CS3800: Theory of Computation — Summer II '22 — Drew van der Poel

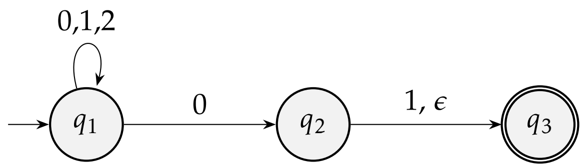
Homework 2
Due Friday, July 22 at 11:59pm via Gradescope

Name:
Collaborators:

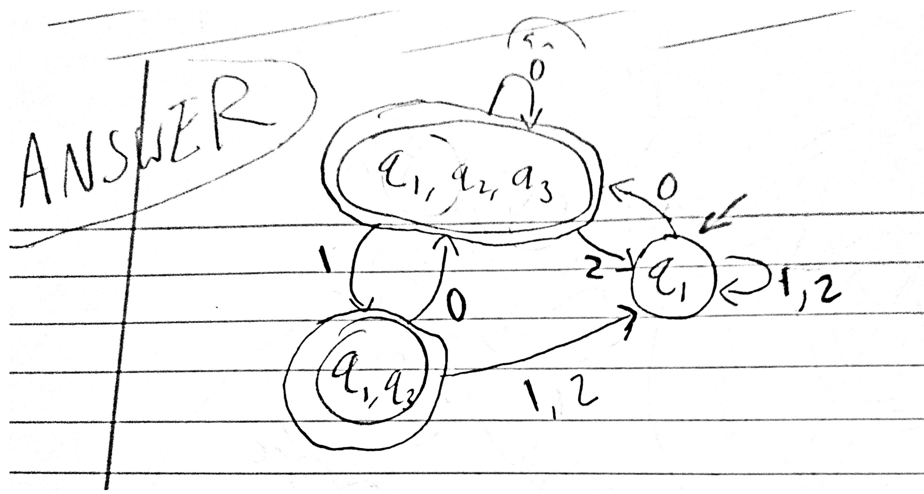
- Make sure to put your name on the first page. If you are using the LaTeX template we provided, then you can make sure it appears by filling in the `yourname` command.
- This assignment is due Friday, July 22 at 11:59pm via Gradescope. No late assignments will be accepted. Make sure to submit something before the deadline.
- Solutions must be typeset. If you need to draw any diagrams, you may draw them by hand as long as they are embedded in the PDF. I recommend using the source file for this assignment to get started.
- I encourage you to work with your classmates on the homework problems. *If you do collaborate, you must write all solutions by yourself, in your own words.* Do not submit anything you cannot explain. Please list all your collaborators in your solution for each problem by filling in the `yourcollaborators` command.
- Finding solutions to homework problems on the web, or by asking students not enrolled in the class is strictly forbidden.

Problem 1. *Converting NFAs to DFAs (*6 points*)*

Convert the following NFA to a DFA. Your DFA should be as simple as possible (e.g. any states which cannot be reached from the initial state should be removed). Note $\Sigma = \{0, 1, 2\}$.



Solution:



WORK FOR PROBLEM ON NEXT PAGE

δ	0	1	2	ε
q_1	$\{q_1,q_2\}$	$\{q_1\}$	$\{q_1\}$	$\emptyset$
q_2	$\emptyset$	$\{q_3\}$	$\emptyset$	$\{q_3\}$
q_3	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

$E(q_1) = \{q_1\}$

$E(q_2) = \{q_2, q_3\}$

$E(q_3) = \{q_3\}$

	0	1	2
q_1, q_2, q_3	q_1, q_2, q_3	q_1, q_3	q_1
$\cancel{q_1, q_2}$	q_1, q_2, q_3	q_1, q_3	q_1
q_1, q_3	q_1, q_2, q_3	q_1	q_1
$\cancel{q_2, q_3}$	$\emptyset$	q_3	$\emptyset$
q_1	q_1, q_2, q_3	q_1	q_1
$\cancel{q_2}$	$\emptyset$	$\{q_3\}$	$\emptyset$
$\cancel{q_3}$	$\emptyset$	$\emptyset$	$\emptyset$
$\cancel{\emptyset}$	$\emptyset$	$\emptyset$	$\emptyset$

$E(\delta(q_1,q_2,q_3),0) = E(\delta(q_1,0)) \cup E(\delta(q_2,0))$
$\cup E(\delta(q_3,0)) = E(q_1,q_2) \cup \emptyset \cup \emptyset$
$= \{q_1, q_2, q_3\}$

$E(\delta(q_1,q_2,q_3),1) = E(\delta(q_1,1) \cup \delta(q_2,1) \cup \delta(q_3,1)$
$E(q_1, q_3, \emptyset) = \{q_1, q_3\}$

