

CSCI 210: Computer Architecture

Lecture 15: Digital Logic

Stephen Checkoway

Oberlin College

Mar. 25, 2022

Slides from Cynthia Taylor

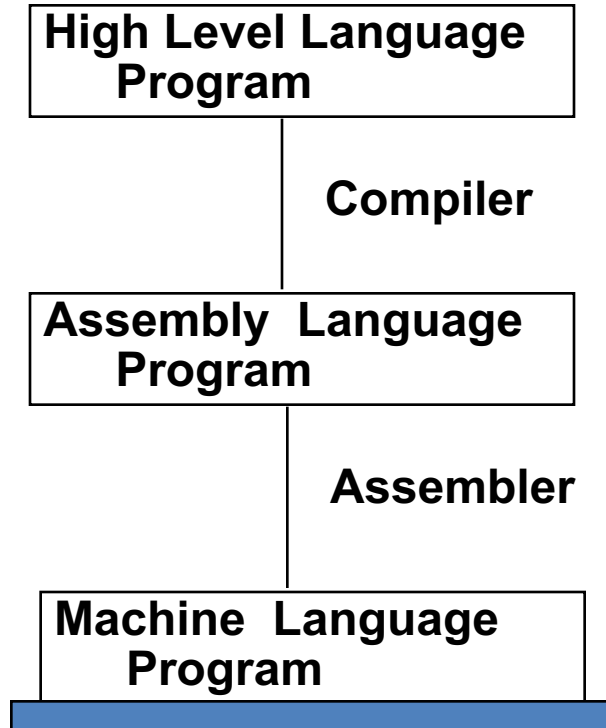
Announcements

- Problem Set 4 due today
- Lab 3 due Sunday!
- Office Hours today 13:30 – 14:30 pm

Creating the Universe from 1 and 0

- We have seen how to build programs from assembly
- Now we'll learn how we implement assembly language instructions using circuits

Machine Interpretation



Machine Interpretation

```
temp = v[k];  
v[k] = v[k+1];  
v[k+1] = temp;
```

```
lw $15, 0($2)  
lw $16, 4($2)  
sw $16, 0($2)  
sw $15, 4($2)
```

```
10001100011000100000000000000000  
10001100111100100000000000000100  
10101100111100100000000000000000  
10101100011000100000000000000100
```

Machine does something!

A digital circuit is comprised of signals, gates, and wires.

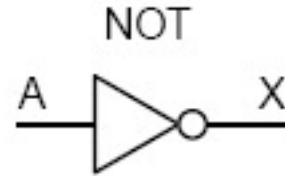
- Signals
 - Voltages applied to wires which generate electric current
- Binary signals are represented by different voltages:
 - 0: 0-1 volts
 - 1: 2-5 volts

A digital circuit is comprised of signals, gates, and wires.

- Gates
 - Devices which perform operations on signals corresponding to basic logic operations: and, or, not, nand, nor, xor
 - Made out of transistors
- Wires
 - Lines over which signals are transmitted between gates

