

# Problem Set 2

## Dynamic Macroeconomics

### Modelling Firm in Vietnam

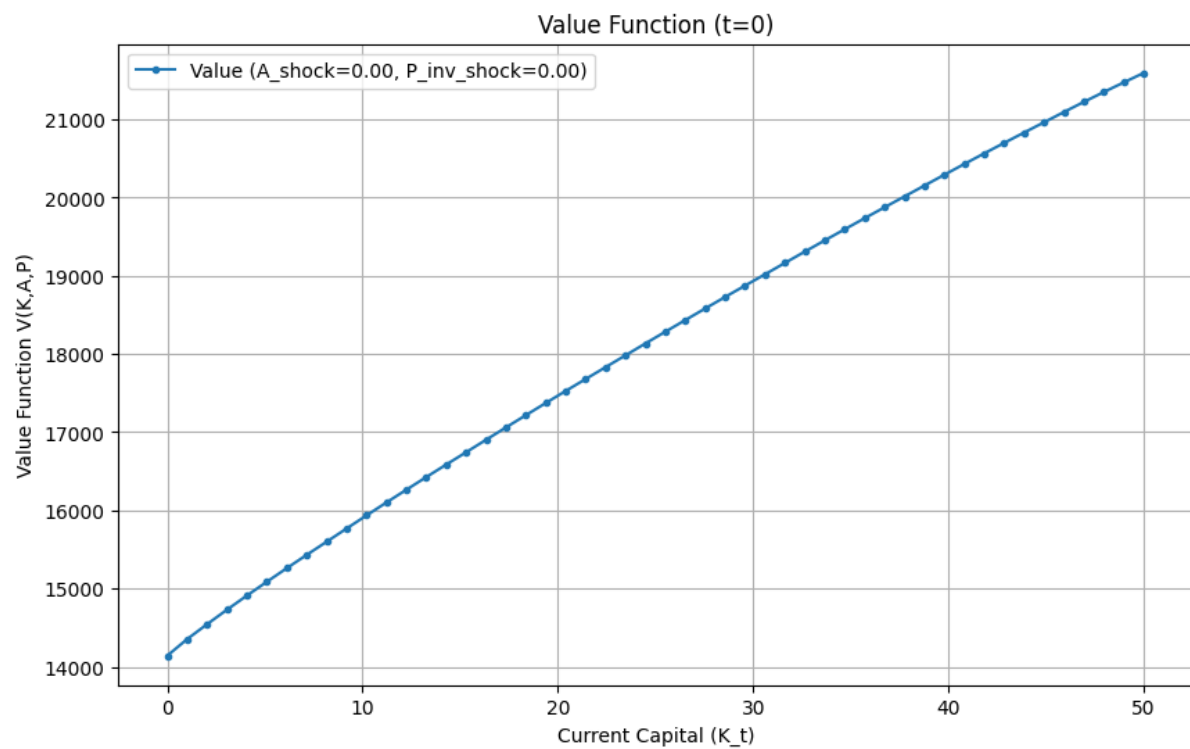
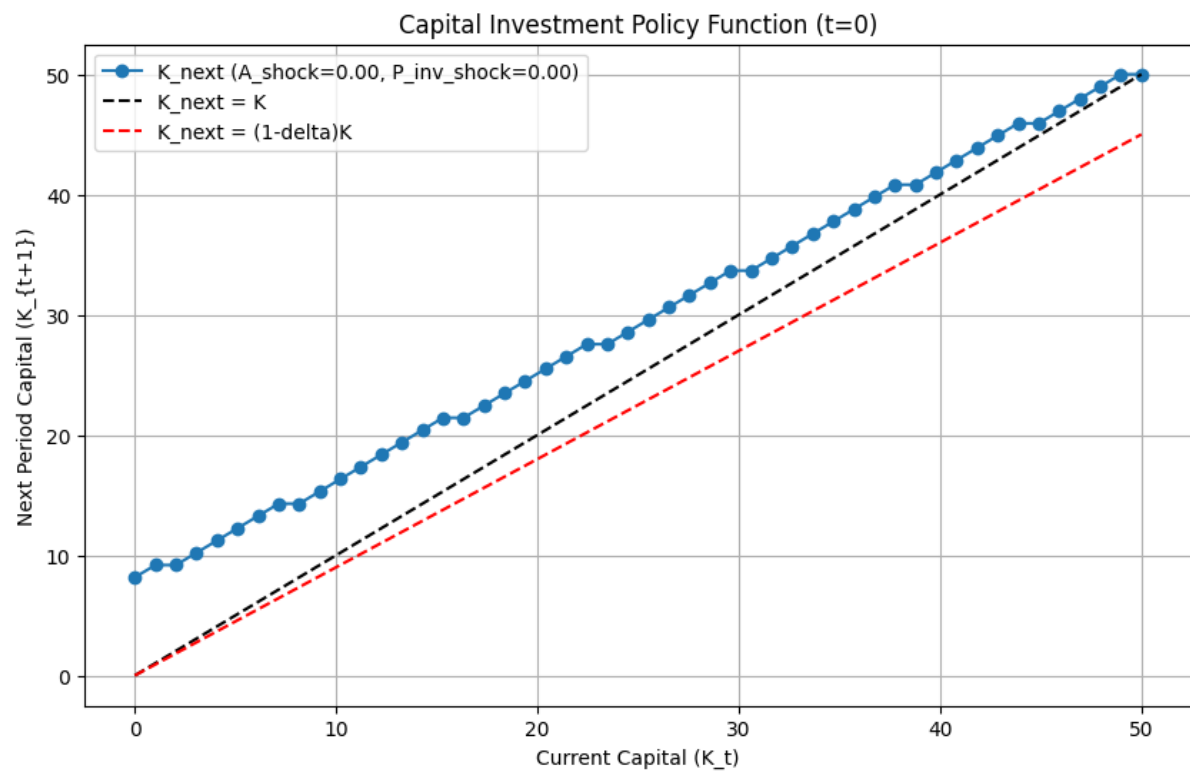
#### Explaining variables:

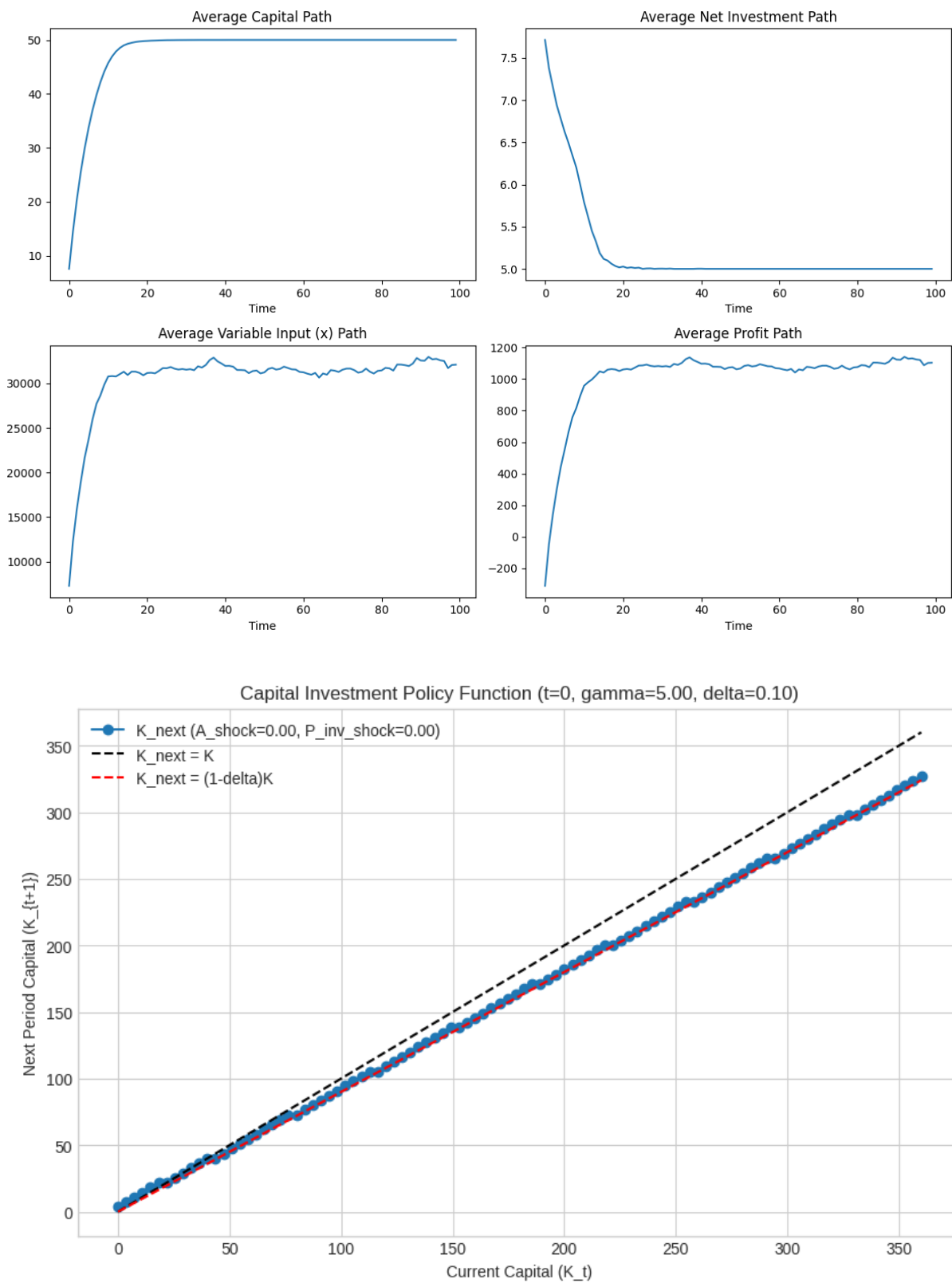
- The large and small firm will be distinguished through the median revenue based on more than 1000 firms.
- The median revenue is 29.3 billion VND (29,292,038,500 VND)
- The mean revenue is 264.7 billion VND (264,748,172,152 VND)
- By calculating the median revenue, I distinguished 515 big firms, and 512 small firms
- The key input variables for production ( $x_t$ ) is the number of full-time workers within a firm. (L1 column in csv)
- The variable costs ( $w_t$ ) is calculated from the sum of labour cost, social security and employment-based tax, electricity, water, internet of each firm then half it to obtain the variable cost. (Median: 1,574,642,500 VND) (n2a, n2a2, n2b, n2k, n2l columns)
- The price of investment in new capital ( $p_t$ ) is calculated from the median sum of purchasing of equipments, machinery, vehicles, land, manufacturing site. (1,348,000,000 VND) (n5a, n5b)
- The key determinants of production choices among Vietnamese firms is the total annual cost of raw materials and intermediate goods used in production (n2e).
- 