$E(\delta(q_1,q_2,q_3),2) = E(\delta(q_1,2) \cup \delta(q_2,2) \cup \delta(q_3,2)) =$
$E(q_1, \emptyset, \emptyset) =$
$\{q_1\}$

$E(\delta(q_1,q_2),0) = E(\delta(q_1,0) \cup \delta(q_2,0))$
$E(\{q_1,q_2\} \cup \emptyset)$
$E(\{q_1,q_2\}) = \{q_1,q_2,q_3\}$

$E(\delta(q_1,q_2),1) = E(\delta(q_1,1) \cup \delta(q_2,1))$
$= E(q_1 \cup q_3)$
$= \{q_1, q_3\}$

$E(\delta(q_1,q_2),2) = E(\delta(q_1,2) \cup \delta(q_2,2))$
$E(q_1 \cup \emptyset) = q_1$

$E(\delta(q_1,q_3),0) = E(\delta(q_1,0) \cup \delta(q_3,0))$
$= E(\{q_1,q_2\} \cup \emptyset)$
$= \{q_1,q_2,q_3\}$

$E(\delta(q_1,q_3),1) = E(\delta(q_1,1) \cup \delta(q_3,1))$
$E(q_1 \cup \emptyset) = \{q_1\}$

$E(\delta(q_1,q_3),2) = E(\delta(q_1,2))$
$E(q_1) = \{q_1\}$

$E(\delta(q_2,q_3),0) = E(\delta(q_2,0) \cup \delta(q_3,0))$
$= \emptyset$

$E(\delta(q_2,q_3),1) = E(q_2,1)) = E(q_3) = \{q_3\}$

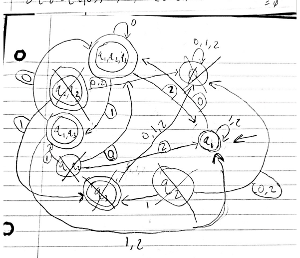
$E(\delta(q_2,q_3),2) = \emptyset$

$E(\delta(q_1),0) = E(q_1,q_2) = \{q_1,q_2,q_3\}$

$E(\delta(q_1),1) = E(q_1) = \{q_1\}$

$E(\delta(q_1),2) = E(q_1) = \{q_1\}$

$E(\delta(q_2,0)) = \emptyset$, $E(\delta(q_2,1)) = q_3$ $E(\delta(q_2,2)) = \emptyset$



Problem 2. *Regular Expressions (*6 points*)*

Give a regular expression for the language of words over $\Sigma = \{0, 1\}$ which have an equal number of copies of 01 and 10 as substrings (note: these copies need not be disjoint). Show why your regular expression accepts the words 10101, 0101110, 111, and 111111011. Explain why it does not generate 0101. Provide brief justification for why your regular expression works as desired.

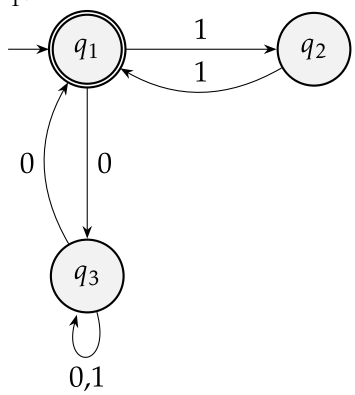
Solution:

((01*0) ∪ (10*1))*

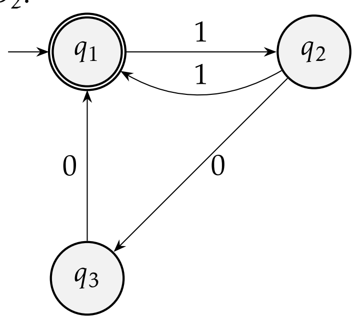
Problem 3. *NFA to RegEx (*9 points*)*

Consider the following NFAs. For each, convert it to a regular expression. Show all intermediate steps when building and simplifying the GNFA. You can omit transitions labeled with $\emptyset$.

D_1:

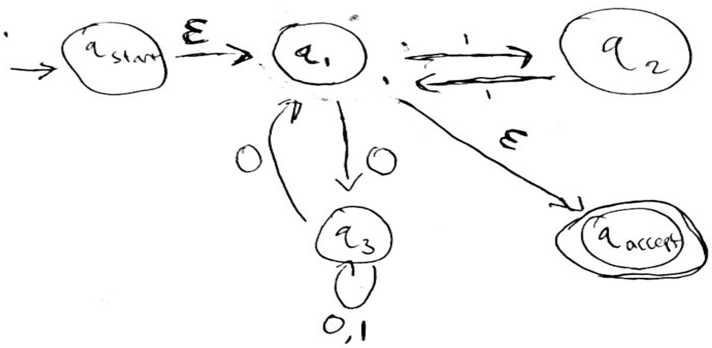


D_2:

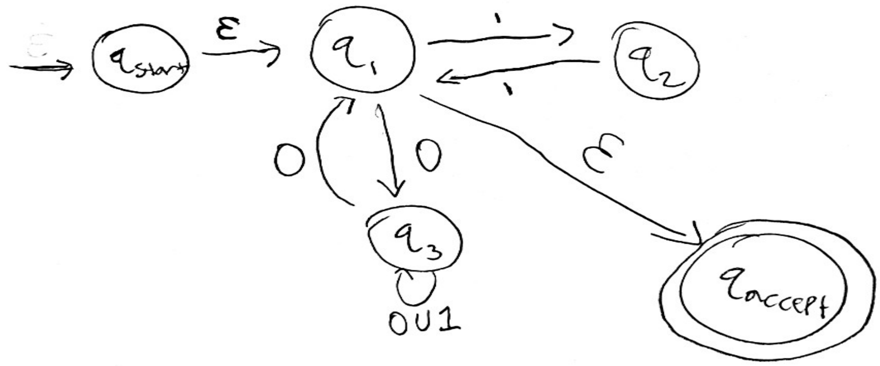


Solution:
D1: $(11 \cup 0(0 \cup 1)^*0)^*$
D2: $(11 \cup 100)^*$
1st image is D1, 2nd image is D2

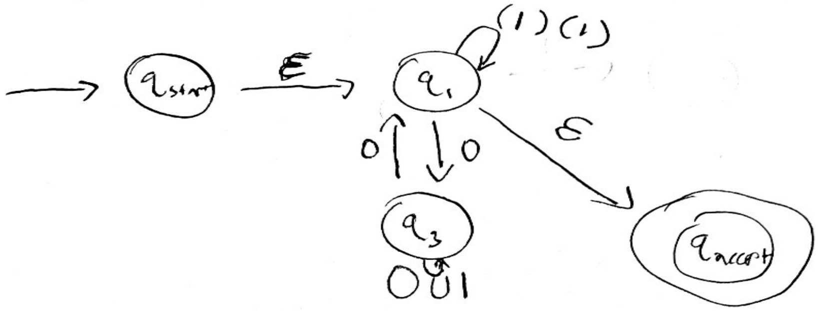
1.


q_{start}
ε
q_1
1
1
q_2
0
0
ε
q_3
q_{accept}
0,1

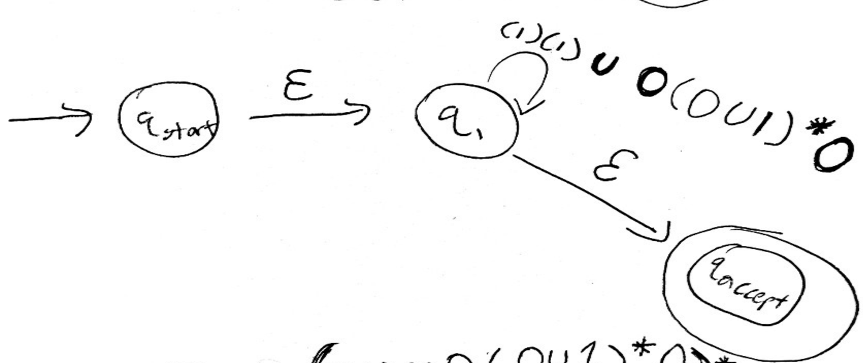
2.


q_{start}
ε
q_1
1
1
q_2
0
0
ε
q_3
q_{accept}
$0 \cup 1$

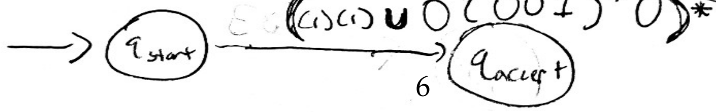
3.


(1)(1)
q_{start}
ε
q_1
0
0
ε
q_3
q_{accept}
$0 \cup 1$

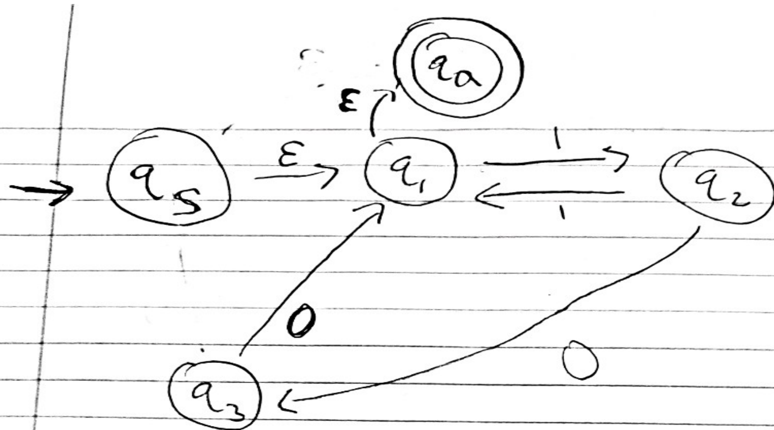
4.


