CS 301
Lecture 26 – Conclusion

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- Decidability helps you think about what problems you cannot solve with computers
- Complexity helps you think about what problems you can solve or verify quickly

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- More computability!
- More complexity! We've only scratched the surface

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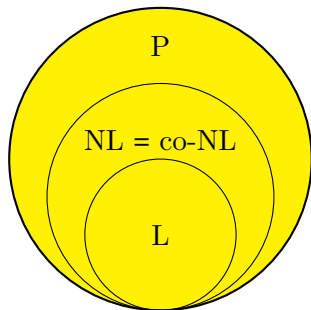- What if instead of TMs, you have circuits?
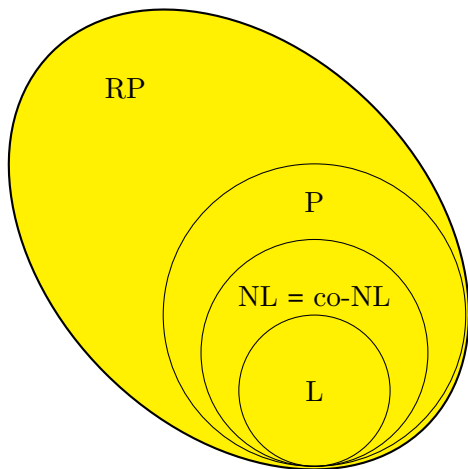
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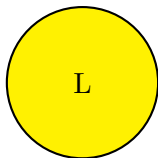