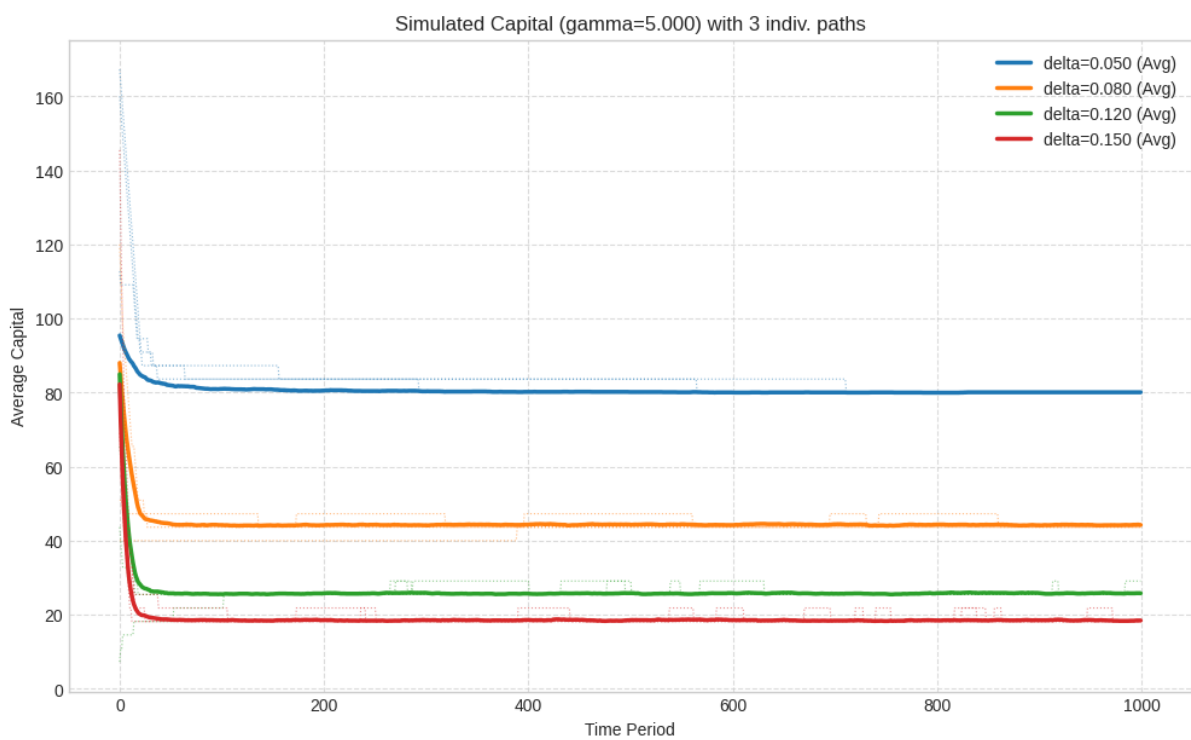
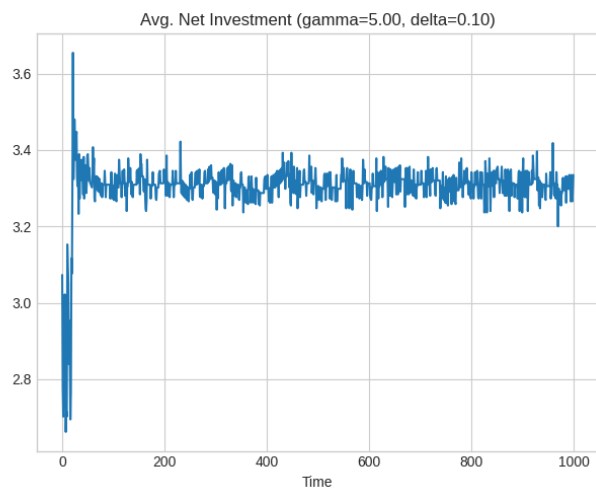
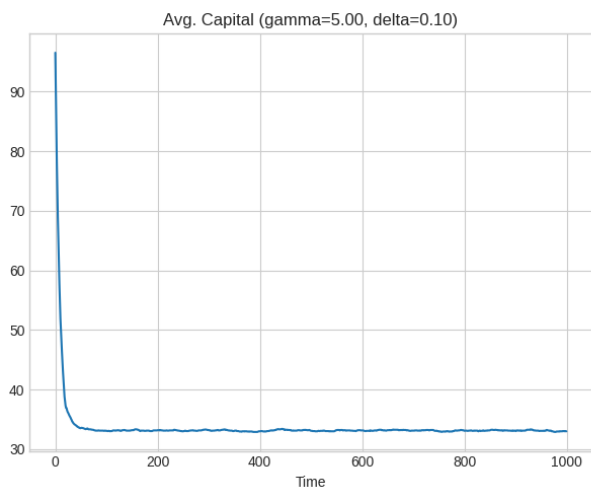
(Empirical relationship between debt, production inputs, finances, etc. will be on page 22)

(Anecdotal evidence will be after the optimisation problem, page 22)

## Small Firm Graphs



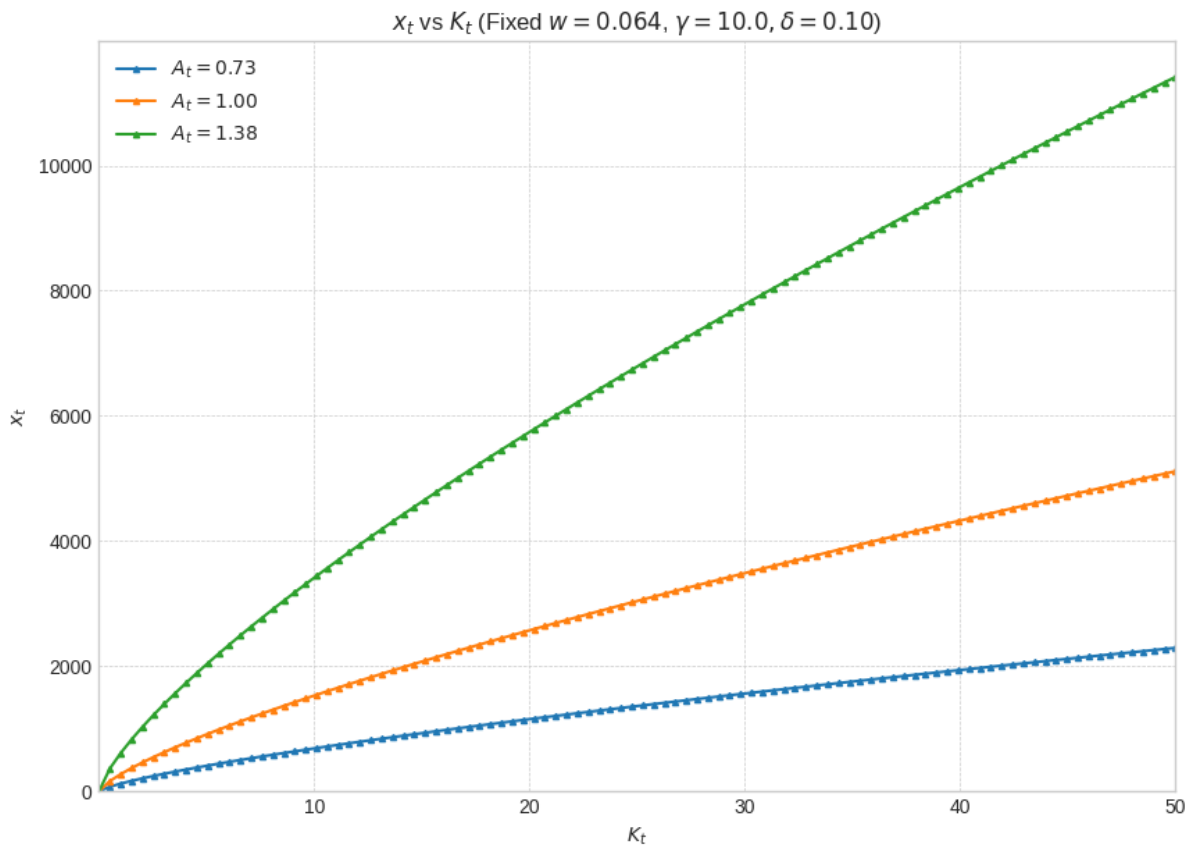
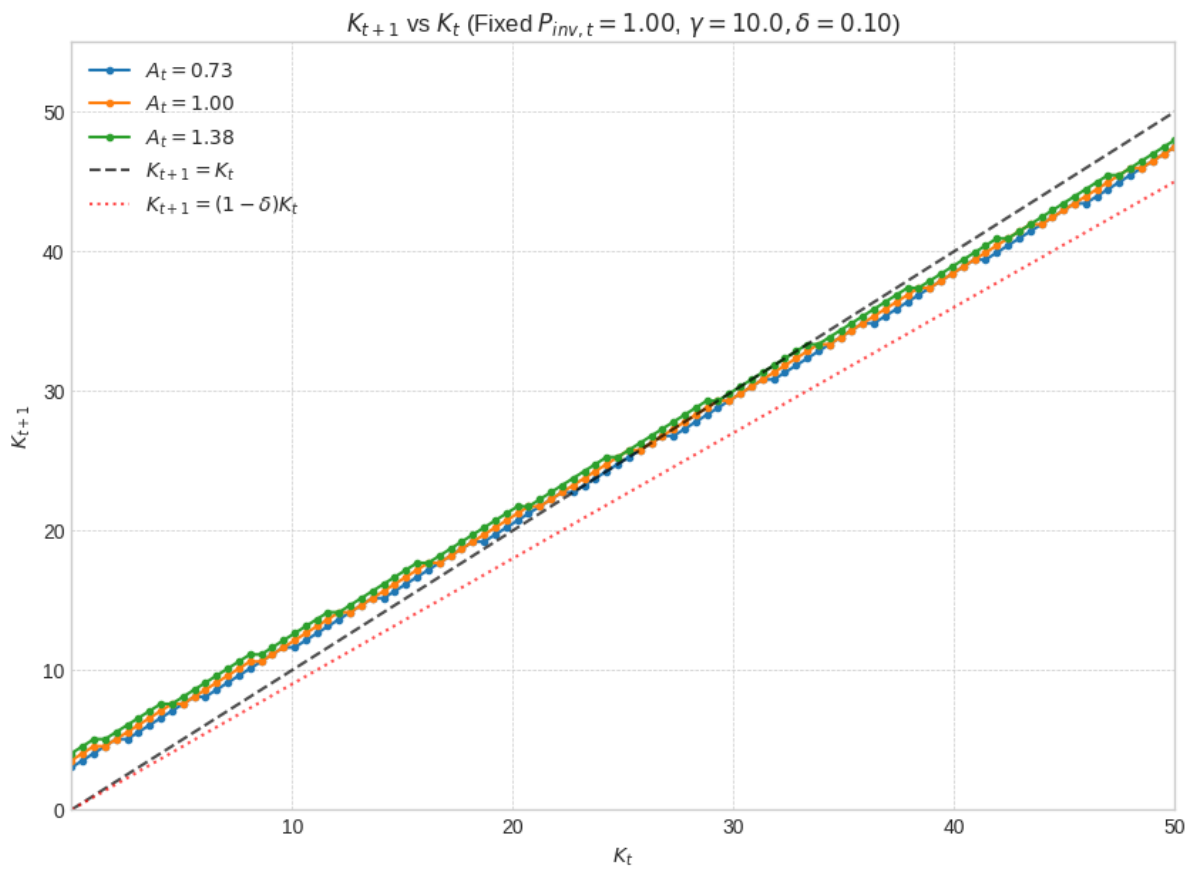




