CS 301

Lecture 06 – Nonregular languages and the pumping lemma

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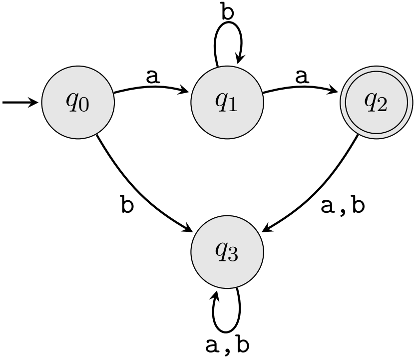
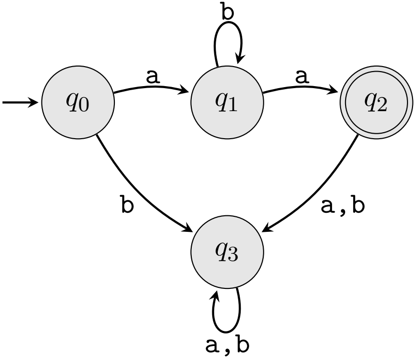
February 5, 2018

DFA M_1



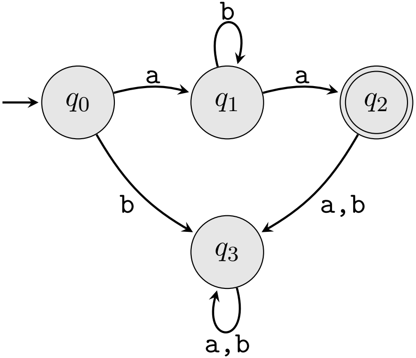
DFA M_1



Strings in the language

- aa
- aba
- abba
- abbba
- $\mathtt{ab}^k\mathtt{a}$ for all $k \geq 0$

DFA M_1



Strings in the language

- aa
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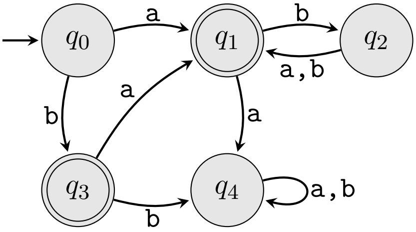
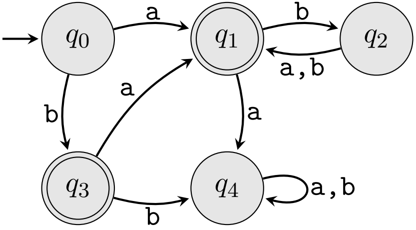
All of the strings $w \in L(M_1)$ s.t. $|w| \geq 3$ have a curious property: w can be written as $w = xyz$ where

1. $|y| > 0$ and
2. $xy^iz \in L(M_1)$ for all $i \geq 0$

DFA M_2



DFA M_2



Strings in the language include

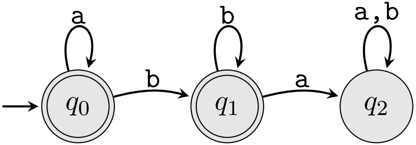
- a
- b
- ba
- aba
- abb
- baba
- abbba
- bababb

Again, strings $w \in L(M_2)$ s.t. $|w| \geq 3$ can be written as $w = xyz$ with $|y| > 0$ and $xy^iz \in L(M_2)$.

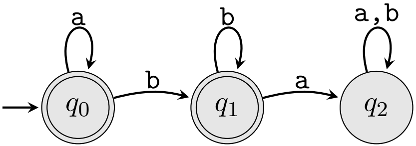
E.g., x = ba, y = ba, $z = \varepsilon$

- xy^0z = ba
- xy^1z = baba
- xy^2z = bababa
- ...

DFA M_3



DFA M_3



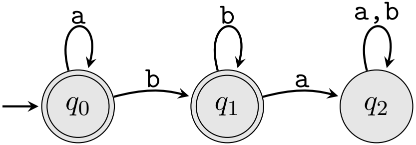
$L(M_3) = \{\mathtt{a}^m\mathtt{b}^n \mid m, n \geq 0\}$

Strings $w \in L(M_3)$ s.t. $|w| \geq 1$ have the same property.

E.g., $x = \varepsilon$, $y = \mathtt{a}$, $z = \mathtt{abb}$

- $xy^0z = \mathtt{abb}$
- $xy^1z = \mathtt{aabb}$
- $xy^2z = \mathtt{aaabb}$
- $xy^iz = \mathtt{a}^{i+1}\mathtt{bb}$

DFA M_3



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Strings $w \in L(M_3)$ s.t. $|w| \geq 1$ have the same property.

E.g., $x = \varepsilon$, $y = \mathtt{a}$, $z = \mathtt{abb}$

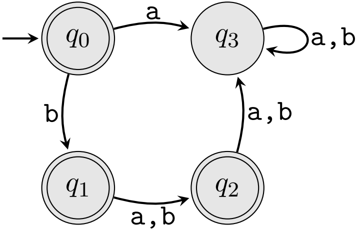
- $xy^0z = \mathtt{abb}$
- $xy^1z = \mathtt{aabb}$
- $xy^2z = \mathtt{aaabb}$
- $xy^iz = \mathtt{a}^{i+1}\mathtt{bb}$

Not every way we split the strings works

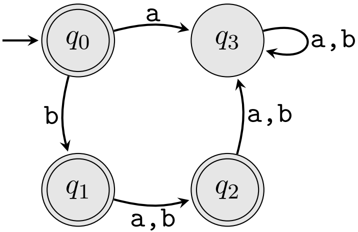
$x = \mathtt{a}$, $y = \mathtt{ab}$, $z = \mathtt{b}$

- $xy^0z = \mathtt{ab} \in L(M_3)$
- $xy^1z = \mathtt{aabb} \in L(M_3)$
- $xy^2z = \mathtt{aababb} \notin L(M_3)$

DFA M_4



DFA M_4

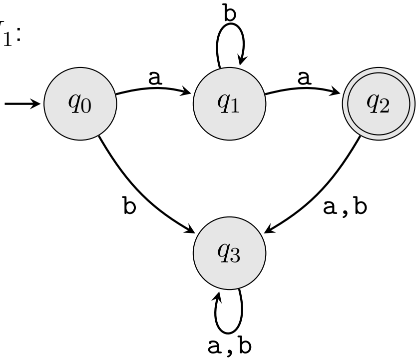


$L(M_4) = \{\varepsilon, \mathtt{b}, \mathtt{ba}, \mathtt{bb}\}$

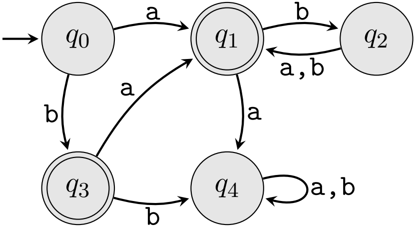
$L(M_4)$ doesn't appear to have this property (unless we say it holds for all strings in $L(M_4)$ with length at least 3 because there are no such strings)

What do M_1, M_2, and M_3 have that M_4 lacks?

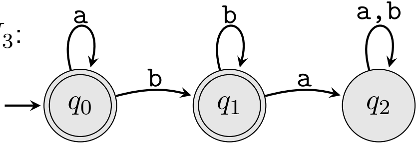
M_1:



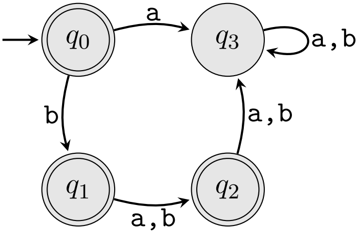
M_2:



M_3:

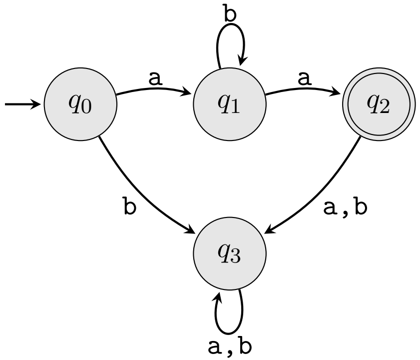


M_4:



Repeated state for some string in the language

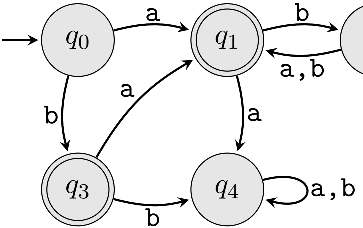
M_1, M_2, and M_3 all have a repeated state in some accepting computation



On input aba, M_1 goes through states q_0, q_1, q_1, q_2

State q_1 is repeated so we can repeat it 0 or more times by following the loop on b

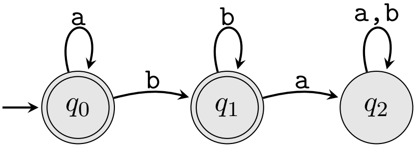
M_2



On input `baba`, M_2 goes through states q_0, q_3, q_1, q_2, q_1

State q_1 is repeated so we can perform the $q_1 \to q_2 \to q_1$ sequence corresponding to input `ba` 0 or more times

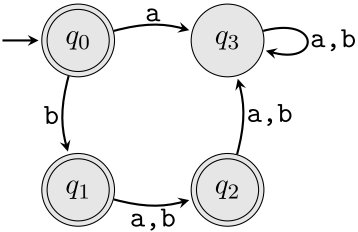
M_3



On input aabb, M_2 goes through states q_0, q_0, q_0, q_1, q_1

State q_0 is repeated so we can perform the $q_0 \rightarrow q_0$ sequence corresponding to input a 0 or more times

M_4



None of the strings in $L(M_4)$ lead to a repeated state

As mentioned, we can "cheat" and say that the property holds for strings of length at least 3 since $L(M_4)$ has no strings of length at least 3

Pumpable languages

A language A is said to be pumpable if there exists an integer $p > 0$ s.t. for all strings $w \in A$ with $|w| \geq p$, there exist strings $x, y, z \in \Sigma^*$ with $w = xyz$ s.t.

1. $xy^iz \in A$ for all $i \geq 0$
2. $|y| > 0$
3. $|xy| \leq p$

The integer p is called the pumping length

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2. $|y| > 0$
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The integer p is called the pumping length

Almost certainly the most complicated mathematical definition you've seen:

$$\exists p > 0.\ \forall w \in A.\ \exists x, y, z \in \Sigma^*.\ \forall i \geq 0.\ [\dots]$$

Contrast with the definition of a continuous function $f : \mathbb{R} \to \mathbb{R}$ from calculus

$$\forall \varepsilon > 0.\ \exists \delta > 0.\ [\dots]$$

A two-player game

Player One ($\exists$)		**Player Two ($\forall$)**
Claims A is pumpable with p.l. p		
	$\xrightarrow{A, p}$	
		Picks $w \in A$ s.t. $\lvert w \rvert \geq p$
	$\xleftarrow{w}$	
Picks x, y, z s.t. $w = xyz$		
	$\xrightarrow{x, y, z}$	
		Checks 3 conditions

Player One "wins" the game if

1. $xy^i z \in A$ for all $i \geq 0$
2. $\lvert y \rvert > 0$
3. $\lvert xy \rvert \leq p$

Player One can win if and only if A is pumpable

Pumping lemma for regular languages

Theorem (Pumping lemma)

Regular languages are pumpable.

Note: The converse is *not* true! There are pumpable languages that are not regular

Proof

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA with $L(M) = A$ and set $p = |Q|$.

If A contains no strings of length at least p, then we're finished since A is pumpable with pumping length p.

Proof

Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA with $L(M) = A$ and set $p = |Q|$.

If A contains no strings of length at least p, then we're finished since A is pumpable with pumping length p.

Otherwise, let w be a string in A of length $n \geq p$.
Write $w = w_1 w_2 \cdots w_n$ where each $w_i \in \Sigma$.

Let $r_0, r_1, \ldots, r_n$ be the accepting computation of M on w.

By the pigeonhole principle, in the first $p + 1$ states $(r_0, r_1, \ldots, r_p)$, there are states $r_j = r_k$ s.t. $0 \leq j < k \leq p$.