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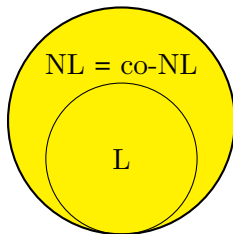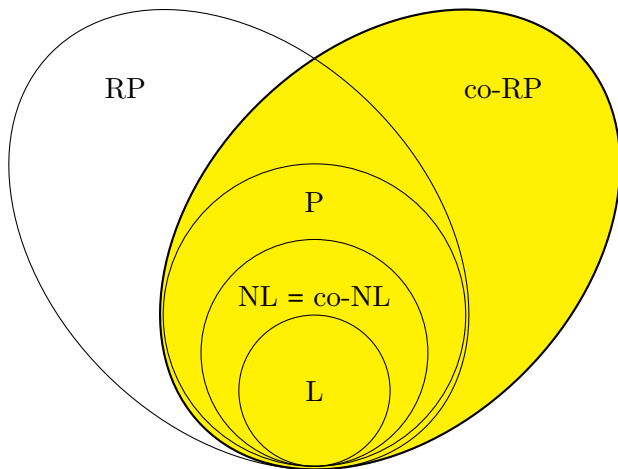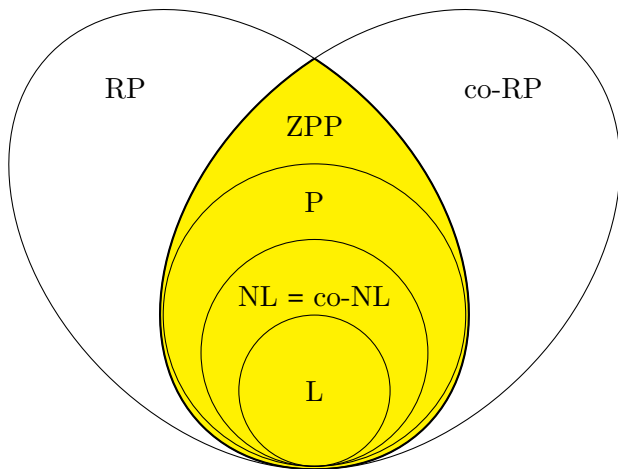
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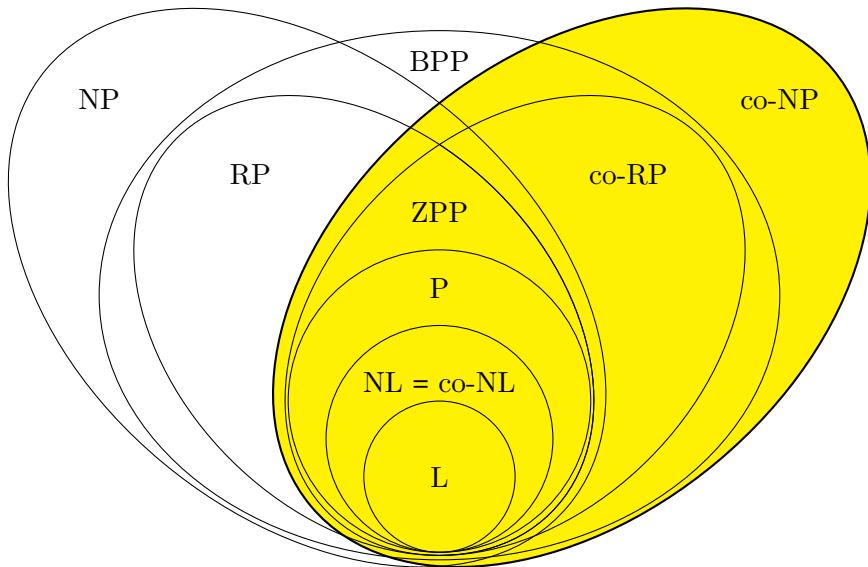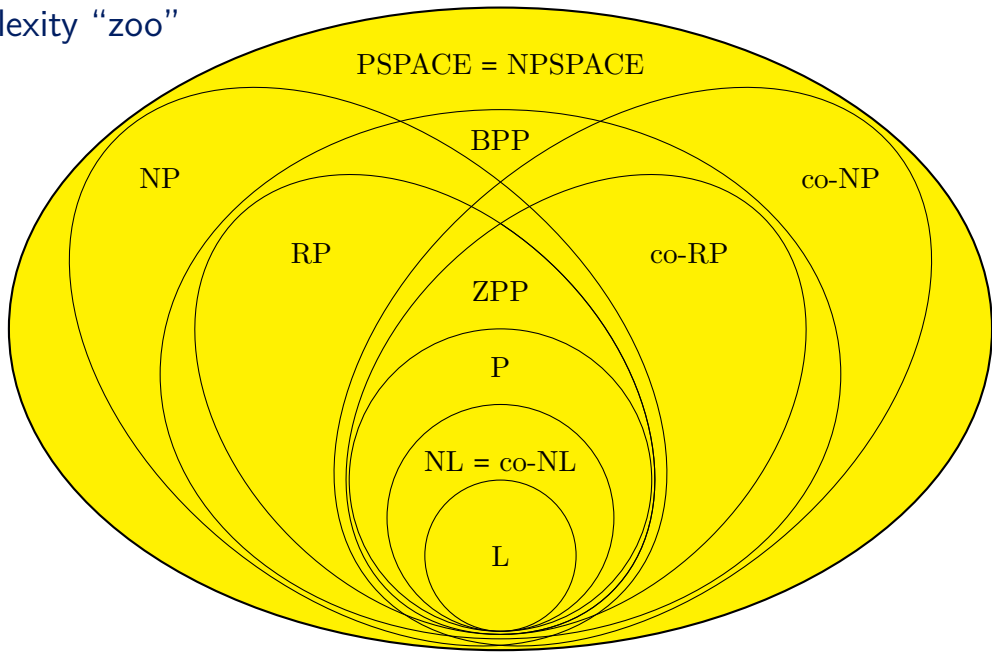
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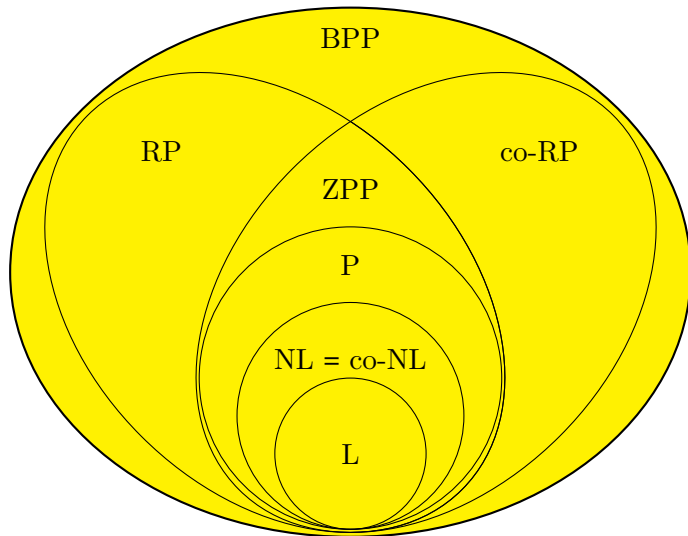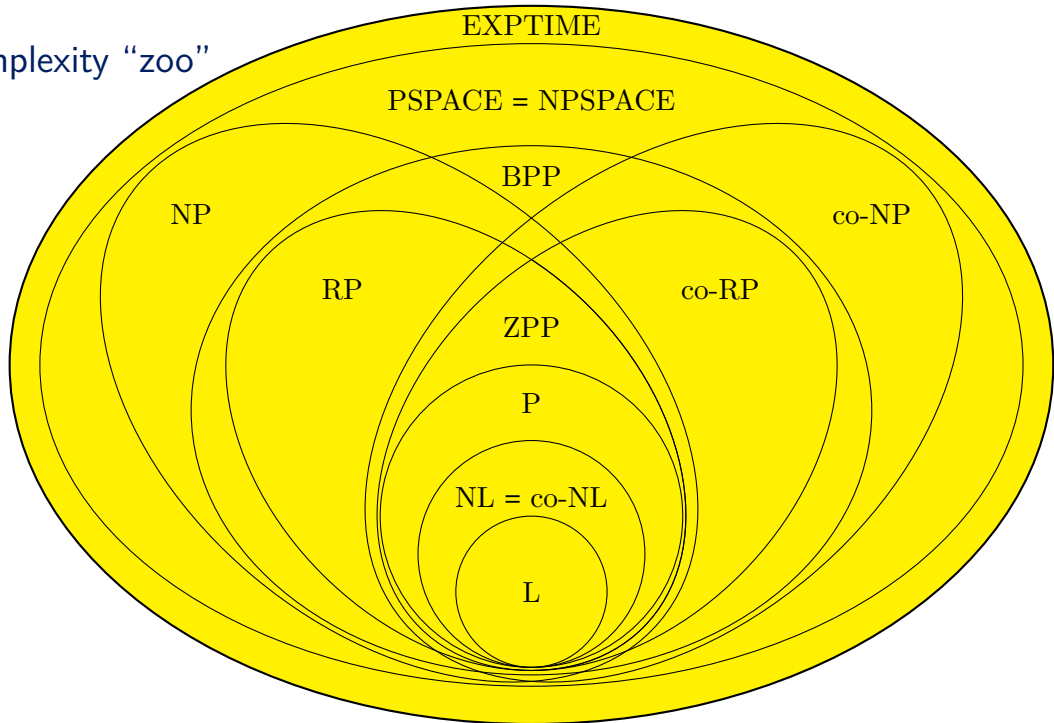
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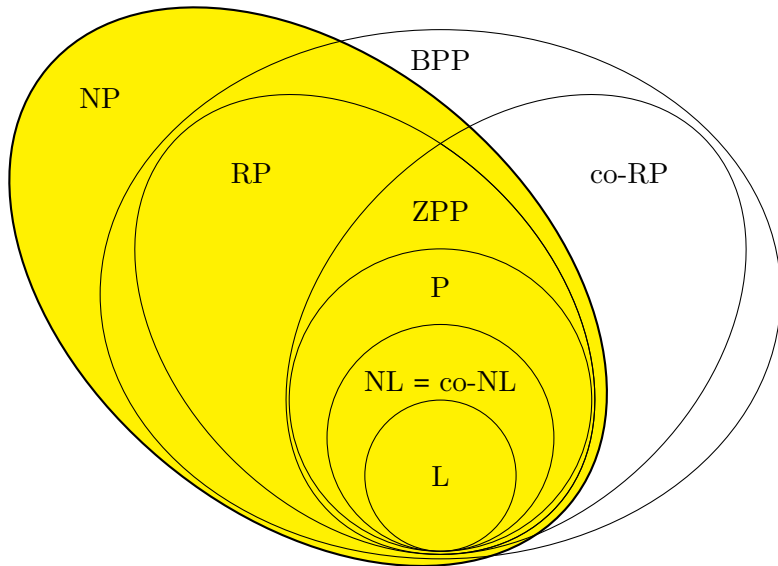
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Exam topics