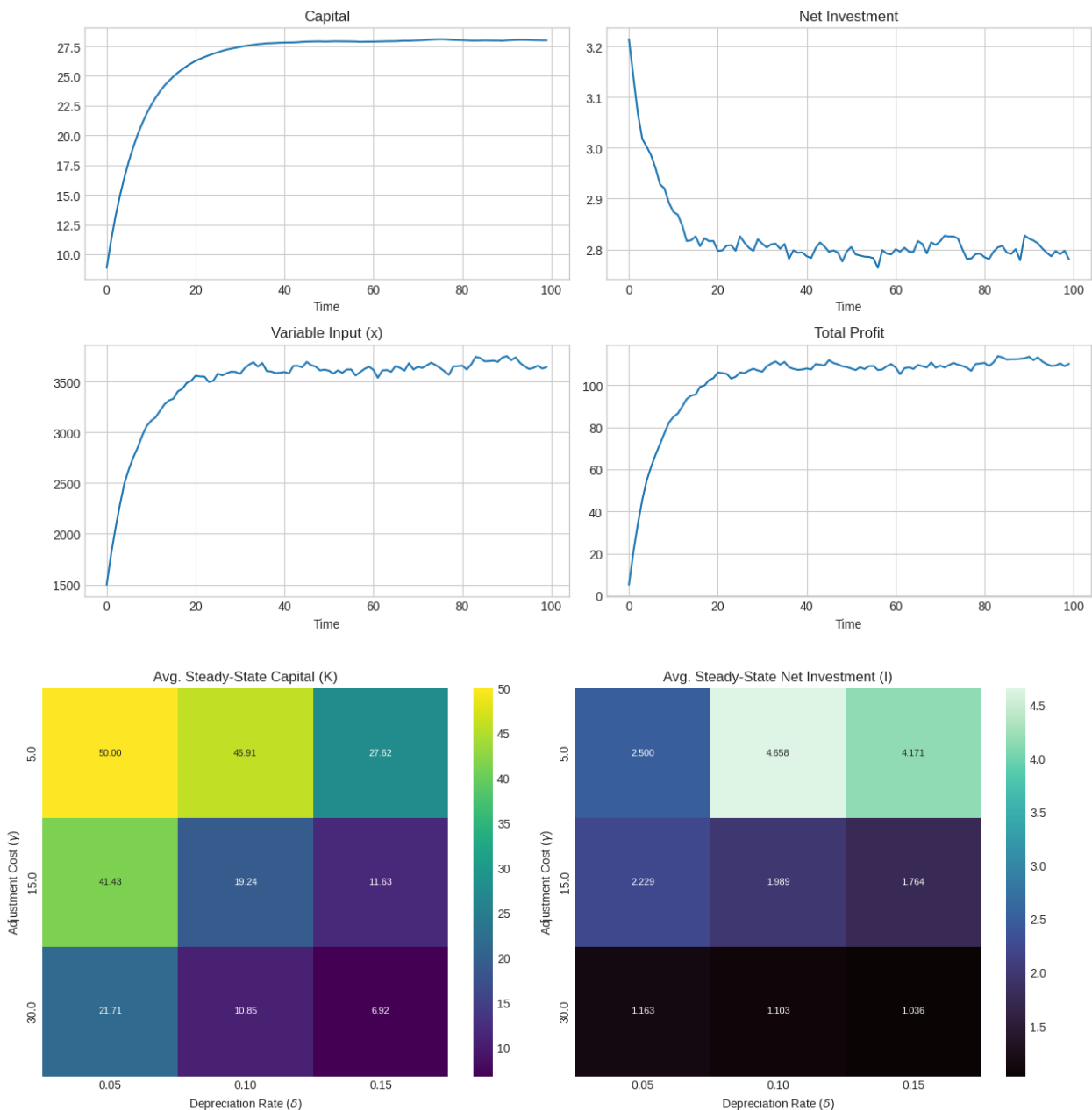


Discussion for Scenario 1 (Varying Delta with dynamic parameters):

- Capital Paths: With higher shock variance ( $\sigma_A$ ) and potentially lower gamma, the average capital paths should now show more fluctuations around their respective mean levels. Higher delta should still lead to a lower mean capital level.
- Investment Paths: Average net investment should also be more volatile. Gross investment will be higher with higher delta. Individual paths will show significant deviations.



Average Simulated Paths ( $\gamma = 10.0, \delta = 0.10$ )



## Discussion of Findings

The plots for the given gamma and delta show typical firm behavior: capital investment increases with current capital (up to a target) and productivity, and decreases with higher investment prices. Variable input use increases with capital and productivity.

The heatmaps illustrate how these dynamics change across different structural parameters:

Average Capital Stock (K) - Heatmap:

- Higher gamma (Y-axis, adjustment cost): Generally leads to lower average capital. Firms are more cautious about building/adjusting capital due to higher friction costs.

- Higher delta (X-axis, depreciation): Generally leads to lower average capital. It's more expensive to maintain any given capital stock as it wears out faster.
- > Lowest capital stocks are expected in the bottom-right (high gamma, high delta). Highest in the top-left (low gamma, low delta).

#### Average Net Investment (I) - Heatmap:

- The interpretation of average net investment over the simulation needs care. In a long-run steady state, average net investment would be zero. The heatmap shows the average over a finite simulation which includes transition dynamics and fluctuations around a stochastic steady state.
- Higher gamma: Makes investment less volatile. The average magnitude of investment spikes might be lower. If high gamma leads to a much lower target  $K$ , there could be initial average negative net investment during adjustment from a common starting point.
- Higher delta: Requires higher gross investment ( $\delta \cdot K$ ) to maintain any  $K$ . If firms settle at a lower  $K$  due to high delta, the net investment to reach that  $K$  could be negative initially. Once at the lower  $K$ , net investment would average around zero, but gross investment is  $\delta \cdot K_{\text{new\_lower}}$ .
- > The heatmap might show that with very high delta, the average net investment is slightly more negative if firms are aggressively disinvesting towards a much lower target, or it might hover closer to zero but representing higher turnover (gross investment). Values near zero are expected if the simulation captures behavior around a steady state.

#### Comparison with 'Baseline Model':

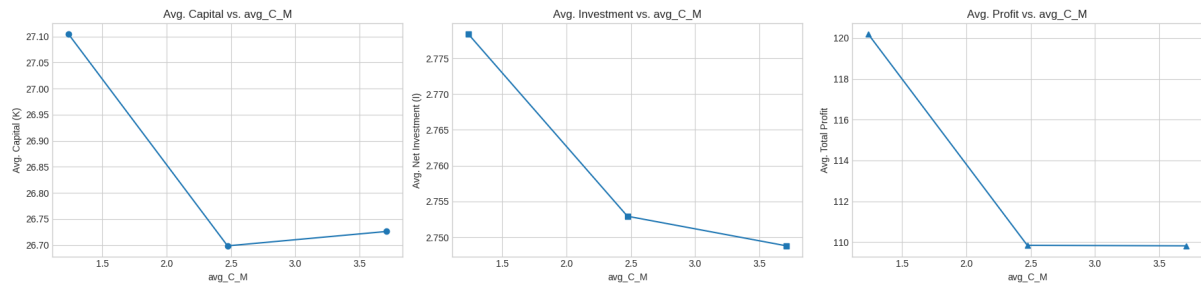
The 'baseline model' in the original problem description (from the image in the first prompt) was a general dynamic profit maximization problem for an infinitely-lived firm. This modified Python model implements a version of it with specific functional forms and the addition of raw material costs.

- The key parameters being varied (gamma, delta) are standard in such investment models. Their effects (higher gamma slows adjustment and can lower  $K$ , higher  $\delta$  lowers  $K$  and increases capital turnover) are consistent with theory.
- The inclusion of stochastic productivity ( $A_t$ ) and investment prices ( $p_t$ ) makes the firm's problem richer and leads to state-dependent policy functions, as observed.
- The new element, 'total annual cost of raw materials' ( $C_M$ ), acts as a reduction in per-period operating profit. Ceteris paribus, this would make any given level of capital less profitable to maintain, potentially leading to lower optimal capital stock and investment compared to a hypothetical baseline that omitted this cost or had a lower  $C_M$ . The specific values of gamma and delta then determine how quickly and to what level the firm adjusts its capital in response to this underlying profitability.

