Homework 1 for Math 6644 : Iterative Methods for Systems of Equations
(Due on February 19)

Please write your answers to each question neatly.

(1) Construct a 3×3 dense matrix G with $\rho(G) > 1$. Define an iteration,

$$\vec{x}_{k+1} = G\vec{x}_k + \vec{c}.$$

Find an example for $\vec{x}_0$ and $\vec{c}$ such that the iteration does not converge.

(2) Give the matrix expression for the symmetric Gauss-Seidel iterations.

(3) Assume A is a $n \times n$ symmetric positive definite matrix with all eigenvalues in the intervals $(1, 1.1) \cup (10, 10.2)$. Assume that the cost of a matrix vector multiplication is about $4n$ floating point multiplications. Estimate the number of floating point operations reduce the $A-$norm of the error by a factor of 10^{-3} using CG iterations.

(4) Assume that A is a $n \times n$ symmetric positive definite matrix. If $\kappa(A) = O(n)$, give a rough estimate of the number of CG iterations required to reduce the relative residual to $O(1/n)$.

(5) Discreteize the following differential equation

$$\begin{cases} -u'' + 4u = 0 & x \in [0, 1] \\ u(0) = -1, & u(1) = 2 \end{cases}$$

by the central difference scheme. Write your linear system of equations (you must give the matrix A and $\vec{b}$). Solve the system by using classical iteration such as Jacobi, Gauss-Seidel and SOR with $n = 1000$. Test you relaxation parameter in SOR for several values and decide which one is better. You need to discuss your results.