

CSCI 210: Computer Architecture

Lecture 7: Negative Numbers, Overflow

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Announcements

- Problem Set 2 due Friday
- Lab 1 due Sunday

Questions So Far?

How We Store Numbers

- Binary numbers in memory are stored using a finite, fixed number of bits (typically 8, 16, 32, or 64)
 - 8 bits = byte (usually and always in this class)
- Pad extra digits with leading 0s
- A byte representing $4_{10} = 00000100$

A byte (8 bits) can store nonnegative values from 0
up to

- A. 127
- B. 128
- C. 255
- D. 256
- E. None of the above

Java

- A `byte` is 8 bits
- A `char` is 16 bits
- A `short` is 16 bits
- An `int` is 32 bits
- A `long` is 64 bits

Rust

- `bool`s are 1 byte, `char`s are 4 bytes
- Specify size in type for integers
 - `i8`, `i16`, `i32`, etc.
- `isize` or `usize` will be the size of an address on the architecture it's compiled for
 - 32 bits on 32 bit systems, 64 bits on 64 bit systems

In C, an int is

- A. 8 bits
- B. 16 bits
- C. 32 bits

- D. It depends
- E. None of the above

C specifies a *minimum size* for types

- `char`s are 1 byte and must be at least 8 bits (but can be more!)
- `short`s and `int`s must be at least 16 bits
- `long`s are at least 32 bits
- `long long`s are at least 64 bits
- `sizeof(type)` tells us how many bytes type is
- $$\begin{aligned} 1 &= \text{sizeof}(\text{char}) \leq \text{sizeof}(\text{short}) \\ &\leq \text{sizeof}(\text{int}) \leq \text{sizeof}(\text{long}) \\ &\leq \text{sizeof}(\text{long long}) \end{aligned}$$

So how do I know?

- Use `sizeof(int)` to check
- Or use C99 types like `int16_t` or `int32_t`

How do we indicate a negative number?

- Sign and magnitude (History; and for floating point numbers)
- Ones' Compliment (History)
- Two's Compliment (Modern Systems)

Sign and Magnitude

- Have a separate bit for sign
- Set it to 0 for positive, and 1 for negative
- Can represent from -127 to 127 in 8 bits
- With n bits, can represent $-(2^{n-1} - 1)$ to $2^{n-1} - 1$