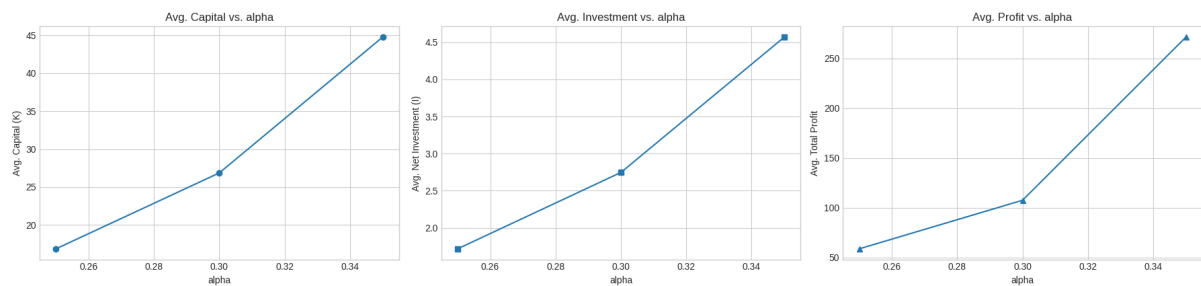


Done.

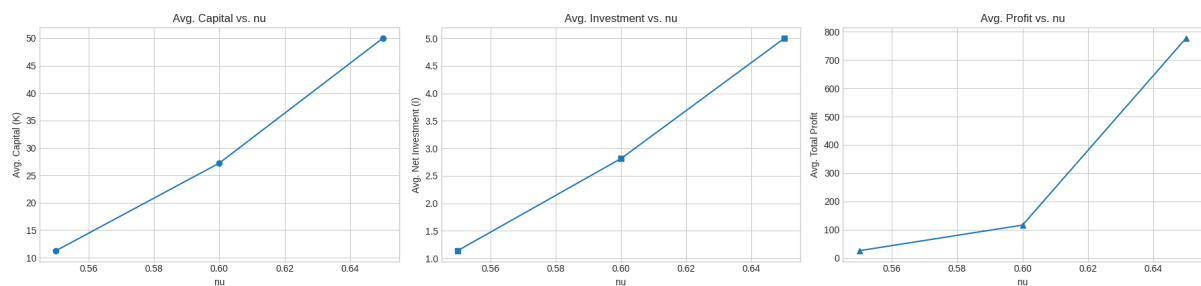
Sensitivity Analysis: Impact of avg\_C\_M (Fixed  $\gamma = 10.0$ ,  $\delta = 0.10$ )



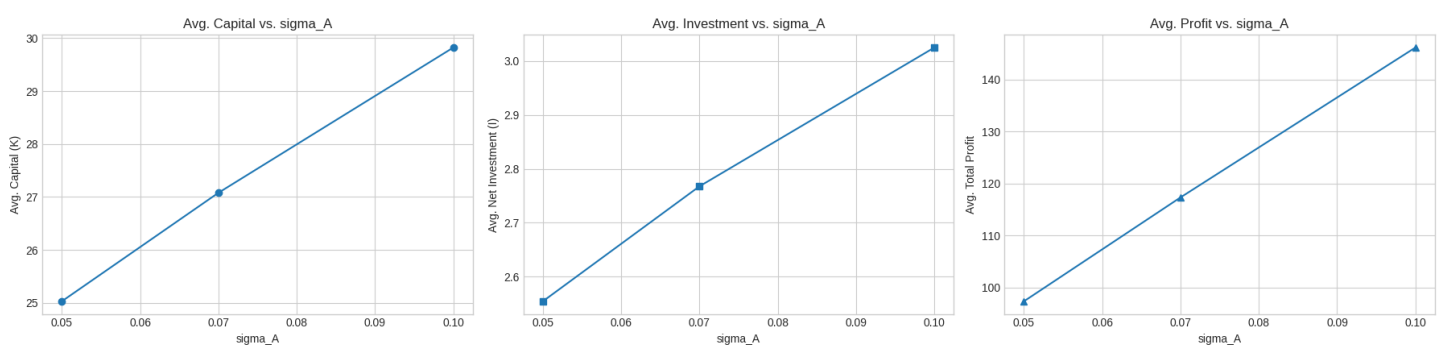
Sensitivity Analysis: Impact of alpha (Fixed  $\gamma = 10.0$ ,  $\delta = 0.10$ )



Sensitivity Analysis: Impact of nu (Fixed  $\gamma = 10.0$ ,  $\delta = 0.10$ )



Sensitivity Analysis: Impact of sigma\_A (Fixed  $\gamma = 10.0$ ,  $\delta = 0.10$ )



--- Comparing Summary Statistics: Real Data vs. Simulated Data ---

Statistic	Real Data	Simulated Data
-----		
StdDev_VarInput	35.386	N/A
StdDev_VarInput	N/A	1706.120
Avg_InvOutRatio	0.395	0.009
Avg_CapIncRatio	N/A	0.086
VarRatio_InvOut	20.273	0.000
-----		

Discussion of Sensitivity and Comparison (Based on Output)

Comparison of Summary Statistics (Observed):

StdDev\_VarInput: Data vs. Sim = 35.38556502215425 vs. 1706.1202826523593

Avg\_InvOutRatio: Data vs. Sim = 0.39472715393416563 vs. 0.009241036148919439

Avg\_CapIncRatio (Sim only): 0.08604099627666495

VarRatio\_InvOut: Data vs. Sim = 20.272696245222456 vs. 3.936673519102559e-06

[Discuss specific discrepancies and consistencies. For instance, is investment more/less volatile in sim vs data? Is I/Y ratio similar?]

Attempting 'Best Fit' (Parameter Tweaks):

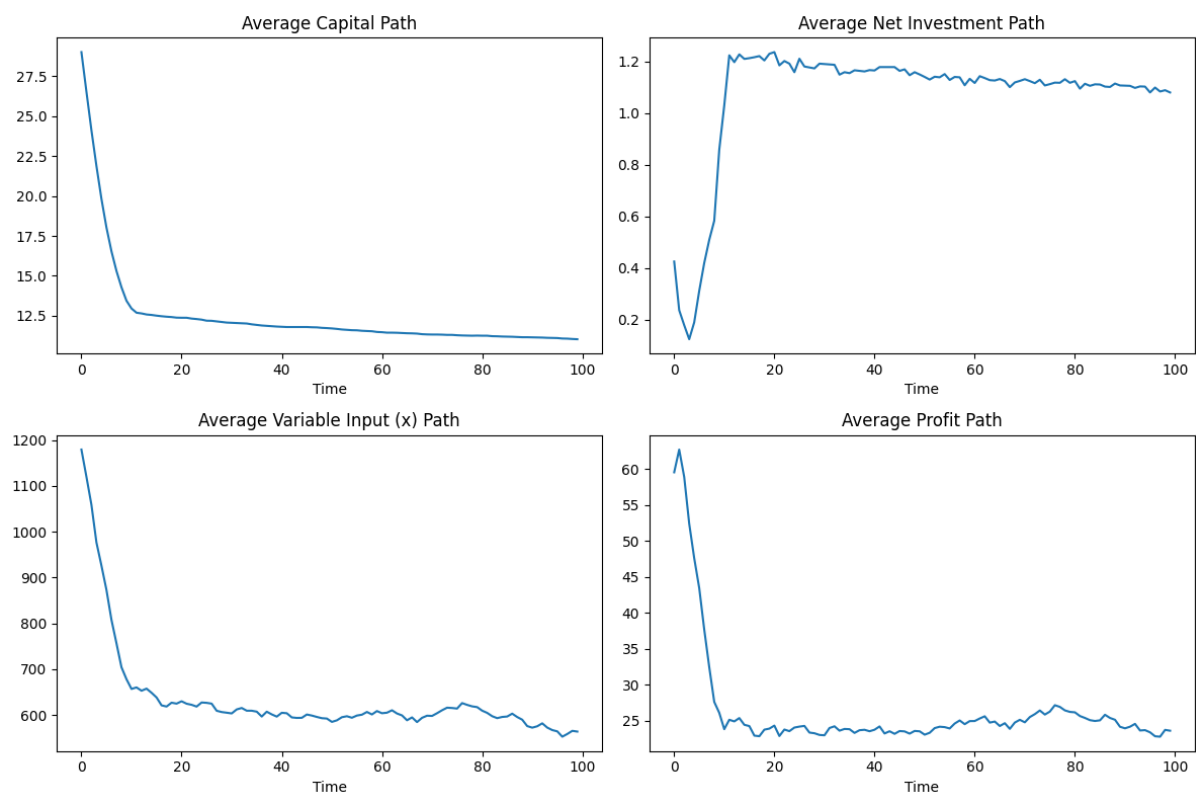
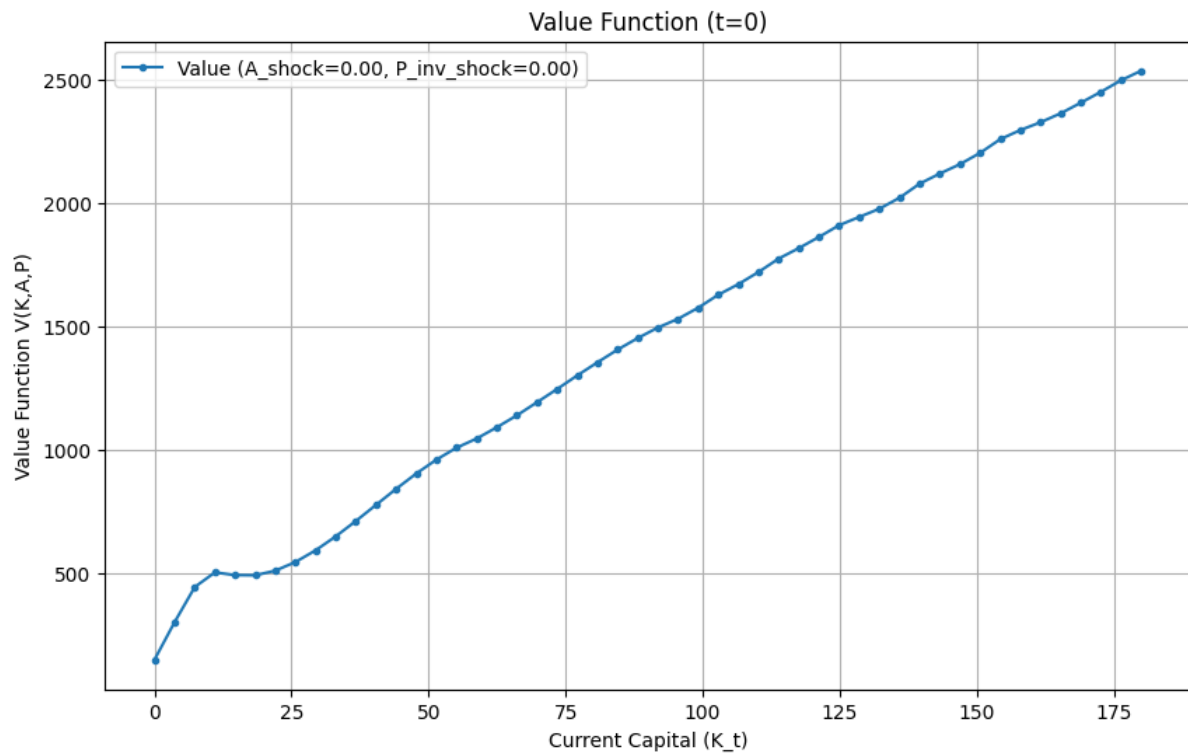
[Based on the comparison, suggest parameter changes. E.g., 'If simulated investment volatility is too low compared to data ( $3.936673519102559e-06 < 20.272696245222456$ ), reducing gamma or increasing  $\sigma_A/\sigma_P$  could help. If simulated K is too high, increasing  $\text{avg\_}C_M$ , w, or delta, or decreasing alpha might be considered.']

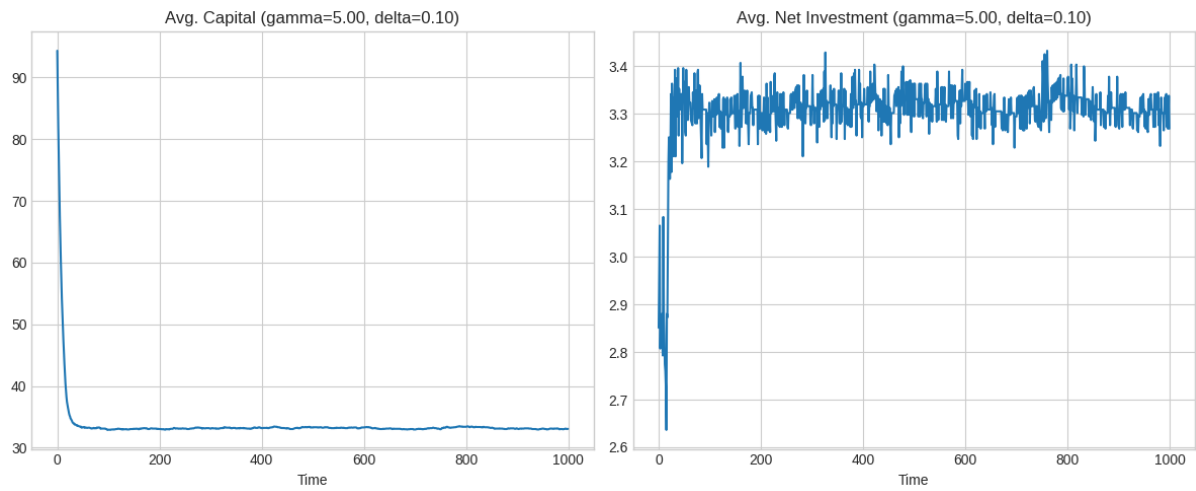
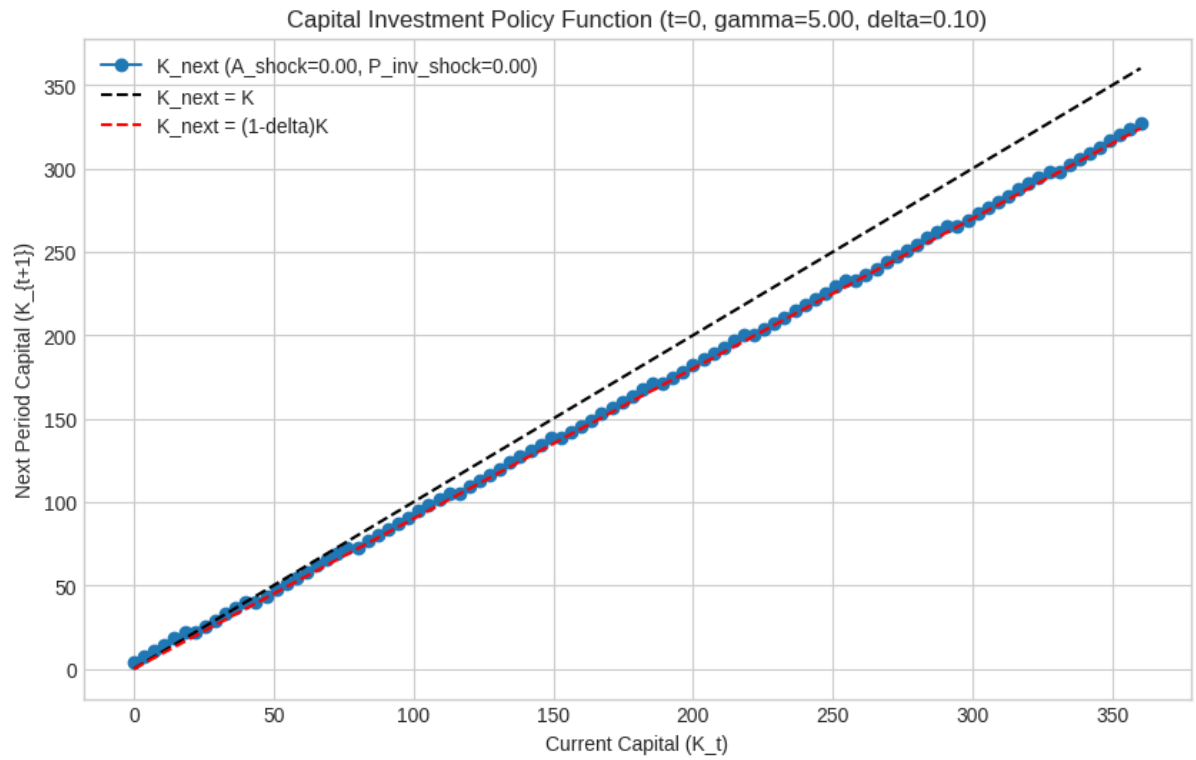
It's an iterative process. For example, if the simulated I/Y ratio is much lower than the data, we might need a combination of higher target capital (e.g., higher alpha) and perhaps a slightly lower delta (though delta also drives replacement). If simulated output is less volatile than data, increasing  $\sigma_A$  would be a first step.

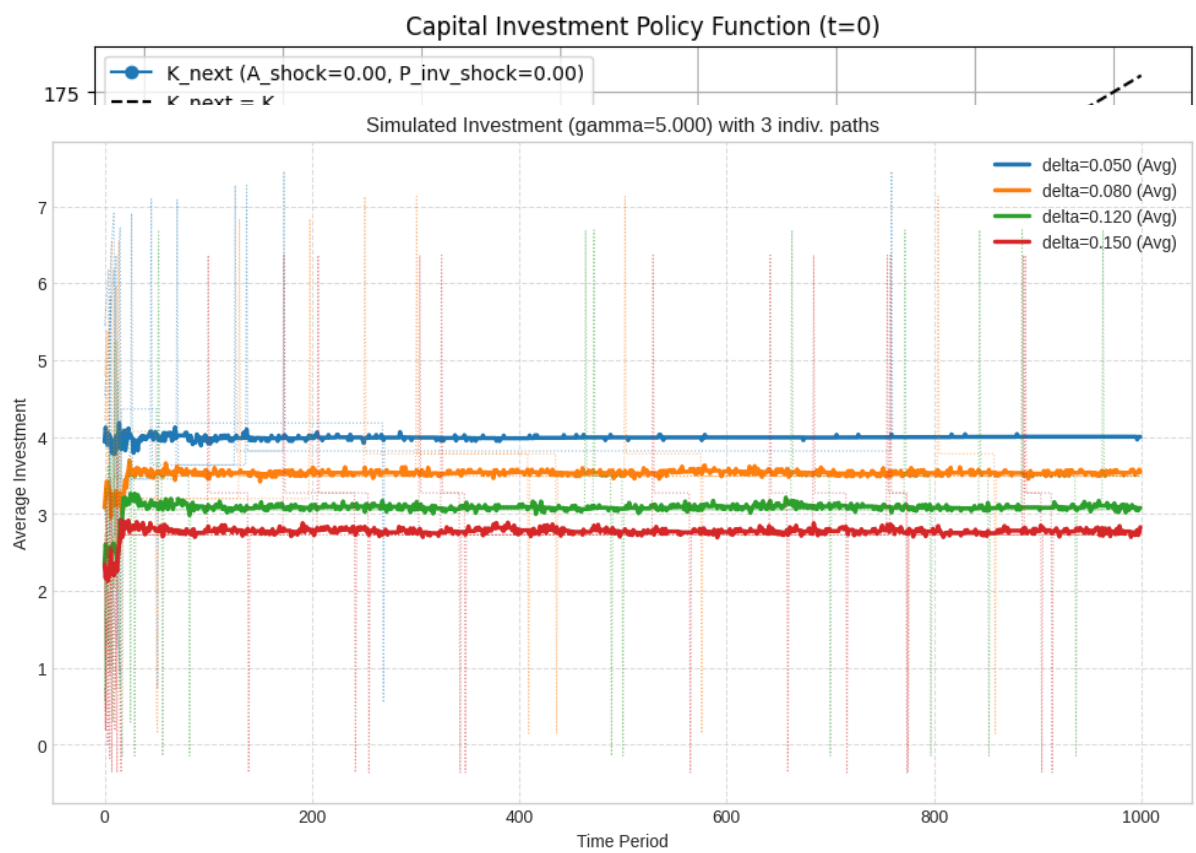
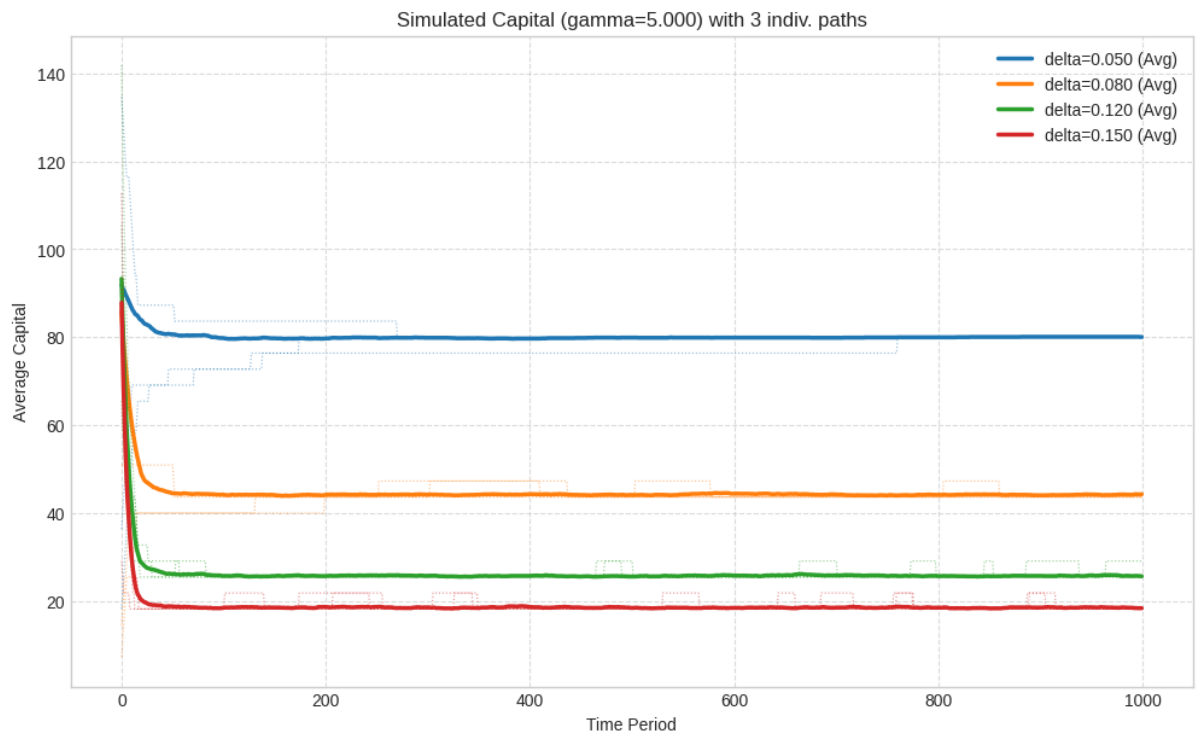
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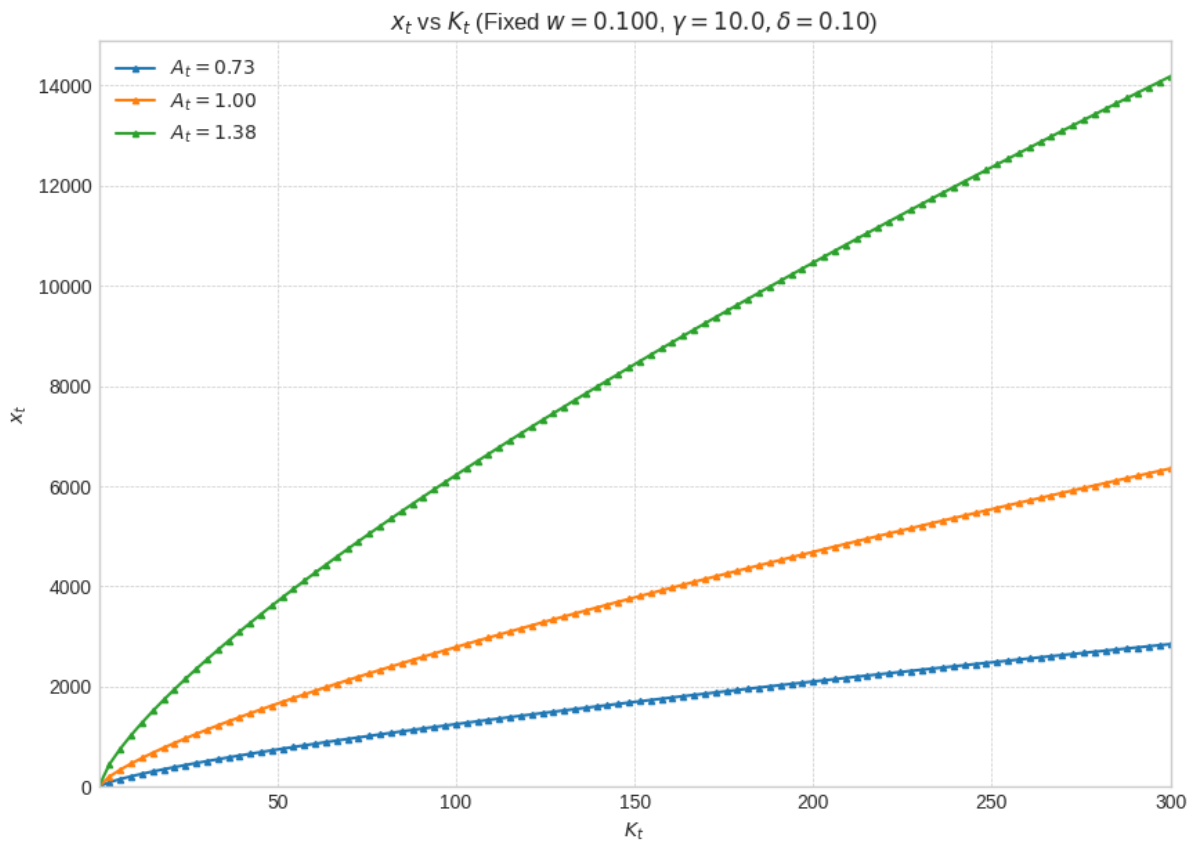
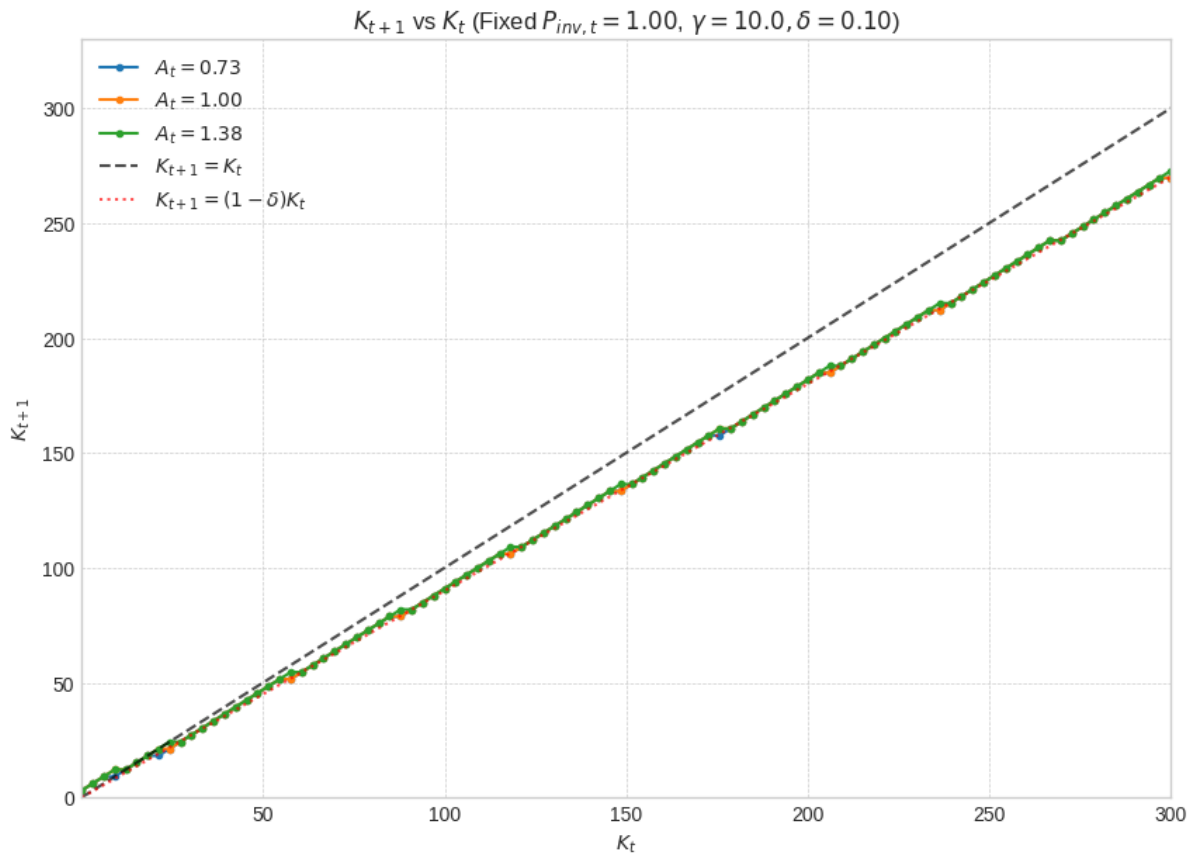
A key challenge is that the firm-level data is cross-sectional and likely very heterogeneous, while the model is for a 'representative' firm over time. Matching moments requires careful thought about what the model represents.