Representation of Logic Gates

- Symbol



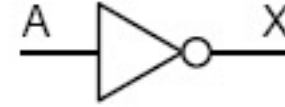
- Truth Table

A	X
0	1
1	0

- Algebraic Representation

\bar{A}

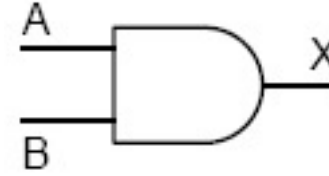
Not



A	X
0	1
1	0

- Inverts the input
- Algebraic representation: \bar{A}

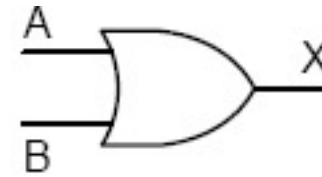
And



A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

- Algebraic representation: AB or $A \cdot B$

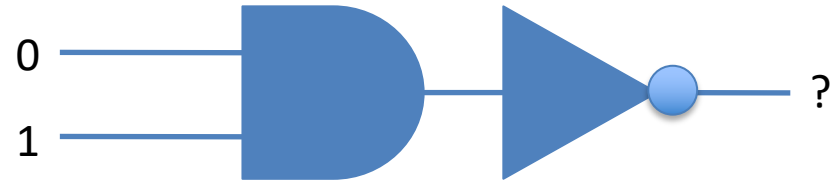
Or



A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

- Algebraic representation: $A+B$

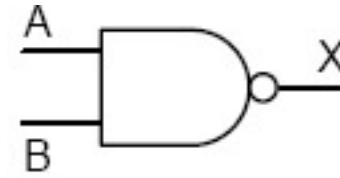
And and Not



A. 0

B. 1

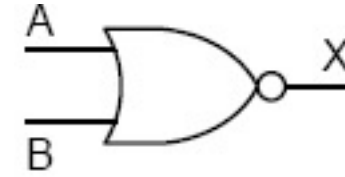
Nand



A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

- Algebraic representation: $\overline{(A \cdot B)}$

Nor



A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

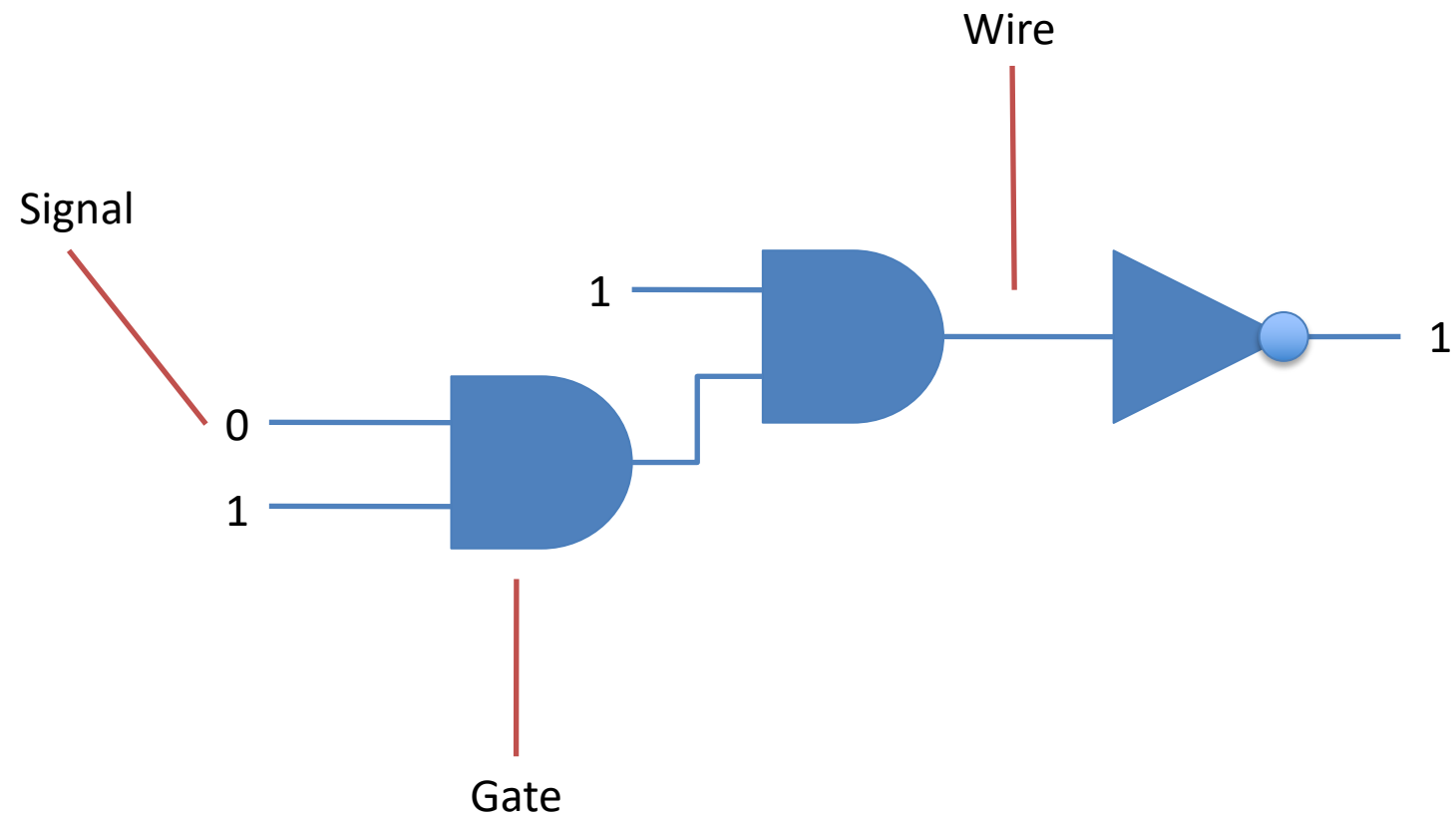
- Algebraic representation: $\overline{(A + B)}$

Xor

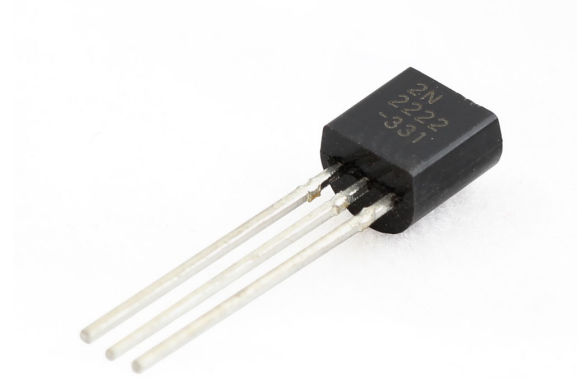
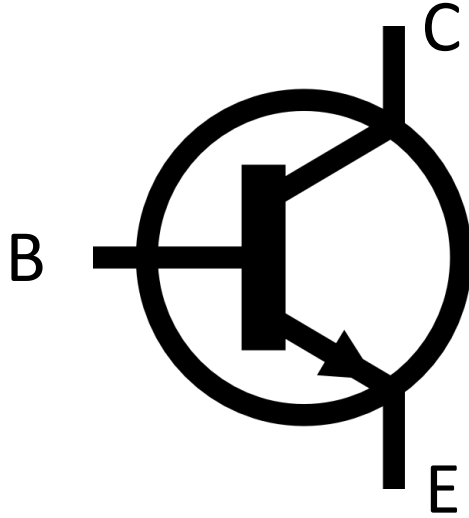


A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

- Algebraic representation: $A \wedge B$ or $A \oplus B$



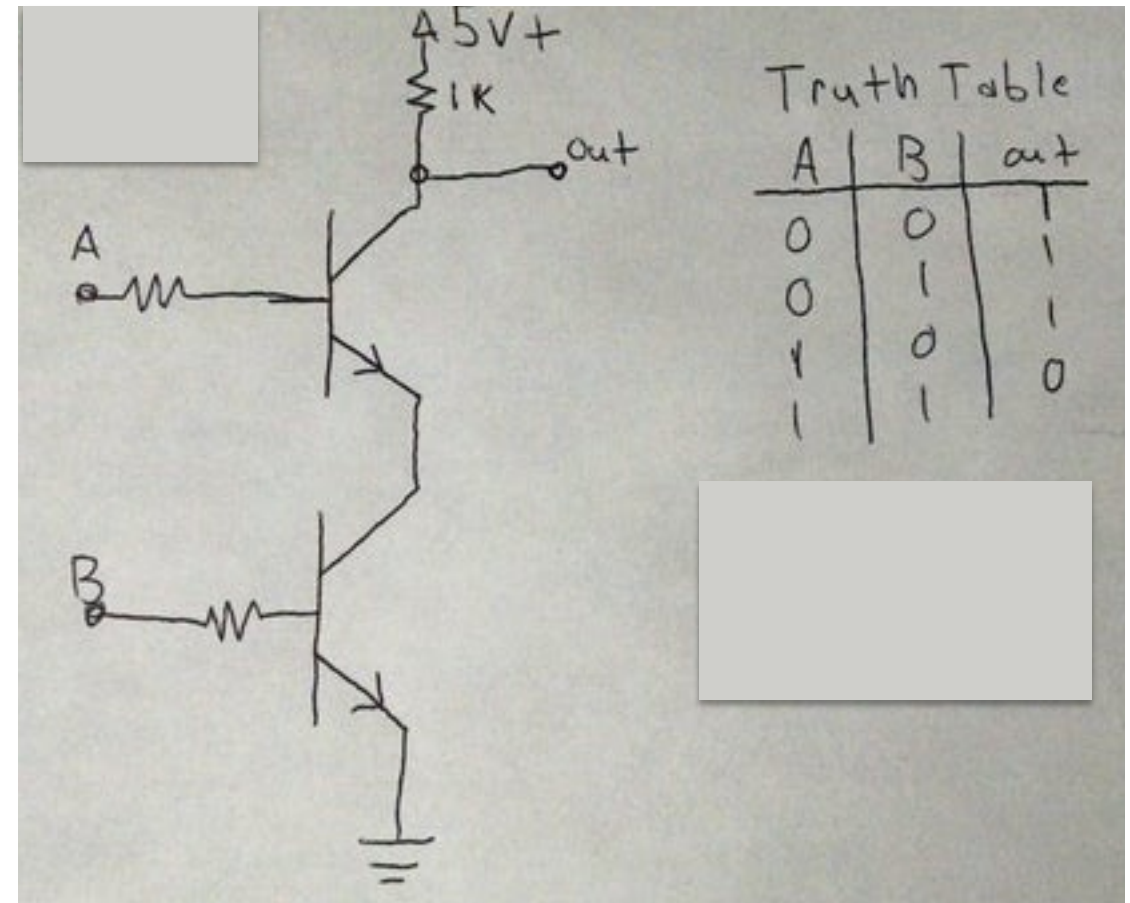
Our Friend the Transistor



- The basic electronic component from which all gates are created; there are many types, this is an NPN transistor
- When the base (B) has a high voltage, current can flow from the collector (C) to the emitter (E)
- This creates an on/off switch

Building gates out of switches

- Two inputs A and B
- One output out
- When A or B are 1, the other two electrodes (collector and emitter) are connected
- When A and B are both 1, out is connected to ground (logic value 0)
- When either A or B is 0, out is not connected to ground and current can flow from 5V to out

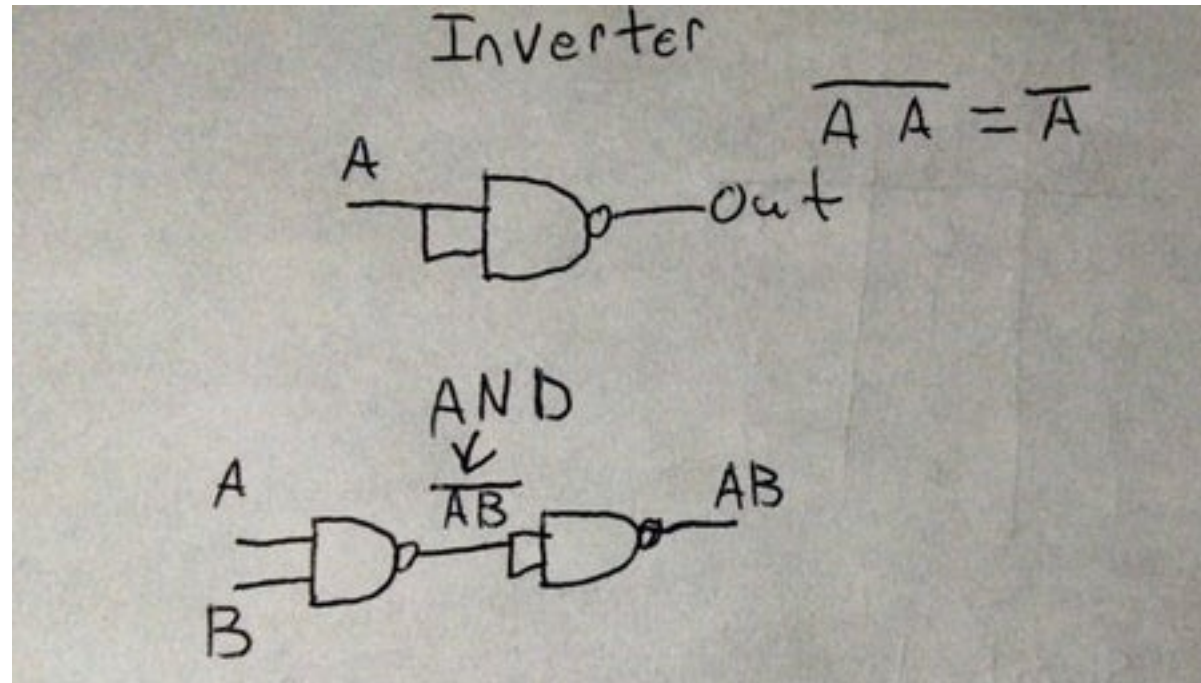


What Gate Does This Match?

- If A and B are high voltage (logical 1), out will be low voltage (logical 0)
- Otherwise, out is high voltage

- A. AND
- B. OR
- C. NAND
- D. NOR

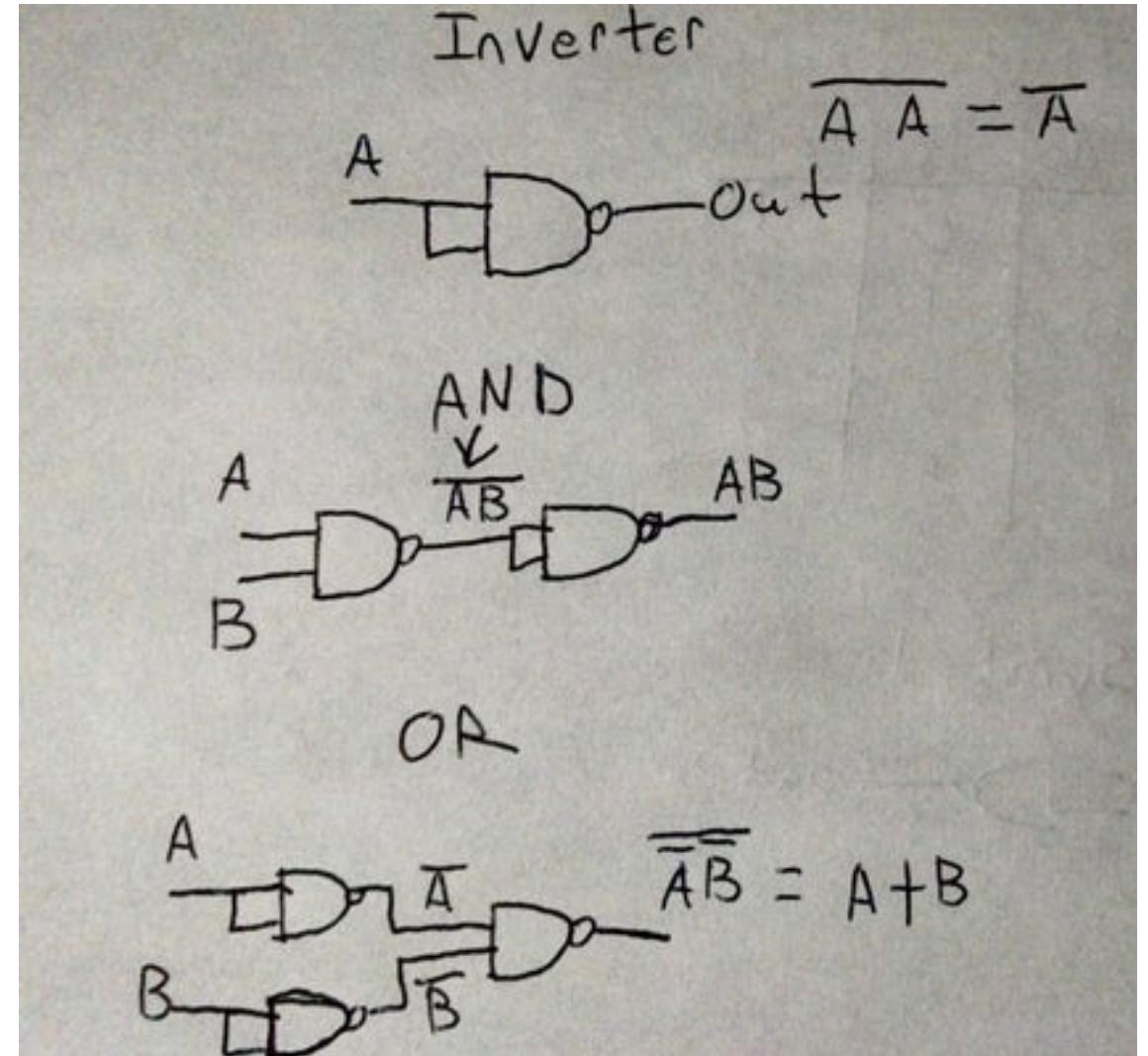
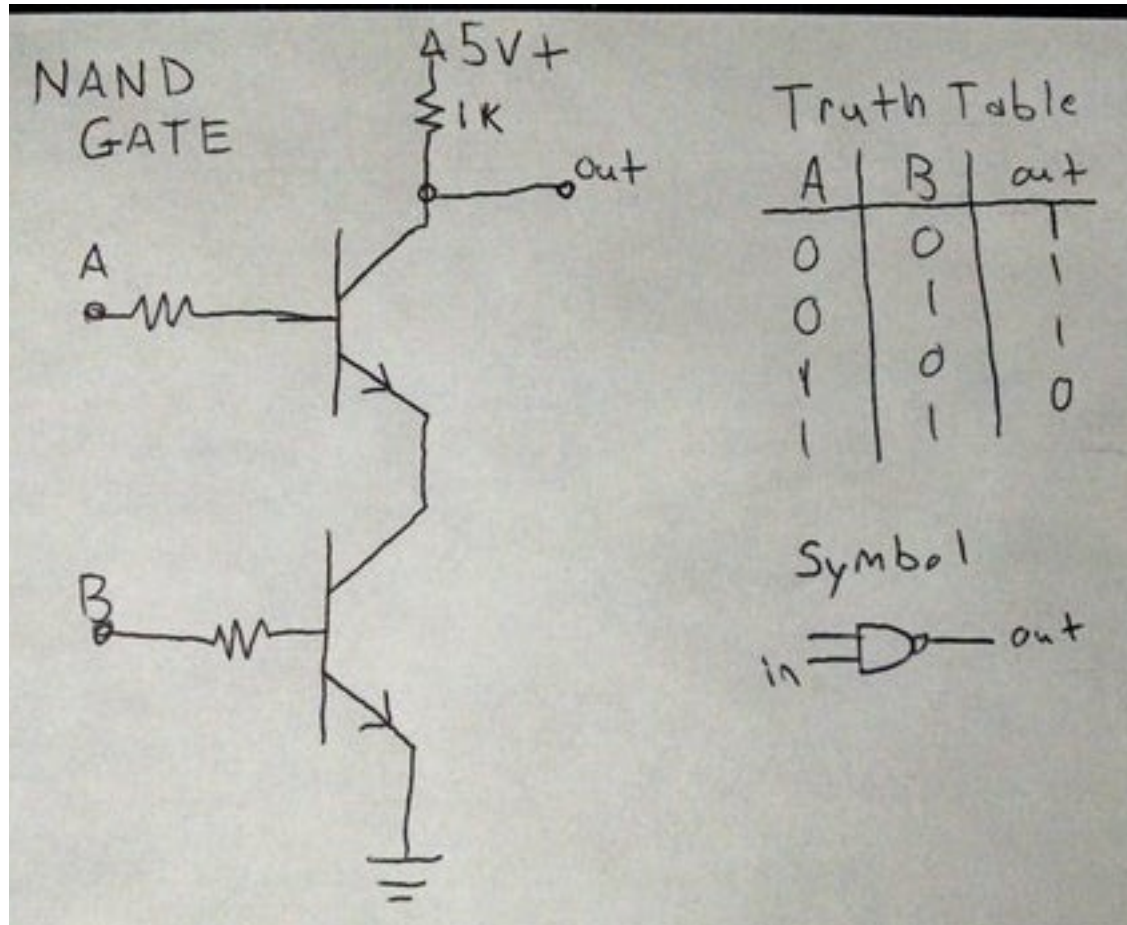
All Other Gates Can Be Created From NAND