Proof

Set

$$x = w_1 w_2 \cdots w_j$$
$$y = w_{j+1} w_{j+2} \cdots w_k$$
$$z = w_{k+1} w_{k+2} \cdots w_n.$$

$$\begin{array}{lccccccccccccc} \text{input:} & & \overbrace{w_1 & w_2 & \cdots & w_j}^{x} & \overbrace{w_{j+1} & w_{j+2} & \cdots & w_k}^{y} & \overbrace{w_{k+1} & w_{k+2} & \cdots & w_n}^{z} \\ \text{states:} & r_0 & r_1 & r_2 & \cdots & \color{red}{r_j} & r_{j+1} & r_{j+2} & \cdots & \color{red}{r_k} & r_{k+1} & r_{k+2} & \cdots & r_n \end{array}$$

Remember $\delta(r_{m-1}, w_m) = r_m$ for all $1 \leq m \leq n$

2. $|y| = k - j > 0$
3. $|xy| \leq p$ because $k \leq p$

Proof

1. $xy^i z \overset{?}{\in} A$

$$\boxed{\delta(r_{m-1}, w_m) = r_m \quad \forall m}$$

Proof

1. $xy^i z \overset{?}{\in} A$

$$\delta(r_{m-1}, w_m) = r_m \quad \forall m$$

$i = 0$

$$\begin{array}{ccccccccc} & \multicolumn{4}{c}{\overbrace{w_1 \quad w_2 \quad \cdots \quad w_j}^{x}} & \multicolumn{4}{c}{\overbrace{w_{k+1} \quad w_{k+2} \quad \cdots \quad w_n}^{z}} \\ r_0 & r_1 & r_2 & \cdots & \color{red}{r_j} & & & & \end{array}$$

Proof

1. $xy^i z \overset{?}{\in} A$

$$\delta(r_{m-1}, w_m) = r_m \quad \forall m$$
$$\delta(r_j, w_{k+1}) = \delta(r_k, w_{k+1}) = r_{k+1}$$

$i = 0$

$$\begin{array}{ccccccccc} & \multicolumn{4}{c}{\overbrace{w_1 \quad w_2 \quad \cdots \quad w_j}^{x}} & \multicolumn{4}{c}{\overbrace{w_{k+1} \quad w_{k+2} \quad \cdots \quad w_n}^{z}} \\ r_0 & r_1 & r_2 & \cdots & r_j & & & & \end{array}$$

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1. $xy^i z \overset{?}{\in} A$

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$$\delta(r_j, w_{k+1}) = \delta(r_k, w_{k+1}) = r_{k+1}$$

$i = 0$

$$\begin{array}{ccccccccc} & \multicolumn{4}{c}{\overbrace{\hphantom{w_1 \; w_2 \; \cdots \; w_j}}^{x}} & \multicolumn{4}{c}{\overbrace{\hphantom{w_{k+1} \; w_{k+2} \; \cdots \; w_n}}^{z}} \\ & w_1 & w_2 & \cdots & w_j & w_{k+1} & w_{k+2} & \cdots & w_n \\ r_0 & r_1 & r_2 & \cdots & r_j & r_{k+1} & r_{k+2} & \cdots & r_n \end{array}$$

Proof

1. $xy^i z \overset{?}{\in} A$

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$$\delta(r_j, w_{k+1}) = \delta(r_k, w_{k+1}) = r_{k+1}$$

$i = 0$

$$\begin{array}{ccccccccc} & \multicolumn{4}{c}{\overbrace{w_1 \quad w_2 \quad \cdots \quad w_j}^{x}} & \multicolumn{4}{c}{\overbrace{w_{k+1} \quad w_{k+2} \quad \cdots \quad w_n}^{z}} \\ r_0 & r_1 & r_2 & \cdots & r_j & r_{k+1} & r_{k+2} & \cdots & r_n \end{array}$$

$i = 1$

$$\begin{array}{ccccccccccccc} & \multicolumn{4}{c}{\overbrace{w_1 \quad w_2 \quad \cdots \quad w_j}^{x}} & \multicolumn{4}{c}{\overbrace{w_{j+1} \quad w_{j+2} \quad \cdots \quad w_k}^{y}} & \multicolumn{4}{c}{\overbrace{w_{k+1} \quad w_{k+2} \quad \cdots \quad w_n}^{z}} \\ r_0 & r_1 & r_2 & \cdots & r_j & r_{j+1} & r_{j+2} & \cdots & r_k & r_{k+1} & r_{k+2} & \cdots & r_n \end{array}$$

Proof

1. $xy^iz \overset{?}{\in} A$

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$i = 0$

$$\begin{array}{ccccccccc} & \multicolumn{4}{c}{\overbrace{w_1 \quad w_2 \quad \cdots \quad w_j}^{x}} & \multicolumn{4}{c}{\overbrace{w_{k+1} \quad w_{k+2} \quad \cdots \quad w_n}^{z}} \\ r_0 & r_1 & r_2 & \cdots & r_j & r_{k+1} & r_{k+2} & \cdots & r_n \end{array}$$

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$$\begin{array}{ccccccccccccc} & \multicolumn{4}{c}{\overbrace{w_1 \quad w_2 \quad \cdots \quad w_j}^{x}} & \multicolumn{4}{c}{\overbrace{w_{j+1} \quad w_{j+2} \quad \cdots \quad w_k}^{y}} & \multicolumn{4}{c}{\overbrace{w_{k+1} \quad w_{k+2} \quad \cdots \quad w_n}^{z}} \\ r_0 & r_1 & r_2 & \cdots & r_j & r_{j+1} & r_{j+2} & \cdots & r_k & r_{k+1} & r_{k+2} & \cdots & r_n \end{array}$$

$i = 2$

$$\begin{array}{ccccccccccccccccc} & \multicolumn{4}{c}{\overbrace{w_1 \quad w_2 \quad \cdots \quad w_j}^{x}} & \multicolumn{4}{c}{\overbrace{w_{j+1} \quad w_{j+2} \quad \cdots \quad w_k}^{y}} & \multicolumn{4}{c}{\overbrace{w_{j+1} \quad w_{j+2} \quad \cdots \quad w_k}^{y}} & \multicolumn{4}{c}{\overbrace{w_{k+1} \quad w_{k+2} \quad \cdots \quad w_n}^{z}} \\ r_0 & r_1 & r_2 & \cdots & r_j & r_{j+1} & r_{j+2} & \cdots & r_k & & & & & & & & \end{array}$$

Proof

1. $xy^iz \stackrel{?}{\in} A$

$$\boxed{\begin{array}{l} \delta(r_{m-1}, w_m) = r_m \quad \forall m \\ \delta(r_j, w_{k+1}) = \delta(r_k, w_{k+1}) = r_{k+1} \\ \delta(r_k, w_{j+1}) = \delta(r_j, w_{j+1}) = r_{j+1} \end{array}}$$

$i = 0$

$$\begin{array}{cccccccccc} & \overbrace{w_1 \quad w_2 \quad \cdots \quad w_j}^{x} & \overbrace{w_{k+1} \quad w_{k+2} \quad \cdots \quad w_n}^{z} \\ r_0 & r_1 \quad r_2 \quad \cdots \quad r_j & r_{k+1} \quad r_{k+2} \quad \cdots \quad r_n \end{array}$$

$i = 1$

$$\begin{array}{cccc} & \overbrace{w_1 \quad w_2 \quad \cdots \quad w_j}^{x} & \overbrace{w_{j+1} \quad w_{j+2} \quad \cdots \quad w_k}^{y} & \overbrace{w_{k+1} \quad w_{k+2} \quad \cdots \quad w_n}^{z} \\ r_0 & r_1 \quad r_2 \quad \cdots \quad r_j & r_{j+1} \quad r_{j+2} \quad \cdots \quad r_k & r_{k+1} \quad r_{k+2} \quad \cdots \quad r_n \end{array}$$

$i = 2$

$$\begin{array}{ccccc} & \overbrace{w_1 \quad w_2 \quad \cdots \quad w_j}^{x} & \overbrace{w_{j+1} \quad w_{j+2} \quad \cdots \quad w_k}^{y} & \overbrace{w_{j+1} \quad w_{j+2} \quad \cdots \quad w_k}^{y} & \overbrace{w_{k+1} \quad w_{k+2} \quad \cdots \quad w_n}^{z} \\ r_0 & r_1 \quad r_2 \quad \cdots \quad r_j & r_{j+1} \quad r_{j+2} \quad \cdots \quad r_k & & \end{array}$$

Proof

1. $xy^iz \overset{?}{\in} A$

$$
\boxed{\begin{array}{l}
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\delta(r_j, w_{k+1}) = \delta(r_k, w_{k+1}) = r_{k+1} \\
\delta(r_k, w_{j+1}) = \delta(r_j, w_{j+1}) = r_{j+1}
\end{array}}
$$

$i = 0$

$$
\begin{array}{ccccccccc}
 & \multicolumn{4}{c}{\overbrace{\hphantom{w_1 \quad w_2 \quad \cdots \quad w_j}}^{x}} & \multicolumn{4}{c}{\overbrace{\hphantom{w_{k+1} \quad w_{k+2} \quad \cdots \quad w_n}}^{z}} \\
 & w_1 & w_2 & \cdots & w_j & w_{k+1} & w_{k+2} & \cdots & w_n \\
r_0 & r_1 & r_2 & \cdots & r_j & r_{k+1} & r_{k+2} & \cdots & r_n
\end{array}
$$

$i = 1$

$$
\begin{array}{ccccccccccccc}
 & \multicolumn{4}{c}{\overbrace{\hphantom{w_1 \quad w_2 \quad \cdots \quad w_j}}^{x}} & \multicolumn{4}{c}{\overbrace{\hphantom{w_{j+1} \quad w_{j+2} \quad \cdots \quad w_k}}^{y}} & \multicolumn{4}{c}{\overbrace{\hphantom{w_{k+1} \quad w_{k+2} \quad \cdots \quad w_n}}^{z}} \\
 & w_1 & w_2 & \cdots & w_j & w_{j+1} & w_{j+2} & \cdots & w_k & w_{k+1} & w_{k+2} & \cdots & w_n \\
r_0 & r_1 & r_2 & \cdots & r_j & r_{j+1} & r_{j+2} & \cdots & r_k & r_{k+1} & r_{k+2} & \cdots & r_n
\end{array}
$$

$i = 2$

$$
\begin{array}{ccccccccccccccccc}
 & \multicolumn{4}{c}{\overbrace{\hphantom{w_1 \quad w_2 \quad \cdots \quad w_j}}^{x}} & \multicolumn{4}{c}{\overbrace{\hphantom{w_{j+1} \quad w_{j+2} \quad \cdots \quad w_k}}^{y}} & \multicolumn{4}{c}{\overbrace{\hphantom{w_{j+1} \quad w_{j+2} \quad \cdots \quad w_k}}^{y}} & \multicolumn{4}{c}{\overbrace{\hphantom{w_{k+1} \quad w_{k+2} \quad \cdots \quad w_n}}^{z}} \\
 & w_1 & w_2 & \cdots & w_j & w_{j+1} & w_{j+2} & \cdots & w_k & w_{j+1} & w_{j+2} & \cdots & w_k & w_{k+1} & w_{k+2} & \cdots & w_n \\
r_0 & r_1 & r_2 & \cdots & r_j & r_{j+1} & r_{j+2} & \cdots & r_k & r_{j+1} & r_{j+2} & \cdots & r_k & & & &
\end{array}
$$

Proof

1. $xy^iz \overset{?}{\in} A$

$$
\boxed{
\begin{array}{l}
\delta(r_{m-1}, w_m) = r_m \quad \forall m \\
\delta(r_j, w_{k+1}) = \delta(r_k, w_{k+1}) = r_{k+1} \\
\delta(r_k, w_{j+1}) = \delta(r_j, w_{j+1}) = r_{j+1}
\end{array}
}
$$

$i = 0$

$$
\begin{array}{ccccccccc}
 & \overbrace{w_1 \quad w_2 \quad \cdots \quad w_j}^{x} & \overbrace{w_{k+1} \quad w_{k+2} \quad \cdots \quad w_n}^{z} \\
r_0 & r_1 \quad r_2 \quad \cdots \quad r_j & r_{k+1} \quad r_{k+2} \quad \cdots \quad r_n
\end{array}
$$

