# CS3800: Theory of Computation — Summer II '22 — Drew van der Poel

Homework 2

Due Friday, July 22 at 11:59pm via Gradescope

Name:
Collaborators:

- Make sure to put your name on the first page. If you are using the LaTeX template we provided, then you can make sure it appears by filling in the `yourname` command.

- This assignment is due Friday, July 22 at 11:59pm via Gradescope. No late assignments will be accepted. Make sure to submit something before the deadline.

- Solutions must be typeset. If you need to draw any diagrams, you may draw them by hand as long as they are embedded in the PDF. I recommend using the source file for this assignment to get started.

- I encourage you to work with your classmates on the homework problems. *If you do collaborate, you must write all solutions by yourself, in your own words.* Do not submit anything you cannot explain. Please list all your collaborators in your solution for each problem by filling in the `yourcollaborators` command.

- Finding solutions to homework problems on the web, or by asking students not enrolled in the class is strictly forbidden.

**Problem 1.** *Converting NFAs to DFAs (6 points)*

Convert the following NFA to a DFA. Your DFA should be as simple as possible (e.g. any states which cannot be reached from the initial state should be removed). Note $\Sigma = \{0, 1, 2\}$.
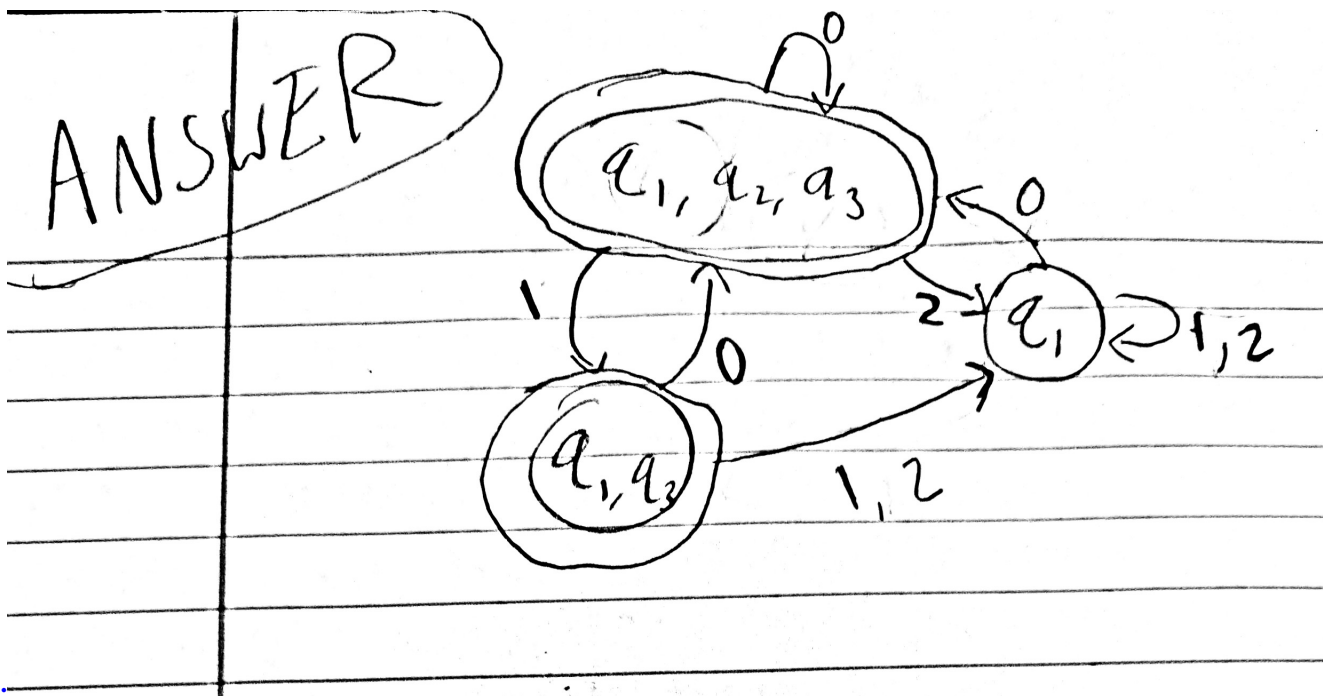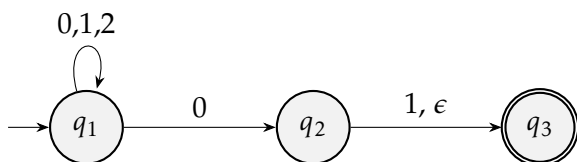


