CS 241: Systems Programming
Lecture 13. Bits and Bytes 2

Spring 2020
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Internal data representation

Data are stored in binary

32 bit unsigned integer values are:

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```
00000000 00000000 00000000 00000000 = 0
```

Internal data representation

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```
00000000 00000000 00000000 00000000 = 0
00000000 00000000 00000000 00000001 = 1
```

Internal data representation

Data are stored in binary

32 bit unsigned integer values are:

```
00000000 00000000 00000000 00000000 = 0
00000000 00000000 00000000 00000001 = 1
00000000 00000000 00000000 00000010 = 2
```

Internal data representation

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32 bit unsigned integer values are:

```
00000000 00000000 00000000 00000000 = 0
00000000 00000000 00000000 00000001 = 1
00000000 00000000 00000000 00000010 = 2
00000000 00000000 00000000 00000011 = 3
```

Internal data representation

Data are stored in binary

32 bit unsigned integer values are:

```
00000000 00000000 00000000 00000000 = 0
00000000 00000000 00000000 00000001 = 1
00000000 00000000 00000000 00000010 = 2
00000000 00000000 00000000 00000011 = 3
```

…

11111111 11111111 11111111 11111111 = $2^{32}-1$

Bitwise operators

Binary operators apply the operation to the corresponding bits of the operands

- `x & y` — bitwise AND
- `x | y` — bitwise OR
- `x ^ y` — bitwise XOR

Unary operator applies the operation to each bit

- `~x` — one's complement (flip each bit)

Boolean logic

`A`	`B`	`~A`	`A&B`	`A|B`	`A^B`
0	0	1	0	0	0
0	1	1	0	1	1
1	0	0	0	1	1
1	1	0	1	1	0

What is the value of `0x4E & 0x1F`?

A. 0xE

B. 0x51

C. 0x5F

D. 0xB1

E. 0xE0

Hex	Binary	Hex	Binary
0	0000	8	1000
1	0001	9	1001
2	0010	A	1010
3	0011	B	1011
4	0100	C	1100
5	0101	D	1101
6	0110	E	1110
7	0111	F	1111

Bit shifting

Bit shifting

Manipulates the position of bits

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- Left shift fills with 0 bits

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```
x << 2; // shifts bits of x two positions left
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- Same as multiplying by 4

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x << 2;  // shifts bits of x two positions left
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- Same as multiplying by 4

```
x >> 3;  // shifts bits of x three positions right
```

Bit shifting

Manipulates the position of bits

- Left shift fills with 0 bits
- Right shift of unsigned variable fills with 0 bits
- Right shift of signed variable fills with sign bit (Actually implementation defined if negative!)

```
x << 2; // shifts bits of x two positions left
```

- Same as multiplying by 4

```
x >> 3; // shifts bits of x three positions right
```

- Same as dividing by 8 (if x is unsigned)

What does the following do?

```
x = ((x >> 2) << 2);
```

A. Changes x to be positive

B. Sets the least significant two bits to 0

C. Sets the most significant two bits to 0

D. Gives an integer overflow error

E. Implementation-defined behavior

Testing if a bit is set (i.e., is 1)

```c
#include <stdbool.h>

// Returns true if the nth bit of x is 1.
bool is_bit_set(unsigned int x, unsigned int n) {
  return x & (1u << n); // 1u is an unsigned int with value 1.
}
```

Testing if a bit is set (i.e., is 1)

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}
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`1u << n` gives an integer with only the nth bit set