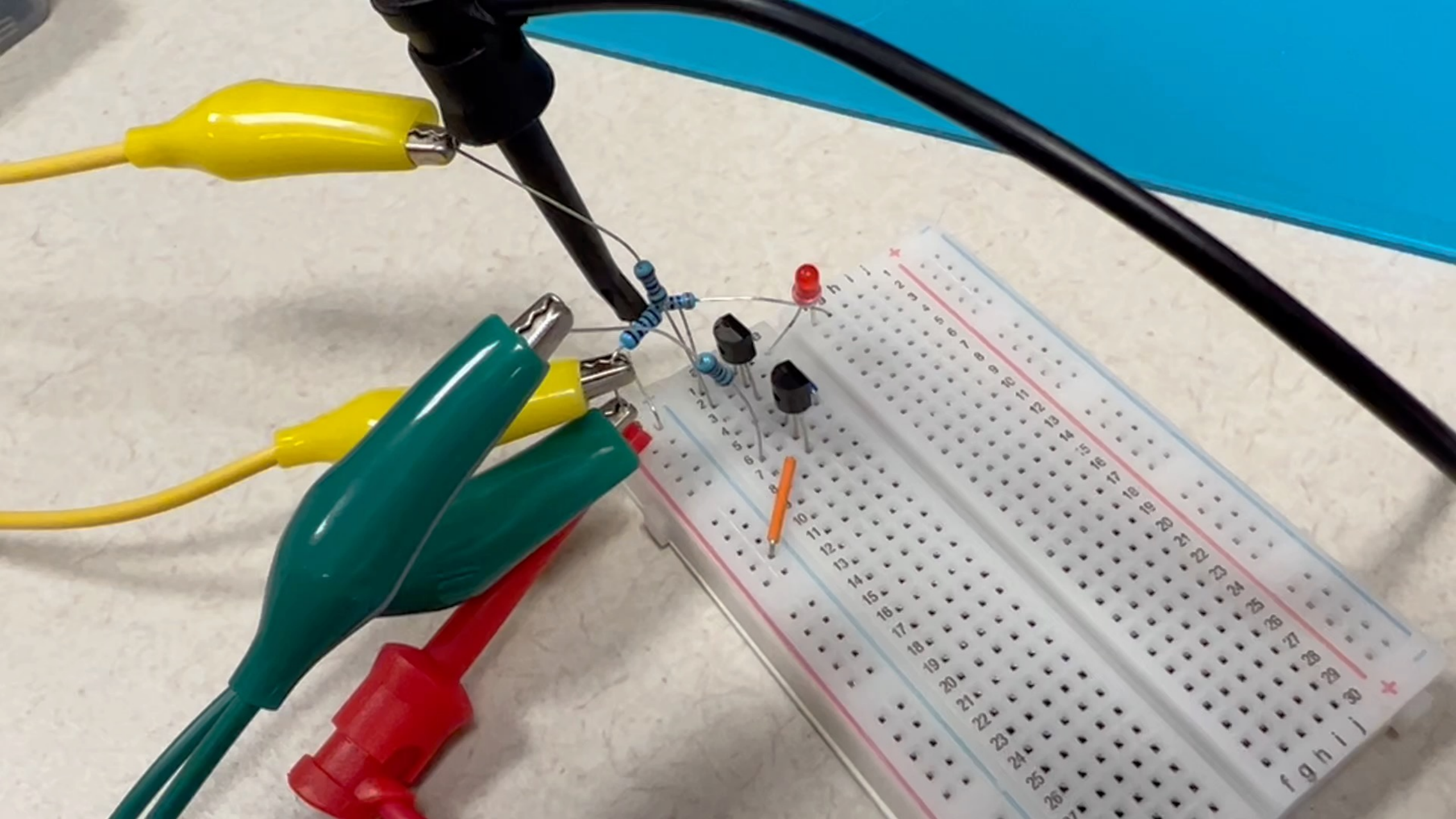


Which is equivalent to $A \text{ OR } B$?

- A. $A \text{ NAND } B$
- B. $\text{NOT } (A \text{ NAND } B)$
- C. $(\text{NOT } A) \text{ NAND } (\text{NOT } B)$
- D. $\text{NOT } ((\text{NOT } A) \text{ NAND } (\text{NOT } B))$
- E. None of the above

Putting them together





All Other Gates Can Be Created from NOR

- You will show this in Problem Set 5

Which column completes the truth table for

$$F = \overline{X} \cdot (Y + Z) ?$$

X	Y	Z	A	B	C	D
0	0	0	0	0	1	1
0	0	1	1	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	0	0	0	0
1	0	1	0	1	0	1
1	1	0	0	1	0	1
1	1	1	0	1	0	1

Diagram: $F = \overline{X} \cdot (Y + Z)$

$$F = \overline{A} + (B(AC + \overline{A}\overline{B}))$$

Truth Table

A	B	C	AC	$\overline{A}\overline{B}$	$B(AC + \overline{A}\overline{B})$	F
0	0	0				
0	0	1				
0	1	0				
0	1	1				
1	0	0				
1	0	1				
1	1	0				
1	1	1				

Reading

- Next lecture: Boolean Algebra
 - 3.3
- Problem Set 4 due today
- Lab 3 due Sunday