$(1)(1) \cup 0(0\cup 1)^*0$
q_{start}
ε
q_1
ε
q_{accept}

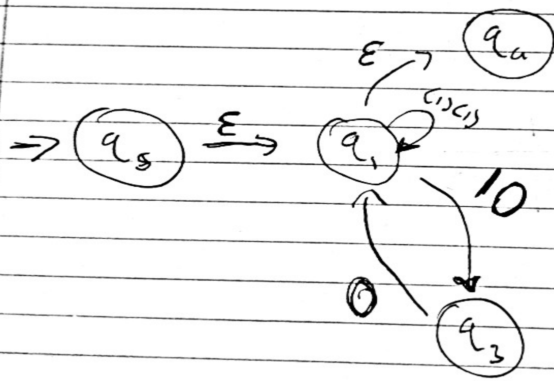
5.


$((1)(1) \cup 0(0\cup 1)^*0)^*$
q_{start}
q_{accept}

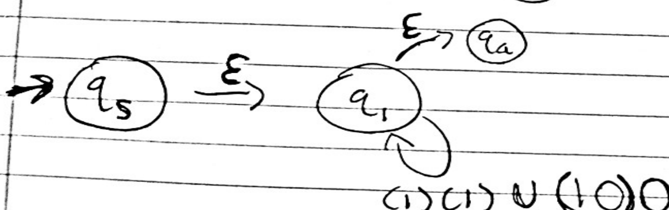
1.


qa
ε
qs
ε
q1
1
1
q2
0
0
q3

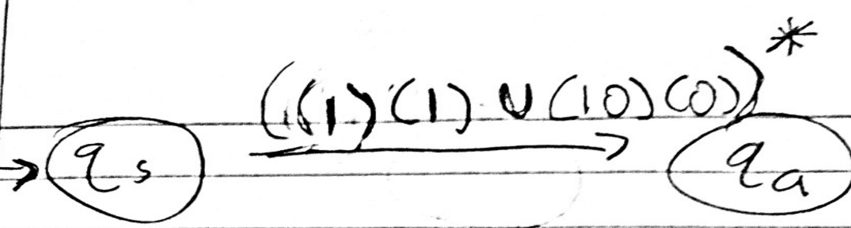
2.


ε
qa
qs
ε
q1
(1)(1)
10
0
q3

3.


ε
qa
qs
ε
q1
(1)(1) ∪ (10)0

4.


qs
((1)(1) ∪ (10)(0))*
qa

Problem 4. *Non-Regular Languages (9 points)*

For each of the following languages over $\Sigma = \{g, q\}$, state whether or not it is regular. If it is, give a regular expression that describes it. If not, prove why using the Pumping Lemma. Note that we assume $\mathbb{N}$ (the natural numbers) to be the non-negative integers ($\{0, 1, 2, \dots\}$).

(a) **[3 pts.]** $L_1 = \{w | w \in \{0,1\}^*;$number of 00 substrings is the same as the number of 11 substrings$\}$

Solution:
Assume true, that L_1 is a regular language so the Pumping Lemma applies to L_1. Let P be the pumping length. Let $S = (00)^P(11)^P$. We know that via the Pumping Lemma, $|xy| \leq P$ which means that y must consist of some number of pairs of only zeroes for string S. However, we also know that the Pumping Lemma states $xy^iz \subseteq L_1$ which is not true when $i = 3$ for S since $xyyyz \not\subseteq L_1$ ($xy = (00)^P$ and $z = (11)^P$ and we know that y must only consist of pairs of zeroes which means that adding more ys means that there will be more pairs of zeroes thans ones). Thus, by contradiction, L_1 does not uphold the Pumping Lemma which means that L_1 is not regular.

(b) **[3 pts.]** $L_2 = \{0^n 1^m | n, m \geq 5; n, m \in \mathbb{N}\}$

Solution:

(00000)(0)*(11111)(1)*

(c) **[3 pts.]** $L_3 = \{w | w \in \{0,1\}^*;$the length of w is odd and contains a 1 as the middle character$\}$

Solution:
Assume true, that L_3 is a regular language so the Pumping Lemma applies to L_3. Let P be the pumping length. Let $S = (00)^P 1 (00)^P$. We know that via the Pumping Lemma, $|xy| \leq P$, which means that y must consist of some number of pairs of only zeroes for string S. However, we also know that the Pumping Lemma states $xy^iz \subseteq L_3$ which is not true when $i = 3$ for S since adding any extra zeroes on the left side of S will result in the left side having more 0s than the right side, thus, not preserving the 1 being in the middle criteria. Thus, by contradiction, L_3 does not uphold the Pumping Lemma which means that L_3 is not regular.