**Bivariate Analysis: Correlation and Linear Regression between Delay and paHR**

This analysis investigates the relationship between Delay (X) and paHR (Y) using both Pearson and Spearman correlations, along with a linear regression model to assess predictability and strength of association. The dataset consists of 10 paired observations, with X (Delay) values ranging from 0 to 70 seconds. The mean of X is 38, and the sum of squares (SSX) is 4610. Using the paired data from Assignment 2, paHR values are matched to delay times. The Y values used are: 122, 125, 127, 130, 131, 132, 135, 137, 138, and 140. The mean of Y is calculated as 131.7, and the sum of squares (SSY) is 331.10.

**Pearson Correlation Analysis**

The deviations (X - µX) and (Y - µY) were calculated for each paired value. The Sum of Products (SP) was computed by multiplying each paired deviation and summing the results, resulting in SP = 1104.6. The product of SSX and SSY is 15260.1. Taking the square root gives √(SSX × SSY) = 123.498. The Pearson correlation coefficient is then r = SP / √(SSX × SSY) = 0.895. This suggests a **strong, positive correlation**. Since r > 0.63, the correlation is statistically significant. The coefficient of determination is r² = 0.80, indicating that 80% of the variance in paHR is explained by Delay time.

**Scatterplot Summary**

The scatterplot of Delay vs. paHR exhibits a clear upward trend, confirming the strong positive correlation. A trendline was added with the regression equation, and it closely aligns with the manually calculated regression line, validating accuracy.

**Linear Regression Equation**

Using SP and SSX, the slope (b) is calculated as b = SP / SSX = 1104.6 / 4610 = 0.2396 (rounded to 0.24). The intercept (a) is found using: a = µY - b(µX) = 131.7 - (0.24 × 38) = 122.58. Thus, the regression equation is **Y’ = 0.24X + 122.58**, which allows prediction of paHR based on Delay.

**Predicted Values and Residuals**

Substituting X values into the regression equation yields predicted paHR values (Y’). The residuals, calculated as (Y - Y’), vary slightly around zero, confirming good model fit. The sum of residuals is approximately 0.1 due to rounding, indicating acceptable accuracy.

**Standard Error of the Estimate (SEE)**

Given the standard deviation of Y is 5.77, r = 0.90, and r² = 0.81, SEE is calculated using SEE = sY × √(1 - r²) = 5.77 × √(0.19) ≈ 5.77 × 0.4359 ≈ 2.52. This means that approximately 68% of observed paHR values fall within ±2.52 of predicted values. For a Delay of 30 seconds, predicted Y = 129.78. The 95% confidence interval is Y ± 2×SEE = 129.78 ± 5.04, or **between 125 and 135** bpm.

**Spearman Correlation**

After ranking the X and Y values, the differences in ranks (D) and their squares (D²) were calculated. The sum of D² is 10. Using the formula rho = 1 - [(6×D²)/(n³ - n)] = 1 - (60 / 990) = 0.939. Spearman’s rho confirms a **strong, positive monotonic relationship**, similar to the Pearson result.

**Conclusion**

Both Pearson r (0.90) and Spearman rho (0.94) show a statistically significant, strong positive correlation between Delay and paHR. The linear regression model (Y = 0.24X + 122.58) demonstrates predictive accuracy, supported by low residuals and a narrow SEE. These results suggest that delay time is a strong predictor of paHR in the observed dataset.