<span style="color:blue">**Solution:**</span>
WORK FOR PROBLEM ON NEXT PAGE

| $\delta$ | 0 | 1 | 2 | $\varepsilon$ |
|---|---|---|---|---|
| $q_1$ | $\{q_1\}$ | $\{q_2\}$ | $\{q_2\}$ | $\emptyset$ |
| $q_2$ | $\emptyset$ | $\{q_3\}$ | $\emptyset$ | $\{q_3\}$ |
| $q_3$ | $\emptyset$ | $\emptyset$ | $\emptyset$ | $\emptyset$ |

$E(q_1) = \{q_1\}$

$E(q_2) = \{q_2, q_3\}$

$E(q_3) = \{q_3\}$

| | 0 | 1 | 2 |
|---|---|---|---|
| $q_1, q_2, q_3$ | $q_1, q_2, q_1$ | $q_1, q_3$ | $q_1$ |
| $q_1, q_2$ | $q_1, q_2, q_3$ | $q_1, q_3$ | $q_1$ |
| $q_1, q_3$ | $q_1, q_2, q_1$ | $q_1$ | $q_1$ |
| $q_2, q_3$ | $\emptyset$ | $q_3$ | $\emptyset$ |
| $q_1$ | $q_1, q_2, q_3$ | $q_1$ | $q_1$ |
| ~~$q_2$~~ | $\emptyset$ | $\{q_3\}$ | $\emptyset$ |
| ~~$q_3$~~ | $\emptyset$ | $\emptyset$ | $\emptyset$ |
| ~~$\emptyset$~~ | $\emptyset$ | $\emptyset$ | $\emptyset$ |

$E(\delta(q_1,q_2,q_3), 0) = E(\delta(q_1,0)) \cup E(\delta(q_2,0))$
$\qquad \cup E(\delta(q_3,0)) = E(q_1,1) \cup \emptyset$
$\qquad = \{q_1, q_2, q_3\}$

$E(\delta(q_1, q_2, q_3), 1) = E(\delta(q_1,1)) \cup \delta(q_2,1) \cup$
$\qquad \delta(q_3,1)$
$\qquad E(q_1, q_3, \emptyset) = \{q_1, q_3\}$

$E(\delta(q_1, q_2, q_3), 2) = E(\delta(q_1,2)) \cup \delta(q_2,2) \cup$
$\qquad \delta(q_3,2) =$
$\qquad E(q_1, \emptyset, \emptyset) =$
$\qquad \{q_1, q_3\}$

$E(\delta(q_1, q_2), 0) = E(\delta(q_1,0) \cup \delta(q_2,0))$
$\qquad E(q_1, q_2, q_3 \cup \emptyset)$
$\qquad E(\{q_1, q_2\}) = \{q_1, q_2, q_3\}$

$E(\delta(q_1, q_2), 1) = E(\delta(q_1,1) \cup \delta(q_2,1))$
$\qquad = E(q_2 \cup q_1)$
$\qquad = \{q_1, q_3\}$

$E(\delta(q_1, q_2), 2) = E(\delta(q_1,2) \cup \delta(q_2,2))$
$\qquad E(q_1 \cup \emptyset) = q_1$

$E(\delta(q_1, q_3), 0) = E(\delta(q_1,0) \cup \delta(q_3,0))$
$\qquad = E(\{q_1, q_3 \cup \emptyset\})$
$\qquad = \{q_1, q_2, q_3\}$

$E(\delta(q_1, q_3), 1) = E(\delta(q_1,1) \cup \delta(q_3,1))$
$\qquad E(q_1 \cup \emptyset) = \{q_1\}$

$E(\delta(q_1, q_3), 2) = E(\delta(q_1,2))$
$\qquad E(q_1) = \{q_1\}$

$E(\delta(q_2, q_3), 0) = E(\delta(q_2,0) \cup \delta(q_3,0))$
$\qquad = \emptyset$

$E(\delta(q_2, q_3), 1) = E(q_1, 1) = E(q_3) = \{q_3\}$

$E(\delta(q_2, q_3), 2) = \emptyset$

$E(\delta(q_1), 0) = E(q_1, q_1) = \{q_1, q_2, q_3\}$
$E(\delta(q_1), 1) = E(q_2) = \{q_1\}$
$E(\delta(q_1), 2) = E(q_2) = \{q_1\}$
$E(\delta(q_2), 0) = \emptyset, \quad E(\delta(q_2,1)) = q_3, \quad E(\delta(q_2,2))$

