

Homework 2

Due: Friday, September 15, 2023

Instructions

For each of the following problems, construct a DFA in **JFLAP** that recognizes the given language. Each DFA should be in its own file. **Each DFA must be a valid DFA! This means they must have one initial state, every state must have a transition for each symbol in the alphabet, and no states may have multiple transitions for the same alphabet symbol. JFLAP does not enforce this.** The name of each file should be the problem number with the `.jff` extension: `1.jff`, `2.jff`, and so on.

Upload your solutions to GradeScope by dragging all of the `.jff` files onto the website. **IMPORTANT: If you do not have the files named exactly as specified, the autograded portion will not work. If you only upload a single file, the autograder will not work.** (If you wish to upload just a single file, for example, `1.jff`, you'll also need to upload some other file such as a blank text file. This is a limitation of GradeScope.) Not all autograded tests will be visible prior to the deadline.

Problem 1 Construct a DFA that recognizes the language $A = \{w \mid w \text{ starts with } \mathbf{a} \text{ and has odd length or starts with } \mathbf{b} \text{ and has even length}\}$. The alphabet is $\Sigma = \{\mathbf{a}, \mathbf{b}\}$.

Problem 2 Construct a DFA that recognizes the language $B = \{w \mid w \text{ does not contain the substring } \mathbf{ab} \text{ and does not contain the substring } \mathbf{ba}\}$. The alphabet is $\Sigma = \{\mathbf{a}, \mathbf{b}\}$.

Problem 3 Construct a DFA that recognizes the language $C = \{w \mid w \text{ is a binary number divisible by } 5\}$. For example, `1001` (9 in base 10) is not divisible by 5 so it would be rejected, whereas `1010` (10 in base 10) is divisible by 5 so it would be accepted. Leading 0's (e.g., `001010`) are permitted and the empty string is taken as a representation for 0. The alphabet is $\Sigma = \{\mathbf{a}, \mathbf{b}\}$.

[Hint: Let the states represent the number read so far, modulo 5. Each new input symbol that's read causes the existing number to be multiplied by 2 and then the input symbol is added. For example, if the DFA has read the string `110` (decimal value 6), and the next input symbol is a `1`, then the string read so far at this point, `1101` has decimal value $6 \cdot 2 + 1 = 13$. Furthermore, note that $6 \bmod 5 = 1$, $13 \bmod 5 = 3$, and $((6 \bmod 5) \cdot 2 + 1) \bmod 5 = 3$. Use this observation to construct the transitions.]

Problem 4 Construct a DFA that recognizes the language $D = \{w \mid w \text{ contains exactly three } \mathbf{a} \text{ symbols and at least two } \mathbf{b} \text{ symbols}\}$. The alphabet is $\Sigma = \{\mathbf{a}, \mathbf{b}\}$.

[Hint: Construct a DFA for the language $D_1 = \{w \mid w \text{ contains exactly three } \mathbf{a} \text{ symbols}\}$ and construct a DFA for the language $D_2 = \{w \mid w \text{ contains at least two } \mathbf{b} \text{ symbols}\}$. Use the product construction described in the book to build a DFA that recognizes $D = D_1 \cap D_2$.]