## Big Firm

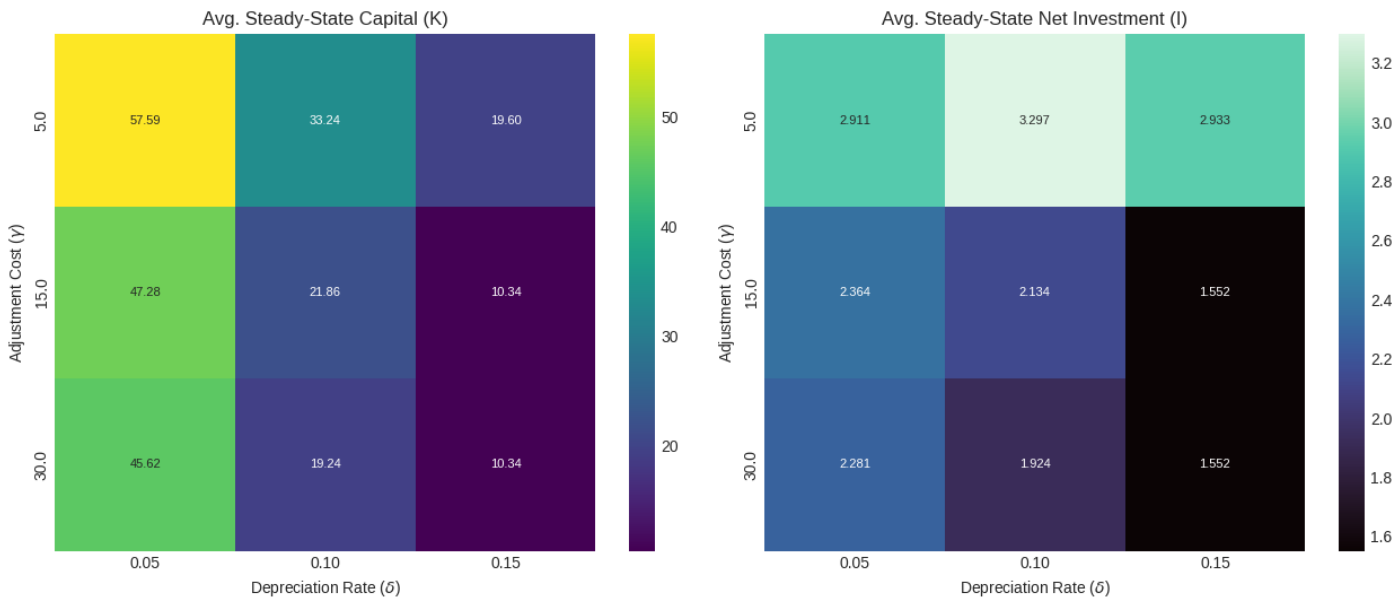
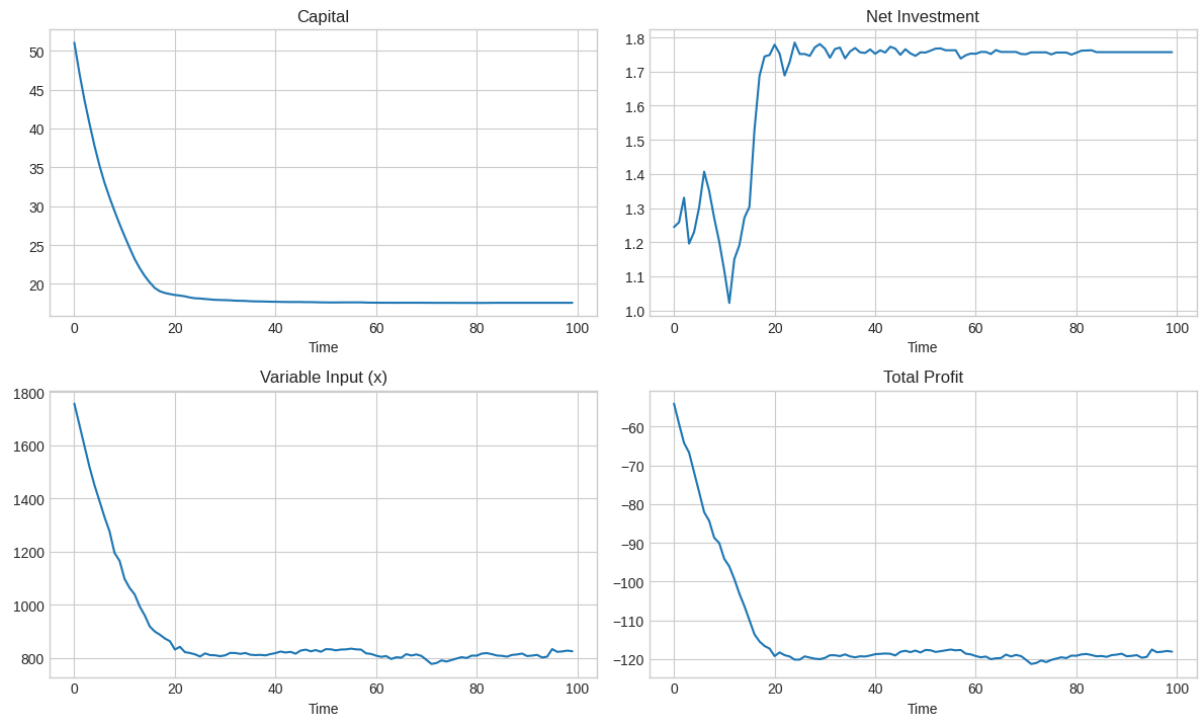








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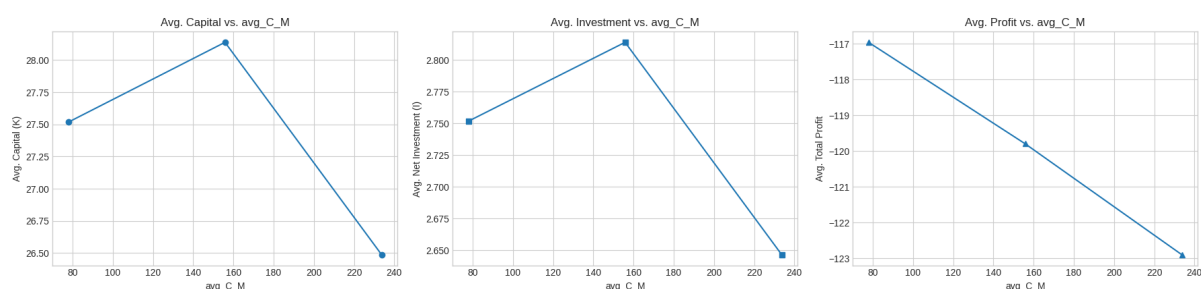
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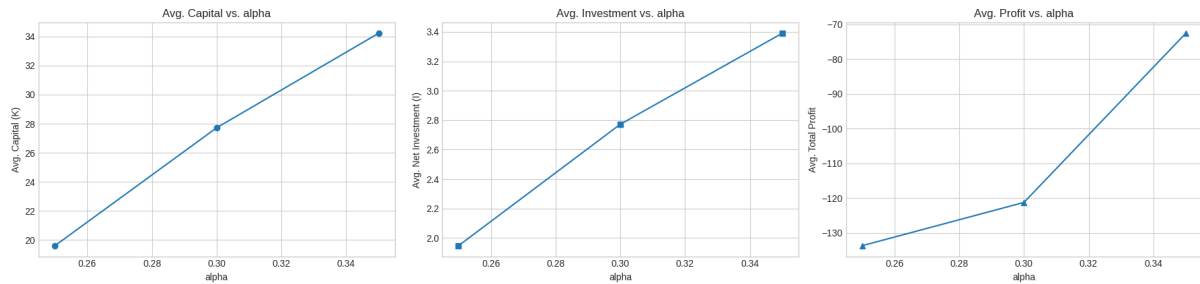
Statistic	Real Data	Simulated Data
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StdDev_VarInput	894.757	N/A
StdDev_VarInput	N/A	449.991
Avg_InvOutRatio	0.069	0.015
Avg_CapIncRatio	N/A	0.156
VarRatio_InvOut	0.014	0.000

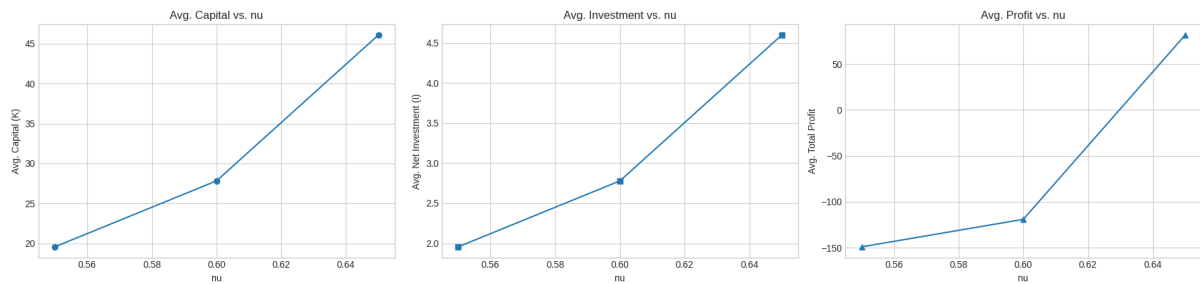
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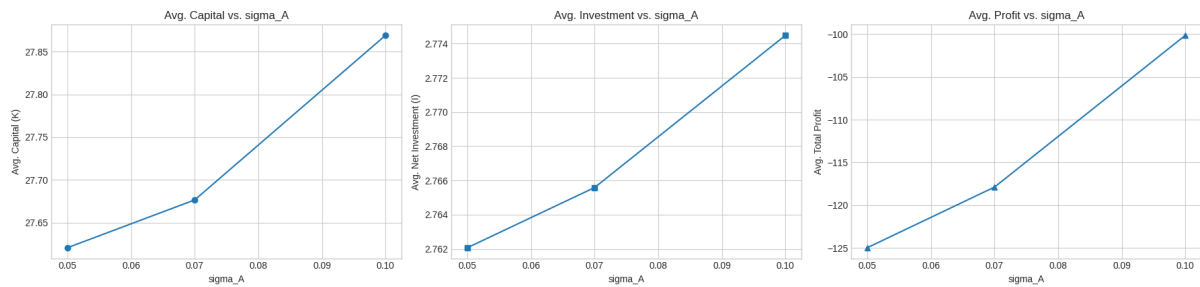
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Sensitivity Analysis: Impact of sigma\_A (Fixed  $\gamma = 10.0$ ,  $\delta = 0.10$ )



--- Discussion of Sensitivity and Comparison (Based on Output) ---

Sensitivity Analysis (Observed Patterns):

- Average Raw Material Cost (avg\_C<sub>M</sub>): [Discuss how K, I, Profit plots changed with avg\_C<sub>M</sub>]
- Output Elasticity of Capital (alpha): [Discuss how K, I, Profit plots changed with alpha]
- Output Elasticity of Labor (nu): [Discuss how K, I, Profit plots changed with nu]
- Productivity Shock StdDev (sigma<sub>A</sub>): [Discuss how K, I, Profit plots changed with sigma<sub>A</sub>, likely increasing volatility of outcomes]

Comparison of Summary Statistics (Observed):

StdDev\_VarInput: Data vs. Sim = 894.7570755973068 vs. 449.9914702041208

Avg\_InvOutRatio: Data vs. Sim = 0.06853068352229302 vs. 0.014622636706452342

Avg\_CapIncRatio (Sim only): 0.15592877866072055

VarRatio\_InvOut: Data vs. Sim = 0.013737517965383968 vs. 3.2643813045169695e-05

### Attempting 'Best Fit' (Parameter Tweaks):

[Based on the comparison, suggest parameter changes. E.g., 'If simulated investment volatility is too low compared to data ( $3.2643813045169695e-05 < 0.013737517965383968$ ), reducing gamma or increasing  $\sigma_A/\sigma_P$  could help. If simulated K is too high, increasing  $\text{avg\_}C_M$ ,  $w$ , or delta, or decreasing alpha might be considered.']

