CS3800: Theory of Computation — Summer II '22 — Drew van der Poel

Homework 3
Due Friday, July 29 at 11:59pm via Gradescope

Name:
Collaborators:

- Make sure to put your name on the first page. If you are using the LaTeX template we provided, then you can make sure it appears by filling in the `yourname` command.
- This assignment is due Friday, July 29 at 11:59pm via Gradescope. No late assignments will be accepted. Make sure to submit something before the deadline.
- Solutions must be typeset. If you need to draw any diagrams, you may draw them by hand as long as they are embedded in the PDF. I recommend using the source file for this assignment to get started.
- I encourage you to work with your classmates on the homework problems. *If you do collaborate, you must write all solutions by yourself, in your own words.* Do not submit anything you cannot explain. Please list all your collaborators in your solution for each problem by filling in the `yourcollaborators` command.
- Finding solutions to homework problems on the web, or by asking students not enrolled in the class is strictly forbidden.

Problem 1. *Context-Free Grammars (*7 points*)*

In the following problems, the alphabet $\Sigma = \{a, b\}$. Give a context-free grammar for each of the following languages.

(a) **[3 pts.]** $L_a = \{a^n b^n | n > 1$ is not a multiple of 3$\}$

Show how to generate *aaaabbbb* with your grammar.

Solution:

A → aabb | aaaAbbb | aaAbb

To generate *aaaabbbb*: A → aaAbb → aaaabbbb

(b) **[4 pts.]** $L_a = \{w | w$ has twice as many a's as b's$\}$

Show how to generate *aababaaab* and *aaaabb* with your grammar.

Solution:

A → AA | aab | aba | baa | aa | bA ε B →

To generate *aababaaab*: A → AA → aabAA → aababaAA → aababaaab

To generate *aaaabb*: A → aaAb → aaAAb → aaaaAbb → aaaabb

Problem 2. *CFGs and PDAs* (7 points)

Consider the following context-free grammar G:

$S \rightarrow aWb|bWa$

$W \rightarrow aW|bW|\epsilon$

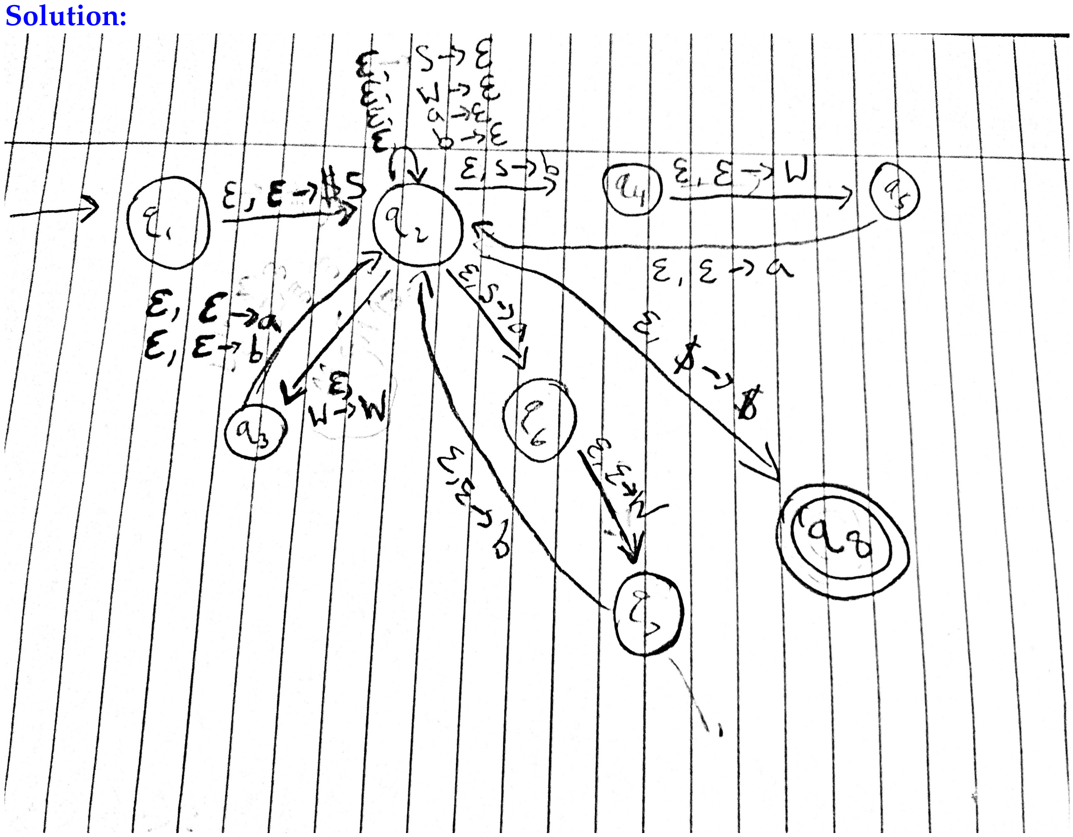
(a) **[2 pts.]** Describe the set of strings which can be generated by G.