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  return x & (1u << n); // 1u is an unsigned int with value 1.
}
```

`1u << n` gives an integer with only the nth bit set

If the *n*th bit is 1, then `x & (1u << n)` is `1u << n` which is nonzero.
If the *n*th bit is 0, then `x & (1u << n)` is 0

Testing if a bit is set (i.e., is 1)

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#include <stdbool.h>

// Returns true if the nth bit of x is 1.
bool is_bit_set(unsigned int x, unsigned int n) {
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If the *n*th bit is 1, then `x & (1u << n)` is `1u << n` which is nonzero.
If the *n*th bit is 0, then `x & (1u << n)` is 0

What happens if n is too large?

Testing if a bit is set (i.e., is 1)

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bool is_bit_set(unsigned int x, unsigned int n) {
  return x & (1u << n); // 1u is an unsigned int with value 1.
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`1u << n` gives an integer with only the nth bit set

If the *n*th bit is 1, then `x & (1u << n)` is `1u << n` which is nonzero.
If the *n*th bit is 0, then `x & (1u << n)` is 0

What happens if n is too large?

- Undefined behavior!

UB

```c
#include <err.h>
#include <stdbool.h>
#include <stdio.h>
#include <stdlib.h>

// Returns true if the nth bit of x is 1.
bool is_bit_set(unsigned int x, unsigned int n) {
  return x & (1u << n);
}

int main(int argc, char **argv) {
  if (argc != 3)
    errx(1, "Usage: %s integer bit", argv[0]);
  unsigned int x = atoi(argv[1]);
  unsigned int n = atoi(argv[2]);
  if (is_bit_set(x, n))
    printf("Bit %u of %u is 1\n", n, x);
  else
    printf("Bit %u of %u is 0\n", n, x);
  return 0;
}
```

UB

```
$ ./bad_shift 3 0
```

```c
#include <err.h>
#include <stdbool.h>
#include <stdio.h>
#include <stdlib.h>

// Returns true if the nth bit of x is 1.
bool is_bit_set(unsigned int x, unsigned int n) {
  return x & (1u << n);
}

int main(int argc, char **argv) {
  if (argc != 3)
    errx(1, "Usage: %s integer bit", argv[0]);
  unsigned int x = atoi(argv[1]);
  unsigned int n = atoi(argv[2]);
  if (is_bit_set(x, n))
    printf("Bit %u of %u is 1\n", n, x);
  else
    printf("Bit %u of %u is 0\n", n, x);
  return 0;
}
```

UB

```
$ ./bad_shift 3 0
Bit 0 of 3 is 1
```

```c
#include <err.h>
#include <stdbool.h>
#include <stdio.h>
#include <stdlib.h>

// Returns true if the nth bit of x is 1.
bool is_bit_set(unsigned int x, unsigned int n) {
  return x & (1u << n);
}

int main(int argc, char **argv) {
  if (argc != 3)
    errx(1, "Usage: %s integer bit", argv[0]);
  unsigned int x = atoi(argv[1]);
  unsigned int n = atoi(argv[2]);
  if (is_bit_set(x, n))
    printf("Bit %u of %u is 1\n", n, x);
  else
    printf("Bit %u of %u is 0\n", n, x);
  return 0;
}
```

UB

```
$ ./bad_shift 3 0
Bit 0 of 3 is 1
$ ./bad_shift 3 1
```

```c
#include <err.h>
#include <stdbool.h>
#include <stdio.h>
#include <stdlib.h>

// Returns true if the nth bit of x is 1.
bool is_bit_set(unsigned int x, unsigned int n) {
  return x & (1u << n);
}

int main(int argc, char **argv) {
  if (argc != 3)
    errx(1, "Usage: %s integer bit", argv[0]);
  unsigned int x = atoi(argv[1]);
  unsigned int n = atoi(argv[2]);
  if (is_bit_set(x, n))
    printf("Bit %u of %u is 1\n", n, x);
  else
    printf("Bit %u of %u is 0\n", n, x);
  return 0;
}
```

UB

```
$ ./bad_shift 3 0
Bit 0 of 3 is 1
$ ./bad_shift 3 1
Bit 1 of 3 is 1
```

```c
#include <err.h>
#include <stdbool.h>
#include <stdio.h>
#include <stdlib.h>

