Mathematical Modelling in LaTeX

Gurobi Optimization

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This document provides an example of how LaTeX can be used to write clean mathematical models. As an example, we consider the famous maximum flow / minimum cut problems. The problems consider a directed graph consisting of a set of nodes and a set of labeled arcs. The arc labels are non-negative values representing a notion of capacity for the arc. In the node set, there exists a source node s and a terminal node t. The amount of flow into one of the intermediary nodes must equal the amount of flow out of the node, i.e., flow is conserved.

The maximum flow question asks: what is the maximum flow that can be transferred from the source to the sink. The minimum cut question asks: which is the subset of arcs, that once removed would disconnect the source node from the terminal node, which has the minimum sum of capacities. For example, removing arcs (v_1, v_3) and (v_2, v_4) from the network in 1 would mean there is no longer a path from s to t and the sum of the capacities of these arcs is $12 + 11 = 23$. It is reasonably straight forward to find a better cut, i.e., a subset of nodes with sum of capacities less than 23.

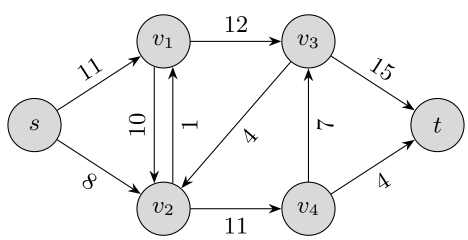

Figure 1: Example network

A complete model is provided for the maximum flow problem, whereas the minimum cut problem is left as a challenge to the reader.

Notation

Sets

Index	Set	Description
i	V	Set of all nodes (s source and t terminal)
(i,j)	A	Set of all arcs

Parameters

Parameter	Description
$c_{i,j}$	Capacity of arc $(i,j) \in A$

Maximum Flow

Variables

Variable	Type	Description
$f_{i,j}$	Cont	flow from i to j in arc $(i,j) \in A$

Model

$$\text{maximise} \sum_{j \in V:(s,j) \in A} f_{s,j} \tag{1a}$$

$$s.t.\ f_{i,j} \leq c_{i,j} \qquad \forall (i,j) \in A \tag{1b}$$

$$\sum_{i \in V:(i,j) \in A} f_{i,j} - \sum_{k \in V:(j,k) \in A} f_{j,k} = 0 \quad \forall j \in V \setminus \{s,t\} \tag{1c}$$

The objective (1a) is to maximise the sum of flow leaving the source node s. Constraints (1b) ensure that the flow in each arc does not exceed the capacity of that arc. Constraints (1c) are continuity constraints, which ensure that the flow into each of the nodes, excluding the source and sink, is equal to the flow out of that node.

Minimum Cut

This section is left for the reader to complete

Variables

Variable	Type	Description
$r_{i,j}$		1 if arc $(i,j) \in A$ is removed
$z_{i,j}$		1 if node $i \in V \setminus \{s,t\}$ connected to s, 0 otherwise

Model

$$\text{to do} \tag{2a}$$