HW 3 **Prof. Caldwell**
Due: 7 September 2006 **COSC 3015**

Recall from class that functions are equal if and only if they are equal on all inputs (this equality is called extensionality.)

Definition 0.1. (extensionality) If $f, g \in A \to B$,

$$f = g \stackrel{\text{def}}{=} \forall x{:}A.\ f(x) = g(x)$$

So, we can prove two functions f and g are equal by choosing an arbitrary $x \in A$ and showing $f(x) = g(x)$.

For example, if $f(x) = |x|$ (the absolute value) and $g(x) = x$ then, $f \neq g$ when we consider them as functions in the type $\mathbb{Z} \to \mathbb{Z}$ since $f(-2) = 2$ and $g(-2) = -2$. But, if we think of these functions as elements of $\mathbb{N} \to \mathbb{N}$, they are equal. To see this, choose an arbitrary $x \in \mathbb{N}$ and argue that $f(x) = g(x)$ *i.e.* that $|x| = x$. But this is trivially true when $x \geq 0$, which follows because $x \in \mathbb{N}$.

Problem 0.1. Create a separate Haskell script called Plus.hs which includes definitions for the following fuctions.

$$\begin{array}{l} plus :: (Integer, Integer) \to Integer \\ plus(x, y) = x + y \\ \\ plusc :: Integer \to (Integer \to Integer) \\ plusc\ x\ y = x + y \end{array}$$

Use *plusc* to create a function of type $(Integer \to Integer)$ that adds 7 to its argument.

$$plusSeven = \ ???$$

Add this function to the Plus module and test it in the interpreter.

Now, consider the following two definitions.

$$\begin{array}{l} compose : (b \to c) \to (a \to b) \to (a \to c) \\ compose\ f\ g\ x\ = f\ (g\ x) \\ id :: a \to a \\ id\ x\ =\ x \end{array}$$

Problem 0.2. Implement these functions in a module than includes the *Plus* module and, in the interpreter, evaluate the following:

$$\begin{array}{l} :t\ compose\ plusSeven\ plusSeven \\ (compose\ plusSeven\ plusSeven)0 \\ (compose\ plusSeven\ plusSeven)1 \\ (compose\ plusSeven\ plusSeven)2 \end{array}$$

We will write $f \circ g$ instead of *compose* $f\ g$.

Problem 0.3. Prove the following theorem.
[**compose-id-right**] For every function f, if $f \in A \to B$ then $f \circ id = f$.

Problem 0.4. Prove the following theorem using extensionailty.
[**compose-id-left**] For every function f, if $f \in A \to B$ then $id \circ f = f$.