// Returns true if the nth bit of x is 1.
bool is_bit_set(unsigned int x, unsigned int n) {
  return x & (1u << n);
}

int main(int argc, char **argv) {
  if (argc != 3)
    errx(1, "Usage: %s integer bit", argv[0]);
  unsigned int x = atoi(argv[1]);
  unsigned int n = atoi(argv[2]);
  if (is_bit_set(x, n))
    printf("Bit %u of %u is 1\n", n, x);
  else
    printf("Bit %u of %u is 0\n", n, x);
  return 0;
}
```

UB

```
$ ./bad_shift 3 0
Bit 0 of 3 is 1
$ ./bad_shift 3 1
Bit 1 of 3 is 1
$ ./bad_shift 3 2
```

```c
#include <err.h>
#include <stdbool.h>
#include <stdio.h>
#include <stdlib.h>

// Returns true if the nth bit of x is 1.
bool is_bit_set(unsigned int x, unsigned int n) {
  return x & (1u << n);
}

int main(int argc, char **argv) {
  if (argc != 3)
    errx(1, "Usage: %s integer bit", argv[0]);
  unsigned int x = atoi(argv[1]);
  unsigned int n = atoi(argv[2]);
  if (is_bit_set(x, n))
    printf("Bit %u of %u is 1\n", n, x);
  else
    printf("Bit %u of %u is 0\n", n, x);
  return 0;
}
```

UB

```
$ ./bad_shift 3 0
Bit 0 of 3 is 1
$ ./bad_shift 3 1
Bit 1 of 3 is 1
$ ./bad_shift 3 2
Bit 2 of 3 is 0
```

```c
#include <err.h>
#include <stdbool.h>
#include <stdio.h>
#include <stdlib.h>

// Returns true if the nth bit of x is 1.
bool is_bit_set(unsigned int x, unsigned int n) {
  return x & (1u << n);
}

int main(int argc, char **argv) {
  if (argc != 3)
    errx(1, "Usage: %s integer bit", argv[0]);
  unsigned int x = atoi(argv[1]);
  unsigned int n = atoi(argv[2]);
  if (is_bit_set(x, n))
    printf("Bit %u of %u is 1\n", n, x);
  else
    printf("Bit %u of %u is 0\n", n, x);
  return 0;
}
```

UB

```
$ ./bad_shift 3 0
Bit 0 of 3 is 1
$ ./bad_shift 3 1
Bit 1 of 3 is 1
$ ./bad_shift 3 2
Bit 2 of 3 is 0
$ ./bad_shift 3 32
```

```c
#include <err.h>
#include <stdbool.h>
#include <stdio.h>
#include <stdlib.h>

// Returns true if the nth bit of x is 1.
bool is_bit_set(unsigned int x, unsigned int n) {
  return x & (1u << n);
}

int main(int argc, char **argv) {
  if (argc != 3)
    errx(1, "Usage: %s integer bit", argv[0]);
  unsigned int x = atoi(argv[1]);
  unsigned int n = atoi(argv[2]);
  if (is_bit_set(x, n))
    printf("Bit %u of %u is 1\n", n, x);
  else
    printf("Bit %u of %u is 0\n", n, x);
  return 0;
}
```

UB

```
$ ./bad_shift 3 0
Bit 0 of 3 is 1
$ ./bad_shift 3 1
Bit 1 of 3 is 1
$ ./bad_shift 3 2
Bit 2 of 3 is 0
$ ./bad_shift 3 32
Bit 32 of 3 is 1
```

```c
#include <err.h>
#include <stdbool.h>
#include <stdio.h>
#include <stdlib.h>

// Returns true if the nth bit of x is 1.
bool is_bit_set(unsigned int x, unsigned int n) {
  return x & (1u << n);
}

int main(int argc, char **argv) {
  if (argc != 3)
    errx(1, "Usage: %s integer bit", argv[0]);
  unsigned int x = atoi(argv[1]);
  unsigned int n = atoi(argv[2]);
  if (is_bit_set(x, n))
    printf("Bit %u of %u is 1\n", n, x);
  else
    printf("Bit %u of %u is 0\n", n, x);
  return 0;
}
```

UB