**Problem 2.** *Regular Expressions (6 points)*

Give a regular expression for the language of words over $\Sigma = \{0, 1\}$ which have an equal number of copies of 01 and 10 as substrings (note: these copies need not be disjoint). Show why your regular expression accepts the words 10101, 0101110, 111, and 111111011. Explain why it does not generate 0101. Provide brief justification for why your regular expression works as desired.
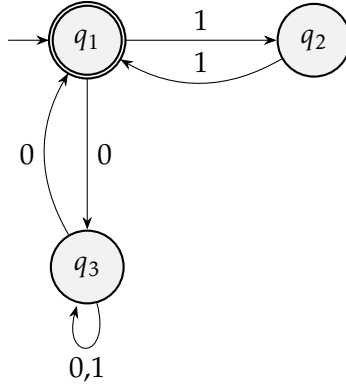
<span style="color:blue">**Solution:**</span>
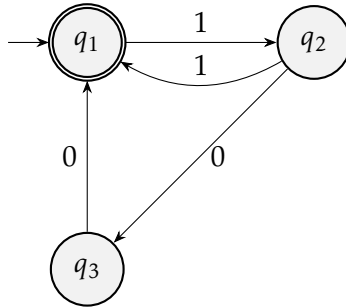$((01^*0) \cup (10^*1))^*$

**Problem 3.** *NFA to RegEx (9 points)*

Consider the following NFAs. For each, convert it to a regular expression. Show all intermediate steps when building and simplifying the GNFA. You can omit transitions labeled with $\emptyset$.
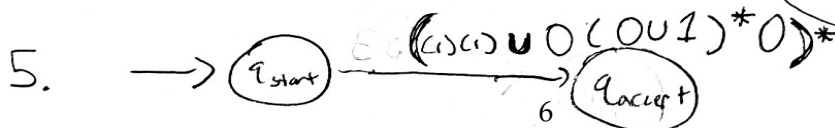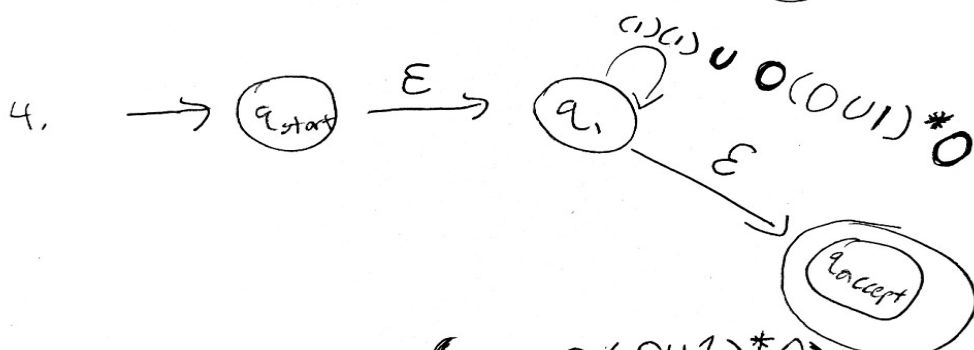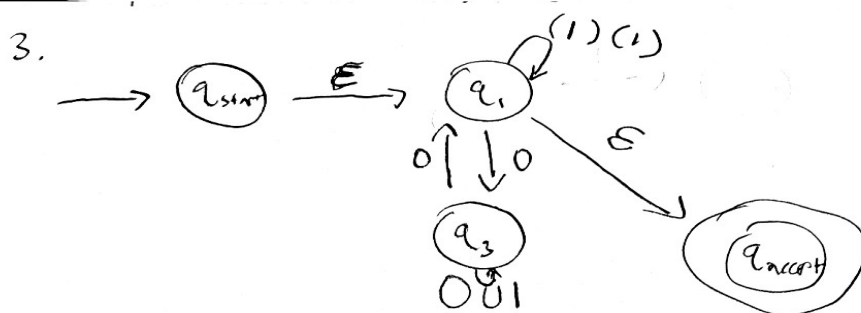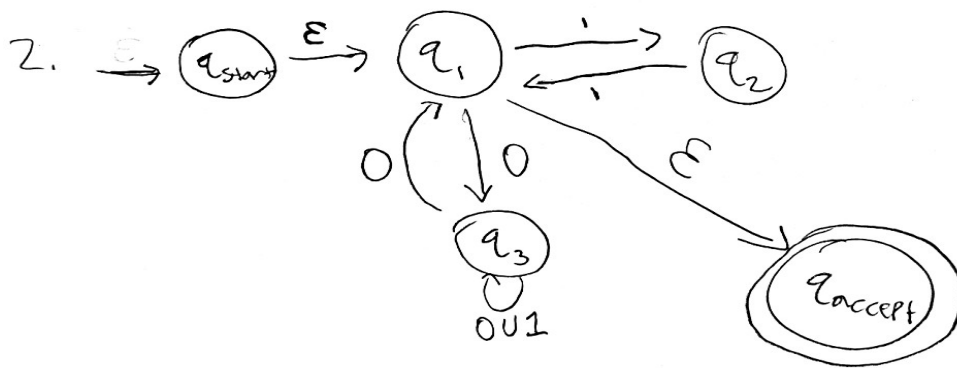
$D_1$:



$D_2$:

**Solution:**
D1: $(11 \cup 0(0 \cup 1)^*0)^*$
D2: $(11 \cup 100)^*$
1st image is D1, 2nd image is D2

**1.**

$q_{start} \xrightarrow{\varepsilon} q_1 \underset{1}{\overset{1}{\rightleftarrows}} q_2$

$q_1 \xrightarrow{\varepsilon} q_{accept}$

$q_1$ with $0$ loop to $q_3$, $0$ back to $q_1$

$q_3$ self-loop $0,1$

**2.**

$q_{start} \xrightarrow{\varepsilon} q_1 \underset{1}{\overset{1}{\rightleftarrows}} q_2$

$q_1 \xrightarrow{\varepsilon} q_{accept}$

$q_1$ with $0$ and $0$ to $q_3$
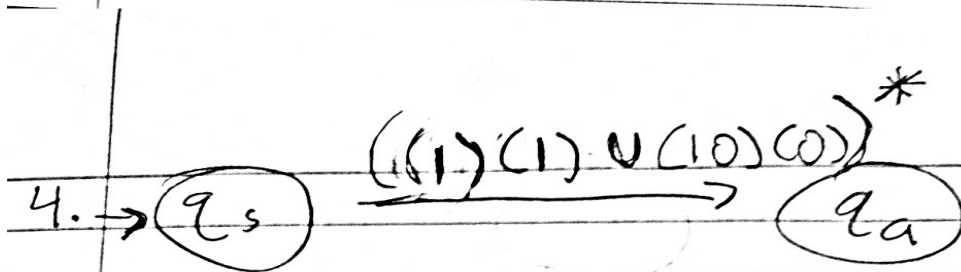
$q_3$ self-loop $0 \cup 1$

**3.**

$q_{start} \xrightarrow{\varepsilon} q_1$ self-loop $(1)(1)$

$q_1 \xrightarrow{\varepsilon} q_{accept}$

$q_1$ with $0$ and $0$ to $q_3$

$q_3$ self-loop $0 \cup 1$

**4.**

$q_{start} \xrightarrow{\varepsilon} q_1$ self-loop $(1)(1) \cup 0(0 \cup 1)^* 0$

$q_1 \xrightarrow{\varepsilon} q_{accept}$

**5.**

$q_{start} \xrightarrow{((1)(1) \cup 0(0 \cup 1)^* 0)^*} q_{accept}$

6

1.



$q_a$

$\varepsilon$

$q_s \xrightarrow{\varepsilon} q_1 \xrightarrow{1} q_2$

$q_1 \xleftarrow{1} q_2$

$0$

$q_3 \longleftarrow$

2.

$q_a$

$\varepsilon \nearrow$

$(1)(1)$

$q_s \xrightarrow{\varepsilon} q_1$

$10$

$0$

$q_3$

3. $\rightarrow q_s \xrightarrow{\varepsilon} q_1 \xrightarrow{\varepsilon} q_a$

$(1)(1) \cup (10)0$

4. $\rightarrow q_s \xrightarrow{((1)(1) \cup (10)(0))^{*}} q_a$

7
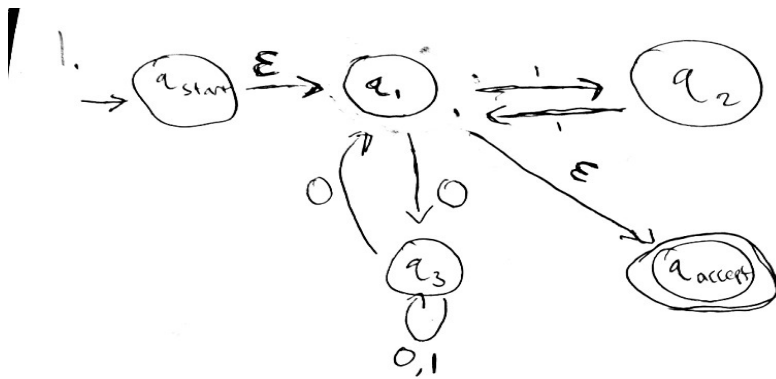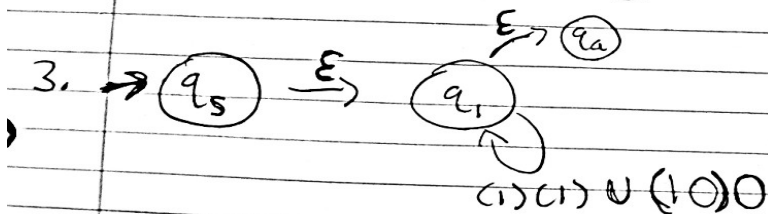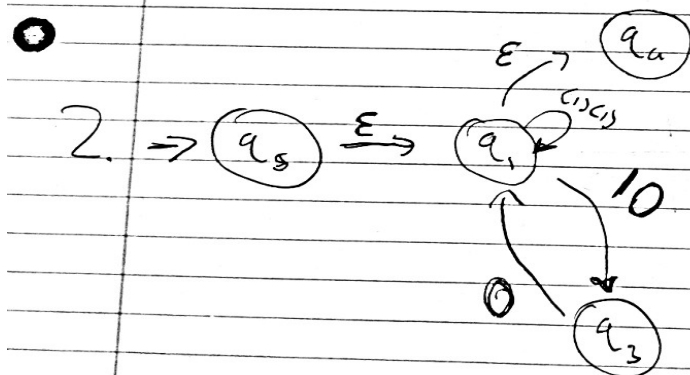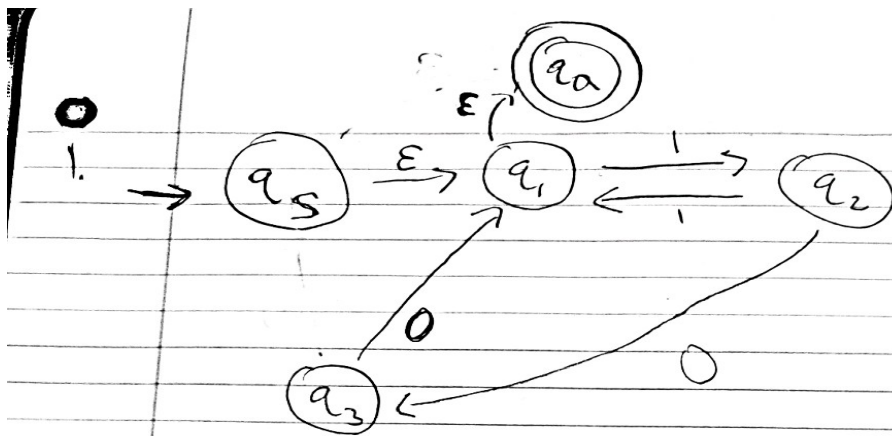
**Problem 4.** *Non-Regular Languages (9 points)*

For each of the following languages over $\Sigma = \{g, q\}$, state whether or not it is regular. If it is, give a regular expression that describes it. If not, prove why using the Pumping Lemma. Note that we assume $\mathbb{N}$ (the natural numbers) to be the non-negative integers ($\{0, 1, 2, \dots\}$).

(a) **[3 pts.]** $L_1 = \{w | w \in \{0, 1\}^*; \text{number of 00 substrings is the same as the number of 11 substrings}\}$

   **Solution:**

(b) **[3 pts.]** $L_2 = \{0^n 1^m | n, m \geq 5; n, m \in \mathbb{N}\}$

   **Solution:**

(c) **[3 pts.]** $L_3 = \{w | w \in \{0, 1\}^*; \text{the length of w is odd and contains a 1 as the middle character}\}$

   **Solution:**