

CS 301

Lecture 21 – Review

Stephen Checkoway

April 11, 2018



Exam topics

Broadly speaking: Everything through decidable languages (Sipser §4.1)

- CFGs, both the mathematical definition as a 4-tuple $G = (V, \Sigma, R, S)$ and as lists of rules
- Converting a CFG to CNF
- PDAs, both the mathematical definition $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$ and diagrams
- Closure properties of CFLs
- TMs, both the mathematical definition $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$ and diagrams
- Turing-recognizable (RE), co-Turing-recognizable (coRE), and decidable languages
- Closure properties of RE and decidable languages
- Decision problems from language theory (**red** are undecidable)
 - Acceptance problems: $A_{\text{DFA}}, A_{\text{NFA}}, A_{\text{REX}}, A_{\text{CFG}}, A_{\text{TM}}$
 - Emptiness problems: $E_{\text{DFA}}, E_{\text{CFG}}, E_{\text{TM}}$
 - Equivalence problems: $EQ_{\text{DFA}}, EQ_{\text{CFG}}, EQ_{\text{TM}}$

Types of exam questions

The questions from the exam fall into these types

- True/false questions with explanation
- Constructions
 - Construct a CFG/PDA for a context-free language
 - Convert a CFG to a PDA
 - Construct a TM diagram to recognize/decide a language
 - Give an implementation-level description of a TM that recognizes/decides a language
 - Give a high-level description of a TM that recognizes/decides a language
- Proofs
 - Prove that a language is/is not context-free using closure properties
 - Prove that a language is decidable
 - Prove that Turing-recognizable/decidable languages are closed under some operation

Exam question break down

- Five true/false questions (4 points each)
- Two constructions (20 points each)
- Two proofs (20 points each)

No pumping lemma for context-free languages questions for this exam (but possibly on the final)

Examples

- 1 Give a CFG that generates the language
 $A = \{w \mid w \in \{a, b\}^* \text{ contains at least 3 as}\}$
- 2 Give a PDA that recognizes the language
 $B = \{w \mid w \in \{a, b\}^* \text{ has odd length and the middle symbol is b}\}$
- 3 Give an implementation-level description of a TM that decides the language
 $C = \{w \mid w \in \{a, b\}^* \text{ does not contain twice as many as as bs}\}$
- 4 Prove that decidable languages are closed under union
- 5 Prove that Turing-recognizable languages are closed under union
- 6 Prove that
 $\text{INFINITE}_{\text{DFA}} = \{\langle M \rangle \mid M \text{ is a DFA and } L(M) \text{ is an infinite language}\}$ is decidable
- 7 Prove that $D = \{\langle G \rangle \mid G \text{ is a CFG over } \{0, 1\} \text{ and } \underline{1}^* \cap L(G) \neq \emptyset\}$ is decidable