```
$ ./bad_shift 3 0
Bit 0 of 3 is 1
$ ./bad_shift 3 1
Bit 1 of 3 is 1
$ ./bad_shift 3 2
Bit 2 of 3 is 0
$ ./bad_shift 3 32
Bit 32 of 3 is 1
$ ./bad_shift 3 33
```

```c
#include <err.h>
#include <stdbool.h>
#include <stdio.h>
#include <stdlib.h>

// Returns true if the nth bit of x is 1.
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  unsigned int x = atoi(argv[1]);
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    printf("Bit %u of %u is 1\n", n, x);
  else
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}
```

UB

```
$ ./bad_shift 3 0
Bit 0 of 3 is 1
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Bit 1 of 3 is 1
$ ./bad_shift 3 2
Bit 2 of 3 is 0
$ ./bad_shift 3 32
Bit 32 of 3 is 1
$ ./bad_shift 3 33
Bit 33 of 3 is 1
```

```c
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    printf("Bit %u of %u is 0\n", n, x);
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```

UB

```
$ ./bad_shift 3 0
Bit 0 of 3 is 1
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Bit 1 of 3 is 1
$ ./bad_shift 3 2
Bit 2 of 3 is 0
$ ./bad_shift 3 32
Bit 32 of 3 is 1
$ ./bad_shift 3 33
Bit 33 of 3 is 1
$ ./bad_shift 3 34
```

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  return 0;
}
```

UB

```
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Bit 0 of 3 is 1
$ ./bad_shift 3 1
Bit 1 of 3 is 1
$ ./bad_shift 3 2
Bit 2 of 3 is 0
$ ./bad_shift 3 32
Bit 32 of 3 is 1
$ ./bad_shift 3 33
Bit 33 of 3 is 1
$ ./bad_shift 3 34
Bit 34 of 3 is 0
```

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    printf("Bit %u of %u is 1\n", n, x);
  else
    printf("Bit %u of %u is 0\n", n, x);
  return 0;
}
```

Testing if a bit is set (i.e., is 1)

```
#include <assert.h>
#include <limits.h>
#include <stdbool.h>

// Returns true if the nth bit of x is 1.
bool is_bit_set(unsigned int x, unsigned int n) {
  // assert(cond) will abort at runtime if cond is false.
  assert(n < CHAR_BIT * sizeof x);
  return x & (1u << n); // 1u is an unsigned int with value 1.
}
```

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  assert(n < CHAR_BIT * sizeof x);
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}
```

E.g., if **CHAR_BIT** is 8 and **sizeof** x is 4, then `n` must be less than 32 or the program aborts

Setting a bit (to 1)

```c
// Returns the value of x with the nth bit set to 1.
unsigned int set_bit(unsigned int x, unsigned int n) {
  assert(n < CHAR_BIT * sizeof x);
  return x | (1u << n);
}
```

Clearing a bit (setting it to 0)

```c
// Returns the value of x with the nth bit set to 0.
unsigned int set_bit(unsigned int x, unsigned int n) {
  assert(n < CHAR_BIT * sizeof x);
  return x & ~(1u << n);
}
```

Clearing a bit (setting it to 0)

```
// Returns the value of x with the nth bit set to 0.
unsigned int set_bit(unsigned int x, unsigned int n) {
  assert(n < CHAR_BIT * sizeof x);
  return x & ~(1u << n);
}
```

`1u << n` gives an integer with just the nth bit set

Clearing a bit (setting it to 0)

```
// Returns the value of x with the nth bit set to 0.
unsigned int set_bit(unsigned int x, unsigned int n) {
  assert(n < CHAR_BIT * sizeof x);
  return x & ~(1u << n);
}
```

`1u << n` gives an integer with just the nth bit set

`~(1u << n)` gives an integer with all bits set *except* the nth bit

Given an unsigned integer `x` with some value, what value should we use for `mask` to clear all of the bits of `x` except for the least significant 5 bits?

```
unsigned int x = /* … */;       // Given some value here,
unsigned int mask = /* … */;    // what value goes here
x = x & mask;                   // to clear the required bits?
```

A. `0x5u`

B. `~0x5u`

C. `0x1Fu`

D. `~0x1Fu`

E. `sizeof x - 5`

Given an unsigned integer `x` with some value, what value should we use for `mask` to clear the 5 least significant bits of `x`?

```
unsigned int x = /* … */;         // Given some value here,
unsigned int mask = /* … */;   // what value goes here
x = x & mask;                     // to clear the required bits?
```

A. `0x5u`

B. `~0x5u`

C. `0x1Fu`

D. `~0x1Fu`

E. `sizeof x - 5`

Combining flags via |

Specify flags via individual bits

Combine flags with |

E.g., set file system permissions via the flags `S_I{R,W,X}{USR,GRP,OTH}`

```
#define S_IRWXU 0000700    /* RWX mask for owner */
#define S_IRUSR 0000400    /* R for owner */
#define S_IWUSR 0000200    /* W for owner */
#define S_IXUSR 0000100    /* X for owner */

#define S_IRWXG 0000070    /* RWX mask for group */
#define S_IRGRP 0000040    /* R for group */
#define S_IWGRP 0000020    /* W for group */
#define S_IXGRP 0000010    /* X for group */

#define S_IRWXO 0000007    /* RWX mask for other */
#define S_IROTH 0000004    /* R for other */
#define S_IWOTH 0000002    /* W for other */
#define S_IXOTH 0000001    /* X for other */

int chmod(char const *path, mode_t mode);
```

Negative numbers

Negative numbers

Usually stored using two's complement

- Take the magnitude of the number
- Invert all of the bits
- Add 1

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representation of -5 in 8 bits

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```
magnitude:   0000_0101
```

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- Take the magnitude of the number
- Invert all of the bits
- Add 1

representation of -5 in 8 bits

```
magnitude:   0000_0101
invert bits: 1111_1010
```

Negative numbers

Usually stored using two's complement

- Take the magnitude of the number
- Invert all of the bits
- Add 1

representation of -5 in 8 bits

```
magnitude:   0000_0101
invert bits: 1111_1010
Add 1:       1111_1011
```

Negative numbers

Usually stored using two's complement
- Take the magnitude of the number
- Invert all of the bits
- Add 1

Computing -x from x (regardless of sign)
- Invert all of the bits
- Add 1

representation of -5 in 8 bits

```
magnitude:   0000_0101
invert bits: 1111_1010
Add 1:       1111_1011
```

Negative numbers

Usually stored using two's complement

- Take the magnitude of the number
- Invert all of the bits
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- Invert all of the bits
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representation of -5 in 8 bits

```
magnitude:   0000_0101
invert bits: 1111_1010
Add 1:       1111_1011
```

-(-5) in 8 bits

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- Invert all of the bits
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representation of -5 in 8 bits

```
magnitude:   0000_0101
invert bits: 1111_1010
Add 1:       1111_1011
```

-(-5) in 8 bits

```
-5:          1111_1011
```

Negative numbers

Usually stored using two's complement

- Take the magnitude of the number
- Invert all of the bits
- Add 1

Computing -x from x (regardless of sign)

- Invert all of the bits
- Add 1

representation of -5 in 8 bits

```
magnitude:   0000_0101
invert bits: 1111_1010
Add 1:       1111_1011
```

-(-5) in 8 bits

```
-5:          1111_1011
invert bits: 0000_0100
```

Negative numbers

Usually stored using two's complement

- Take the magnitude of the number
- Invert all of the bits
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representation of -5 in 8 bits

```
magnitude:   0000_0101
invert bits: 1111_1010
Add 1:       1111_1011
```

-(-5) in 8 bits

```
-5:          1111_1011
invert bits: 0000_0100
Add 1:       0000_0101
```

Negative numbers

Usually stored using two's complement

- Take the magnitude of the number
- Invert all of the bits
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Computing -x from x (regardless of sign)

- Invert all of the bits
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representation of -5 in 8 bits

```
magnitude:   0000_0101
invert bits: 1111_1010
Add 1:       1111_1011
```

-(-5) in 8 bits

```
-5:          1111_1011
invert bits: 0000_0100
Add 1:       0000_0101

0:           0000_0000
```

Negative numbers

Usually stored using two's complement

- Take the magnitude of the number
- Invert all of the bits
- Add 1

Computing -x from x (regardless of sign)

- Invert all of the bits
- Add 1

representation of -5 in 8 bits

```
magnitude:    0000_0101
invert bits:  1111_1010
Add 1:        1111_1011
```

-(-5) in 8 bits

```
-5:           1111_1011
invert bits:  0000_0100
Add 1:        0000_0101

0:            0000_0000
invert bits:  1111_1111
```

Negative numbers

Usually stored using two's complement

- Take the magnitude of the number
- Invert all of the bits
- Add 1

Computing -x from x (regardless of sign)

- Invert all of the bits
- Add 1

representation of -5 in 8 bits

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magnitude:    0000_0101
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-(-5) in 8 bits

```
-5:           1111_1011
invert bits:  0000_0100
Add 1:        0000_0101

0:            0000_0000
invert bits:  1111_1111
Add 1:      1_0000_0000
```

Negative numbers

Usually stored using two's complement

- Take the magnitude of the number
- Invert all of the bits
- Add 1

Computing -x from x (regardless of sign)

- Invert all of the bits
- Add 1

Most significant bit indicates the sign

- 1 indicates a negative number

representation of -5 in 8 bits

```
magnitude:    0000_0101
invert bits:  1111_1010
Add 1:        1111_1011
```

-(-5) in 8 bits

```
-5:           1111_1011
invert bits:  0000_0100
Add 1:        0000_0101
```

```
0:            0000_0000
invert bits:  1111_1111
Add 1:      1_0000_0000
```

Signed numbers in two's complement

```
10000000 00000000 00000000 00000000 = -2^31
10000000 00000000 00000000 00000001 = -2^31+1
…
11111111 11111111 11111111 11111110 = -2
11111111 11111111 11111111 11111111 = -1
00000000 00000000 00000000 00000000 = 0
00000000 00000000 00000000 00000001 = 1
00000000 00000000 00000000 00000010 = 2
00000000 00000000 00000000 00000011 = 3
…
01111111 11111111 11111111 11111110 = 2^31-2
01111111 11111111 11111111 11111111 = 2^31-1
```

Not the only choice

Not the only choice

Sign and magnitude

- Most significant bit represents the sign, remaining bits are the magnitude
- Range $-(2^{n-1} - 1)$ to $2^{n-1} - 1$
- Two different bit patterns for zero: 0 and 0x80000000 (assuming 32-bits)

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Ones' complement

- Negative numbers are the bitwise inverse of positive numbers (-x = ~x)
- Range -(2^{n-1} - 1) to 2^{n-1} - 1
- Two different bit patterns for zero: 0 and 0xFFFFFFFF (assuming 32-bits)

Not the only choice

Sign and magnitude

- Most significant bit represents the sign, remaining bits are the magnitude
- Range $-(2^{n-1} - 1)$ to $2^{n-1} - 1$
- Two different bit patterns for zero: 0 and 0x80000000 (assuming 32-bits)

Ones' complement

- Negative numbers are the bitwise inverse of positive numbers (-x = ~x)
- Range $-(2^{n-1} - 1)$ to $2^{n-1} - 1$
- Two different bit patterns for zero: 0 and 0xFFFFFFFF (assuming 32-bits)

Two's complement

- Negative numbers are ones' complement plus one (-x = ~x + 1)
- Range -2^{n-1} to $2^{n-1} - 1$
- Only one zero

In-class exercise

https://checkoway.net/teaching/cs241/2020-spring/exercises/Lecture-13.html

Grab a laptop and a partner and try to get as much of that done as you can!