Addition and subtraction are a hassle

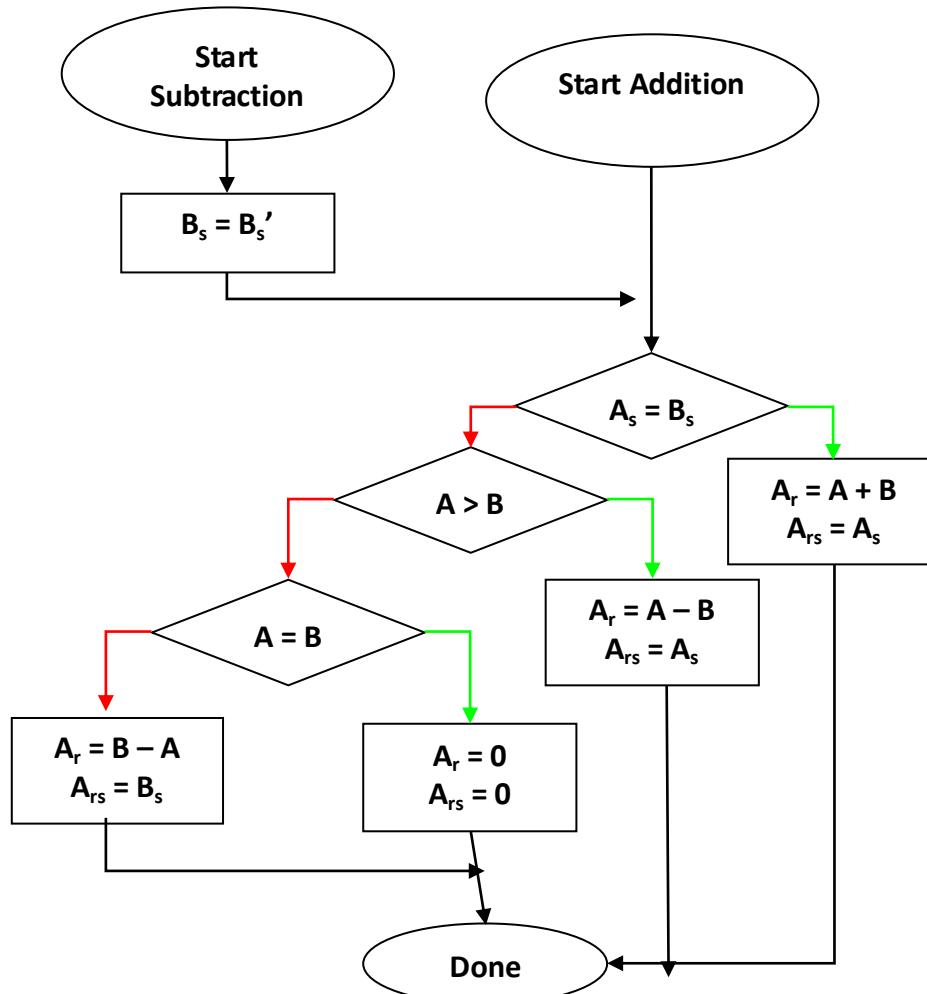


Diagram from Marek Andrzej Perkowski

A byte representing -6_{10} in Sign and Magnitude
(with leftmost sign bit) is

A. 0000 0111

D. 1111 1110

B. 1000 0110

E. None of the above

C. 1000 0111

Which is NOT a drawback of Sign and Magnitude?

- A. There are two zeros
- B. Unclear where to put the sign bit
- C. Complicated arithmetic
- D. Difficult to convert numbers to negative representation
- E. None of the above

Ones' Complement

- To make a number negative, just flip all its bits!
- Need to know how many bits: -5 in
 - 4 bits: $-0101 = 1010$
 - 8 bits: $-0000\ 0101 = 1111\ 1010$

A byte representing -6_{10} in Ones' Complement is

A. 0000 0110

B. 1000 0110

C. 1111 1001

D. 1111 0110

E. None of the above

Ones' complement

- Two zeros: 00000000 and 11111111 (in 8 bits)
- Addition:
 - Perform normal n-bit addition
 - Add the carryout bit back to the result

Ones' complement addition example

-3 + -16 in ones' complement example (in 8 bits)

$$\begin{array}{r} 1111 \ 1100 \\ + \ 1110 \ 1111 \\ \hline \end{array}$$

1 1110 1011

Add the carryout bit **1** back to the result

Ones' complement addition example

-3 + -16 in ones' complement example (in 8 bits)

$$\begin{array}{r}
 & 1111 & 1100 \\
 + & 1110 & 1111 \\
 \hline
 \end{array}$$

$$\begin{array}{r} 1 \quad 1110 \quad 1011 \\ + \qquad \qquad \qquad 1 \\ \hline \end{array}$$

$$1110 \quad 1100 = -19$$

Two's Complement

- To compute $-x$, flip all the bits of x and add 1
- For n bits, the unsigned version of $-x = 2^n - x$
- Can represent -128 to 127 in 8 bits
 - In n bits, can represent -2^{n-1} to $2^{n-1} - 1$
- Only one zero (00000000 in 8 bits)
- Used in modern computers

Short aside

- **ones' complement** involves taking each bit and taking the complement with respect to 1; there are many bits so many complements with respect to 1s hence “ones' complement”
- **two's complement** involves taking a complement with respect to a single power of 2, not bit-by-bit, hence “two's complement” (as unsigned n -bit binary numbers $x + -x = 2^n$)
- Yes. It *is* confusing. No. No one remembers this. The book gets it wrong

-6 in Two's Complement

- A. 1111 0110
- B. 1111 1001
- C. 1111 1010
- D. 1111 1110
- E. None of the above

Two's Complement: 1111 1101₂ = ?₁₀

- A. -2
- B. -3
- C. -4
- D. -5
- E. None of the above

If we multiply $1111\ 0001_2$ by -1, we get _____ $_2$

A. 0000 1110

B. 0000 1111

C. 0001 1110

D. 0111 0001

E. None of the above

Addition and Subtraction

- Positive and negative numbers are handled in the same way.
- The carry out from the most significant bit is ignored.
- To perform the subtraction $A - B$, compute $A + (\text{two's complement of } B)$

