MODULE *TicTacToe*

EXTENDS *Integers*, *Sequences*

VARIABLES *turnState*, *board*

CONSTANTS *PlayerCount*, *BoardSize*

$$
\begin{array}{ll}
Index & \triangleq\ 0\,..\,(BoardSize - 1) \\
TurnState & \triangleq\ 0\,..\,(PlayerCount - 1) \\
First & \triangleq\ 0 \\
Empty & \triangleq\ -1 \\
CellState & \triangleq\ TurnState \cup \{Empty\} \\
CellIndex & \triangleq\ [x : Index,\ y : Index] \\
Board & \triangleq\ [CellIndex \to CellState] \\
EmptyBoard & \triangleq\ [c \in CellIndex \mapsto Empty] \\
NonZeroNat & \triangleq\ Nat \setminus \{0\}
\end{array}
$$

$$
\begin{array}{l}
TypeOK\ \triangleq \\
\quad \land \quad PlayerCount \in NonZeroNat \\
\quad \land \quad BoardSize \in NonZeroNat \\
\quad \land \quad turnState\ \in TurnState \\
\quad \land \quad board \in Board
\end{array}
$$

$$
\begin{array}{l}
BeginNewGame\ \triangleq \\
\quad \land\ turnState' = First \\
\quad \land\ board' = EmptyBoard
\end{array}
$$

$$
\begin{array}{l}
TakeTurn\ \triangleq \\
\quad \land\ turnState' = (turnState + 1)\%PlayerCount \\
\quad \land\ \exists\, c \in CellIndex : \\
\qquad \land\ board[c] = Empty \\
\qquad \land\ board' = [board\ \text{EXCEPT}\ ![c] = turnState]
\end{array}
$$

$$
\begin{array}{l}
GameOver\ \triangleq \\
\quad \lor\ \forall\, c \in CellIndex\ : board[c] \neq Empty \\
\quad \lor\ \exists\, t \in TurnState : \\
\qquad \lor\ \exists\, x \in Index\ : \forall\, y \in Index : board[[x \mapsto x,\ y \mapsto y]] = t \\
\qquad \lor\ \exists\, y \in Index\ : \forall\, x \in Index : board[[x \mapsto x,\ y \mapsto y]] = t \\
\qquad \lor\ \forall\, i \in Index\ : board[[x \mapsto i,\ y \mapsto BoardSize - 1 - i]] = t \\
\qquad \lor\ \forall\, i \in Index\ : board[[x \mapsto i,\ y \mapsto i]] = t
\end{array}
$$

$$Init\ \triangleq\ turnState = First \land board = EmptyBoard$$

$$Next\ \triangleq\ \text{IF}\ GameOver\ \text{THEN}\ BeginNewGame\ \text{ELSE}\ TakeTurn$$