It's an iterative process. For example, if the simulated I/Y ratio is much lower than the data, we might need a combination of higher target capital (e.g., higher alpha) and perhaps a slightly lower delta (though delta also drives replacement). If simulated output is less volatile than data, increasing  $\sigma_A$  would be a first step.

The current parameter values ( $\alpha=0.3$ ,  $\nu=0.6$ ,  $\gamma=10$ ,  $\delta=0.1$ ,  $\sigma_A=0.07$  etc.) are common starting points but likely need adjustment to match specific dataset features.

A key challenge is that the firm-level data is cross-sectional and likely very heterogeneous, while the model is for a 'representative' firm over time. Matching moments requires careful thought about what the model represents.

### Optimisation Problem (Bellman Equation) (for both small and big firm)

Let:

$M_t$  be the quantity of raw materials and intermediate goods used in period  $t$

$p_M$  be the constant price per unit of these materials (we assume it's deterministic and constant for simplicity, but it could also be time-varying or stochastic).

$C_M(M_t) = p_M * M_t$  be the total cost of these materials.

### State Variables:

$K_t$ : Stock of physical capital at the beginning of period  $t$ .

$A_t$ : Productivity shock in period  $t$ .

$P_t$ : Price of investment goods for new capital in period  $t$ .

### Choice Variables in Period $t$ :

$x_t$ : Variable input (e.g., labor).

$M_t$ : Raw materials and intermediate goods.

$K_{t+1}$ : Capital stock for the next period (which determines investment  $I_t$ ).

### Other Given Parameters/Functions: $w_t$ :

Cost per unit of variable input  $x_t$ .

$R(A_t, K_t, x_t, M_t)$ : Revenue function, now also depending on  $M_t$

A common form could be Cobb-Douglas:  $R(A_t, K_t, x_t, M_t) = A_t K_t^\alpha x_t^\nu M_t^\mu$ ,  
where  $\alpha + \nu + \mu$  could be  $\leq 1$  (constant or decreasing returns to scale) or  $> 1$   
(increasing returns).

$\delta$ : Depreciation rate of capital.

$C(K_{t+1}, K_t)$ : Adjustment cost for capital, e.g.,  $(\gamma/2)(K_{t+1} - (1 - \delta)K_t)^2$

$\beta$ : Discount factor.

### The Optimisation Problem (Bellman Equation):

The firm chooses  $x_t, M_t, K_{t+1}$  to maximise the present discounted value of profits.

The value function  $V(A_t, K_t, p_t)$  can be written as:

$$V(A_t, K_t, p_t) = \max_{x_t, M_t, K_{t+1}} \left\{ \Pi_t + \beta \mathbb{E}_{A_{t+1}, p_{t+1} | A_t, p_t} [V(A_{t+1}, K_{t+1}, p_{t+1})] \right\}$$

where  $\Pi_t$  is the current period's profit, defined as:

$$\Pi_t = R(A_t, K_t, x_t, M_t) - w_t x_t - p_M M_t - p_t (K_{t+1} - (1 - \delta)K_t) - \frac{\gamma}{2} (K_{t+1} - (1 - \delta)K_t - \text{Inv}_0)^2$$

**Break down the terms in  $\Pi_t$**  :  $R(A_t, K_t, x_t, M_t)$ : Revenue from production.

$w_t x_t$  : Cost of variable input (e.g., labor).

$p_M M_t$  Cost of raw materials and intermediate goods.

$p_t (K_{t+1} - (1 - \delta)K_t)$ : Cost of gross investment.

Let  $\text{It}_{\text{gross}} = K_{t+1} - (1 - \delta)K_t$ . This is the expenditure on new capital goods needed to reach  $K_{t+1}$  from the depreciated capital stock  $(1 - \delta)K_t$ .

$(\gamma/2)(K_{t+1} - (1 - \delta)K_t - \text{Inv}_0)^2$  : Quadratic adjustment cost for capital.

### Bellman Equation

$$V(A_t, K_t, p_t) = \max_{x_t \geq 0, M_t \geq 0, K_{t+1} \geq 0} \left\{ \underbrace{R(A_t, K_t, x_t, M_t) - w_t x_t - p_M M_t}_{\text{Current Operating Profit}} - p_t (K_{t+1} - (1 - \delta)K_t) - \frac{\gamma}{2} (K_{t+1} - (1 - \delta)K_t)^2 + \underbrace{\beta \mathbb{E}_{A_{t+1}, p_{t+1} | A_t, p_t} [V(A_{t+1}, K_{t+1}, p_{t+1})]}_{\text{Net Investment and Adjustment Costs}} \right\}$$

Laws of motion for  $A_t$  and  $p_t$ :

$$\log A_{t+1} = \rho^A \log A_t + \epsilon^A_{t+1}$$

$$\log p_{t+1} = \rho^p \log p_t + \epsilon^p_{t+1}$$

Bellman Equation (Simplified Form, after optimising static choices):

First, define the maximised current operating profit:

$$\Pi^*(A_t, K_t, w_t, p_M) = \max_{x_t \geq 0, M_t \geq 0} \{R(A_t, K_t, x_t, M_t) - w_t x_t - p_M M_t\}$$

Then, the Bellman equation for the dynamic choice of  $K_{t+1}$  is:

$$V(A_t, K_t, p_t) = \max_{K_{t+1} \geq 0} \left\{ \underbrace{\Pi^*(A_t, K_t, w_t, p_M)}_{\text{Maximized Current Operating Profit}} - \underbrace{p_t (K_{t+1} - (1 - \delta)K_t) + \frac{\gamma}{2} (K_{t+1} - (1 - \delta)K_t)^2}_{\text{Net Investment and Adjustment Costs}} + \beta \mathbb{E}_{A_{t+1}, p_{t+1} | A_t, p_t} [V(A_{t+1}, K_{t+1}, p_{t+1})] \right\}$$

Then, the Bellman equation for the dynamic choice of  $K_{t+1}$  is:

$$V(A_t, K_t, p_t) = \max_{K_{t+1} \geq 0} \left\{ \underbrace{\Pi^*(A_t, K_t, w_t, p_M)}_{\text{Maximized Current Operating Profit}} - \underbrace{p_t (K_{t+1} - (1 - \delta)K_t) - \frac{\gamma}{2} (K_{t+1} - (1 - \delta)K_t)^2}_{\text{Net Investment and Adjustment Costs}} + \beta \mathbb{E}_{A_{t+1}, p_{t+1} | A_t, p_t} [V(A_{t+1}, K_{t+1}, p_{t+1})] \right\}$$

## **Anecdotal Evidence**

Based on my experience, the reason for choosing total annual cost of raw materials and intermediate goods in production for the key determinant is:

- Vital for cost minimisation and efficiency: The optimal usage and timing of intermediate inputs can significantly reduce the total costs and improve productivity for firms.
- Direct impact on profit margin: These cost usually take up a major percentage of the total cost for production of a firm, directly affecting the profits and hence, the firm's value function
- Strong connection with the supply chain and inventory decision of a firm: Firms can adjust their inventories strategy based on these costs, that will be well fitted in the model.

Reasons for other variables for not being included:

- Energy cost: Important but may be counted into intermediate goods if already included.
- Loan amount: In model where capital is optimally chosen, I think that borrowing is a consequence of investment, not an input that drives the decision.
- Salary: May be less flexible in the short run due to contracts or regulations by the gov.

## **Empirical Relationship between production input, debt, finances, etc.**

### **1. Debt and investment in capital (K)**

#### **Leverage reduces investment**

Fazzari, Hubbard, and Petersen (1988) find that firms with high leverage are more sensitive to internal funds (cash flow) when investing. This supports the idea of financial constraints as debt limits flexibility to invest freely.

Firms with low dividend payouts (proxy for financing constraints) are likely to exhibit high sensitivity of investment to cash flow. => Debt constraints limit capital investment.

#### **Debt can enhance investment in some context**

In well-functioning capital markets, debt can be growth-enhancing by enabling firms to invest more than they could with internal funds alone. Leverage may improve efficiency by disciplining managers (agency theory).

### **2. Debt and output**

Access to affordable debt can boost output of firms, especially in growing firms. But in constrained environments, even productive firms may underproduce due to limited access to credit.

Midrigan and Xu (2014) show that financial frictions in developing countries lead to misallocation, therefore productive firms can't borrow enough to fully scale output.

Banerjee and Duflo (2014) stated that when Indian firms gain access to subsidised credit, they increase output and input usage.

### **3. Debt and adjustment costs**

When debt interacts with capital adjustment costs, it can magnify the impact of shocks. High-debt firms may delay or smooth investment to avoid exacerbating financial strain.

Cooper and Haltiwanger (2006) find that capital adjustment is lumpy and costly, and such dynamics are influenced by access to finance.

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