Second Project for Math 6644 : Iterative Methods for Systems of Equations
(Due on April 23)

You are encouraged to design your own projects if you are particularly interested in solving some practical problems using the course materials. Please come to see me if you have any idea in your mind.

Fixed-point and Newton's methods for nonlinear systems

Consider the discrete Chandrasekhar H-equation

$$\mathbf{F}_i(\vec{x}) = x_i - (1 - \frac{c}{2N}\sum_{j=1}^{N}\frac{\mu_i x_j}{\mu_i + \mu_j})^{-1} = 0, \tag{1}$$

where $c \in (0,1)$ is a give constant, $\mu_i = (i - 1/2)/N$ for $1 \le i \le N$, and N is the dimension of the unknown vector $\vec{x}$.

Write your own code and compute the solution of the equation for $N = 200$ and $c = 0.9$ by using

(1) Fixed-point method,
(2) Chord method,
(3) Newton method,
(4) Shamanskii method with $m = 2$.

In all of your computations, the initial guess is taken as $\vec{x} = [1, 1, \cdots, 1]^T$, the stopping condition is that

$$\|\mathbf{F}(\vec{x})\| \le \tau_r r_0 + \tau_a,$$

where $\tau_r = \tau_a = 10^{-6}$.

Compare your computation results (by tabulating or plotting the iteration numbers, error reduction, and cost) and comment on your computations.

Remark: You may use MATLAB code *diffjac* to compute the Jacobian in your computations.