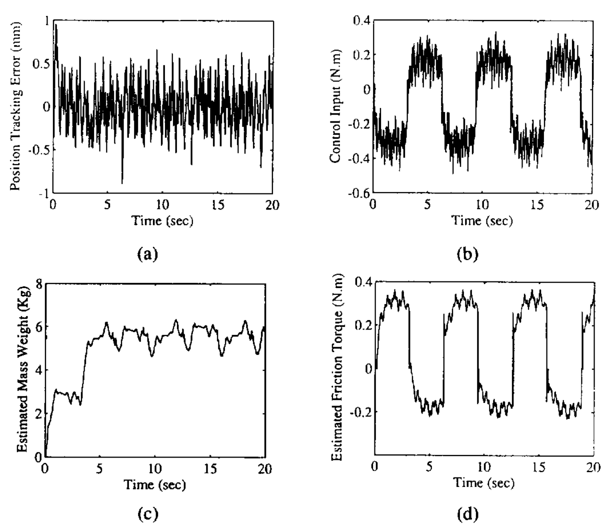

Fig. 5. Adaptive control experiments: slow motion (a) tracking error; (b) control torque; (c) on-line estimated inertia; (d) on-line estimated friction.

very close to the off-line estimated values, as given in Section II.

One notes that the tracking errors of the adaptive controller in experiments, as shown in Figs. 5 and 6, do not converge to zero, unlike those in Fig. 3(b) and 3(d). There are many causes for this difference from the theoretical convergence. This is because that the theoretical convergence property in Section III is proven based on the idealized model in (1) and (2) which does not contain the effects of belt flexibility, the dynamics of motor/drive and time delay in the control computer. The experimental tracking errors in Figs. 5 and 6 were obtained in the presence of all these real-world imperfections. In the case of the high speed tracking, a major cause of residual errors is probably the time delay in the computer (corresponding to 250

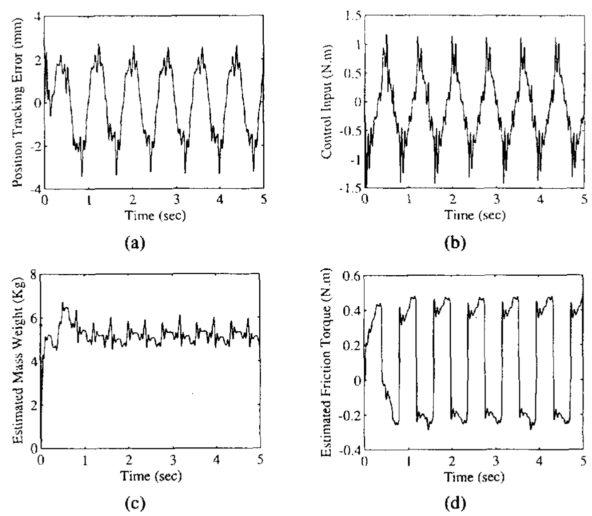

Fig. 6. Adaptive control experiments: fast motion (a) tracking error; (b) control torque; (c) on-line estimated inertia; (d) on-line estimated friction.

Hz sampling rate), which is quite significant for this motion of 3 Hz. The time delay, and the resulting adaptive tracking error, will be much smaller if fast dedicated microprocessors are used to implement this adaptive control algorithm. In industrial motion control systems, high-speed digital signal processors (DSP), including both fixed-point and floating-point ones, are already widely used. A number of controller manufacturers are already marketing motion controllers with sampling rates higher than 10 000 Hz.

V. Conclusion

In this paper, we presented the development and implementation of a new adaptive motion controller for a positioning table. This controller can compensate for uncertainty in both inertial and friction parameters. Due to the direct compensation of friction and inertial forces, high precision control can be achieved using relatively low PD gains, thus resulting in high robustness to unmodeled resonance dynamics. Compared to least-square type self-tuning, this adaptive controller has two advantages. First, the control law and the associated adaptation laws are computationally efficient. Secondly, the adaptive system has guaranteed global stability regardless of the persistent excitation of the reference trajectories. Simulations and experiments confirm the good performance of the adaptive controller in both low speed and high speed motions.

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