Broadly speaking: Everything through today

- Regular languages, context-free languages
- DFAs, NFAs, regular expressions, CFGs, PDAs, TMs
- Conversions between the various machines, grammars, and expressions (where doable).
- Converting a CFG to CNF
- Closure properties of regular, context-free, decidable, and Turing-recognizable languages
- Decision problems from language theory (e.g., A_{DFA}, EQ_{TM}, ALL_{CFG})
- Mapping reductions
- Polynomial time mapping reductions
- P, NP, EXPTIME
- What it means for a language to be NP-complete

Types of exam questions

The questions from the exam fall into these types

- True/false questions with explanation
- Constructions
- Proofs
- **One extra credit problem**

Exam question break down (probably; the exam is still being written)

- Five true/false questions (4 points each)
- Two constructions (20 points each)
- Four proofs (20 points, 15 points, 15 points, 20 points)
- Extra credit (20 points, no partial credit)

Things that won't be on the exam
- Pumping lemma for context-free languages questions
- Proving that a particular language is NP-complete (you may be asked to prove that under some assumptions, some language is NP-complete, but you won't be asked to give a polynomial time reduction)

Examples

1. Regular languages are closed under perfect shuffle

   $\{a_1 b_1 a_2 b_2 \cdots a_n b_n \mid$ each $a_i, b_i \in \Sigma,\ a_1 a_2 \cdots a_n \in A$ and $b_1 b_2 \cdots b_n \in B\}$

2. Turing-recognizable languages are closed under intersection

3. Prove that if $A \leq_{\mathrm{p}} B$ and $B \leq_{\mathrm{p}} C$, then $A \leq_{\mathrm{p}} C$

4. Convert a CFG to a PDA

5. COMPOSITES $= \{\langle n \rangle \mid n > 0$ is a composite integer$\} \in \mathrm{NP}$

6. Any others you want me to do