For n bits, the sum of a number and its negation will
be

- A. $0_{n-1} \dots 0_0$
- B. $1_{n-1} 0_{n-2} \dots 0_0$
- C. $1_{n-1} \dots 1_0$
- D. It will vary
- E. None of the above

$$\begin{array}{r} 1111 \ 0110 \\ + 0000 \ 1100 \\ \hline \end{array}$$

- A. 0000 0010
- B. 0000 1100
- C. 1111 0010
- D. 1111 1110
- E. None of the above

$$\begin{array}{r} 1001 \\ + 1011 \\ \hline \end{array}$$

- A. 0010
- B. 0100
- C. 1000
- D. 1111
- E. None of the above

Overflow

- Overflow occurs when arithmetic results in a value which cannot be represented using the number of bits available
- In that case, the algorithms we have been using produce incorrect results!

What will this Java code print?

- A. -2147483648
- B. 0
- C. 2147483647
- D. 2147483648

```
public static void main(String args[]) {  
    int x = 2147483647;  
    x = x + 1;  
    System.out.println(x);  
}
```

Handling Overflow

- Hardware can detect when overflow occurs
- Software may or may not check for overflow
 - Java guarantees two's complement behavior!
 - In C, signed overflow is “undefined behavior” meaning, it can do anything
 - In Rust, overflow is checked in debug builds (and causes a panic if it occurs) but not optimized builds!

How To Detect Overflow

- On an addition, an overflow occurs if and only if the carry into the sign bit differs from the carry out from the sign bit.
- For example, overflow occurs if
 - adding two negative numbers produces a positive result
 - adding two positive numbers produces a negative result

Overflow (or not) examples (in 8 bits)

- $$\begin{array}{r} \overset{1\ 1\ 1}{0111\ 0001} \\ +\ 0001\ 0100 \\ \hline 1000\ 0101 \end{array}$$
 Carry in = 1, carry out = 0
- $$\begin{array}{r} \overset{1\ 1\ 1\ 1\ 1}{1111\ 1111} \\ +\ 0000\ 0010 \\ \hline 0000\ 0001 \end{array}$$
 Carry in = 1, carry out = 1
- $$\begin{array}{r} \overset{1\ 1\ 1\ 1}{1111\ 0000} \\ +\ 1111\ 1101 \\ \hline 1110\ 1101 \end{array}$$
 Carry in = 1, carry out = 1

Overflow (or not) examples (in 8 bits)

- $$\begin{array}{r} \textcolor{red}{1} & 1111 & 0001 \\ + & 1000 & 0100 \\ \hline 0111 & 0101 \end{array}$$
 Carry in = 0, carry out = 1

Will $0111\ 1111_2 + 0000\ 0101_2$ result in overflow when treated as 8-bit signed integers?

- A. Yes
- B. No
- C. It depends

Overflow with other arithmetic operations

- Addition: add two large positive numbers together
- Subtraction: Subtract a large negative number from a large positive number
- Multiplication: Multiply two mid-sized numbers
- Division: ???

What is $1000\ 0000_2 / -1$ in 8 bits? Does overflow occur?

- A. 0000 0000, no overflow
- B. 0111 1111, no overflow
- C. 1000 0000, no overflow
- D. 0111 1111, overflow
- E. 1000 0000, overflow

Unsigned Numbers

- Some types of numbers, such as memory addresses, will never be negative
- Some programming languages reflect this with types such as “`unsigned int`”, which only hold positive numbers
 - `uint64_t` in C99
 - `u64`, `usize` in Rust
 - Java only has signed types (except for `char` which is unsigned 16-bit)
- In an unsigned byte, values will range from 0 to 255

In MIPS

- add, sub, addi instructions cause exceptions on (signed) overflow
- addu, subu, addiu instructions do not
- Rationale: In C, unsigned types never cause overflow, they're defined to wrap (produce a value modulo 2^n)
- In practice: Since overflow is undefined behavior, it is assumed to never happen so compilers always use addu/subu/addiu

Reading

- Next lecture: How Instructions Are Represented
 - Section 2.5
- Problem Set 2 due Friday
- Lab 1 due Sunday