Solution:

The set of all strings in which the first input is a and the last input is b or the first input is b and the last input is a.

(b) **[5 pts.]** Give a PDA which recognizes the language given by grammar G. You should show all necessary states for "guessing" rule S, but can use shorthand otherwise. Specify Σ and Γ.

Solution:



$\Sigma = \{a, b\}$

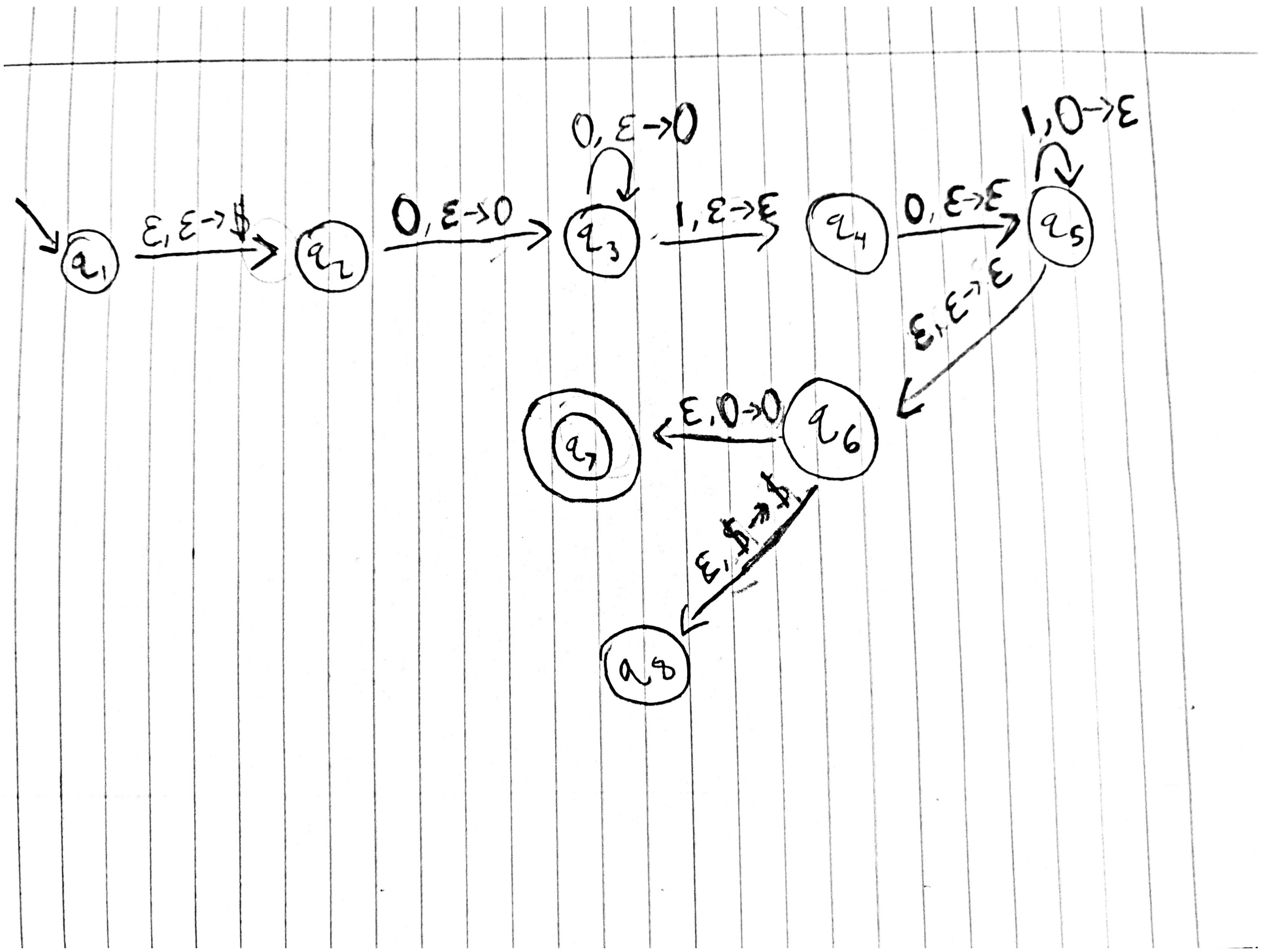
$\Gamma = \{S, W, a, b\}$

Problem 3. *PDAs (*8 points*)*

Consider $\Sigma = \{0,1\}$ and language $L = \{0^n 101^m | n > m; n, m \in \mathbb{N}\}$.

Show that L is context-free by giving a PDA which recognizes it. You should give a complete PDA. Your machine should accept strings 00000001011, 010, 00101, and 00000010, but not 10 0010111, or 0101 . Explain why it accepts 00101 and why it does not accept 0101.

Solution:



00101: We start at $q1$, and epsilon transition while initializing the stack to $q2$. At $q2$, we read the first 0 and push it onto the stack to transition to $q3$. At $q3$, we read the next 0 and push it onto the stack, staying at $q3$. Again, at $q3$, the next input is 1, so we take the 1 transition to $q4$ while pushing nothing onto the stack. At $q4$, our next input is 0 so we transition to $q5$ and push nothing to the stack. At $q5$ we break into two branches, one where read the last input, 1, stay at $q5$ and remove a 0 off of the stack or we can take the epsilon transition to $q7$: If we take the epsilon transition, then we would end up at $q6$ and then since the top of our stack is a 0, we can take the epsilon transition to $q7$. Since in this case we still have a 1 to read and there are no transitions in $q7$ for the 1 then this branch fails. If we take the $q5$ transition and remove a 0 off of the stack, then we can take the transition $q6$ and then since the top of our stack is still a 0, we can take the epsilon transition to $q7$. At $q7$, since we are finished with reading the string and we are in an accept state, the string is accepted.

0101: We start at $q1$, and epsilon transition while initializing the stack to $q2$. At $q2$, we read the first 0 and push it onto the stack to transition to $q3$. At $q3$ we read the 1 to transition to $q4$ and then the 0 to transition to $q5$, pushing nothing onto the stack in both transitions. At $q5$, we once again break into two branches: one where we take the epsilon branch and one where we stay at $q5$, read the 1, and pop a 0 off of the stack. For the same reason, taking the epsilon transition would result in the branch rejecting at $q7$. In the other branch, after we read the 1 and popped the top 0 off of the stack, then we can take the epsilon transition to $q6$. At $q6$, since our stack is now empty, we take the transition to $q8$ where the string will be rejected.

Problem 4. *Non Context-Free Languages (*4+4=8 points*)*

(a) Prove that language $L = \{w | w \in \{a, b, c\}^*$ and the number of a's is equal to the number of b's and the number of a's is greater than the number of c's$\}$ is not context-free.

Solution:

(b) Prove that language $L = \{a^l b^{l^2} | l \in \mathbb{N}\}$ is not context-free.

Solution: