Worst-Case Analysis and Time Complexity in Insertion Sort

1. Filling in the Summation

- When analyzing an algorithm like insertion sort, you derive the summation to represent the total number of operations (e.g., comparisons or shifts) in the worst-case scenario.
- For insertion sort, the worst-case occurs when the array is sorted in reverse order, requiring the maximum number of shifts for each element.

2. Worst-Case Analysis and Summation

- The summation you derive, such as:

$$\sum_{j=2}^{n} j = \frac{n(n+1)}{2} - 1$$

  represents the total number of operations in the worst case.
- This sum grows quadratically as n increases because the highest order term in the expansion is n^2.

3. Quadratic Function and Time Complexity

- The expression $an^2 + bn + c$ represents the total number of operations as a function of n, where:
  - a, b, and c are constants that depend on the specific costs of the operations.
  - The n^2 term dominates as n becomes large.
- Therefore, the time complexity is considered $O(n^2)$, meaning that the algorithm's running time increases quadratically with the size of the input array n.

4. Asymptotic Notation ($O(n^2)$)

- The $O(n^2)$ notation is used to describe the upper bound of the time complexity in the worst-case scenario.
- It ignores lower-order terms (like bn and c) and constant factors because, asymptotically, the n^2 term dominates as n becomes large.

Putting It Together

- **Insertion Sort's Worst-Case**: The worst-case time complexity for insertion sort is $O(n^2)$, which means that the number of operations grows quadratically with the size of the input.
- **Why Quadratic**: Because the total number of operations can be expressed as a quadratic function $an^2 + bn + c$, the n^2 term dominates for large n, leading to $O(n^2)$.

Summary

- **Summation**: Used to calculate the total number of operations in the algorithm.
- **Quadratic Function**: Represents the worst-case scenario.
- **Time Complexity**: $O(n^2)$, indicating the running time grows quadratically with the input size.