$i = 1$

$$
\begin{array}{cccc}
 & \overbrace{w_1 \quad w_2 \quad \cdots \quad w_j}^{x} & \overbrace{w_{j+1} \quad w_{j+2} \quad \cdots \quad w_k}^{y} & \overbrace{w_{k+1} \quad w_{k+2} \quad \cdots \quad w_n}^{z} \\
r_0 & r_1 \quad r_2 \quad \cdots \quad r_j & r_{j+1} \quad r_{j+2} \quad \cdots \quad r_k & r_{k+1} \quad r_{k+2} \quad \cdots \quad r_n
\end{array}
$$

$i = 2$

$$
\begin{array}{ccccc}
 & \overbrace{w_1 \quad w_2 \quad \cdots \quad w_j}^{x} & \overbrace{w_{j+1} \quad w_{j+2} \quad \cdots \quad w_k}^{y} & \overbrace{w_{j+1} \quad w_{j+2} \quad \cdots \quad w_k}^{y} & \overbrace{w_{k+1} \quad w_{k+2} \quad \cdots \quad w_n}^{z} \\
r_0 & r_1 \quad r_2 \quad \cdots \quad r_j & r_{j+1} \quad r_{j+2} \quad \cdots \quad r_k & r_{j+1} \quad r_{j+2} \quad \cdots \quad r_k & r_{k+1} \quad r_{k+2} \quad \cdots \quad r_n
\end{array}
$$

Proof

Starting in state r_j, when M reads y, it ends in state $r_k = r_j$.

Therefore, when M runs on $xy^i z$, it

1. starts in state $r_0 = q_0$ and after reading x is in state r_j;
2. for each of the i copies of y, it is in state r_j, reads y, and moves to state $r_k = r_j$; and
3. from state r_k, it reads z and ends in state $r_n \in F$

Therefore M accepts $xy^i z$ so

1. $xy^i z \in A$
2. $|y| > 0$
3. $|xy| \le p$.

Therefore, A is pumpable. $\square$

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5. This contradicts the pumping lemma so our assumption must be false, namely A is not regular

Example

Let's prove $A = \{0^n 1^n \mid n \geq 0\}$ is not regular

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Let $w = 0^p1^p$ which has length $2p \geq p$

Consider $xyz = w$ such that $|xy| \leq p$ and $|y| > 0$
We got to choose w, but we don't get to choose x, y, and z
We have to consider all possible choices!

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We have to consider all possible choices!

What are the possible values of x, y, and z?
x and y consist solely of 0s and z has the rest of the p 0s followed by p 1s:
$x = 0^m$, $y = 0^n$, $z = 0^{p-m-n} 1^p$ where $n > 0$

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Example

$x = 0^m$, $y = 0^n$, $z = 0^{p-m-n}1^p$ where $n > 0$

Now we need to find an $i \geq 0$ such that $xy^iz \notin A$

What value of i should we choose?
In this case any $i \neq 1$ works, so let's go with $i = 0$ ("pumping down")

$$xy^0z = xz = 0^{p-n}1^p$$

Since $n > 0$, $p - n \neq p$ so $xy^0z \notin A$ and thus A is not regular $\square$

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Let $w = 0^p 1 0^p$

Thus, $x = 0^m$, $y = 0^n$, and $z = 0^{p-m-n}10^p$

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Thus, $x = 0^m$, $y = 0^n$, and $z = 0^{p-m-n}10^p$

Let's "pump up" this time and try $i = 2$

$$xy^2z = 0^{p+n}10^p \notin B$$

Therefore, B is not regular $\square$

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Since A is regular, there is a DFA M that recognizes A
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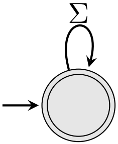
It's missing the fact that for B to be regular there needs to be a DFA M' that accepts every string in B and rejects every string not in B



accepts every string in Σ^*

More nonregular languages

- $C = \{0^m 1^n 0^m \mid m, n \geq 0\}$
- $D = \{0^m 1^n \mid m \leq n\}$
- $E = \{w \mid w \in \{0,1\}^*$ and w has the same number of 0s and 1s$\}$