Sequent Proof Rules

$$\frac{}{\Gamma_1, \phi, \Gamma_2 \vdash \Delta_1, \phi, \Delta} \text{ (Ax)}$$

$$\frac{}{\Gamma_1, \bot, \Gamma_2 \vdash \Delta} \text{ ($\bot$Ax)}$$

$$\frac{\Gamma_1, \phi, \psi, \Gamma_2 \vdash \Delta}{\Gamma_1, (\phi \wedge \psi), \Gamma_2 \vdash \Delta} \text{ ($\wedge$L)}$$

$$\frac{\Gamma \vdash \Delta_1, \phi, \Delta_2 \qquad \Gamma \vdash \Delta_1, \psi, \Delta_2}{\Gamma \vdash \Delta_1, (\phi \wedge \psi), \Delta_2} \text{ ($\wedge$R)}$$

$$\frac{\Gamma_1, \phi, \Gamma_2 \vdash \Delta \qquad \Gamma_1, \psi, \Gamma_2 \vdash \Delta}{\Gamma_1, (\phi \vee \psi), \Gamma_2 \vdash \Delta} \text{ ($\vee$L)}$$

$$\frac{\Gamma \vdash \Delta_1, \phi, \psi, \Delta_2}{\Gamma, \vdash \Delta_1, (\phi \vee \psi), \Delta_2} \text{ ($\vee$R)}$$

$$\frac{\Gamma_1, \Gamma_2 \vdash \phi, \Delta \qquad \Gamma_1, \psi, \Gamma_2 \vdash \Delta}{\Gamma_1, (\phi \Rightarrow \psi), \Gamma_2 \vdash \Delta} \text{ ($\Rightarrow$L)}$$

$$\frac{\Gamma, \phi \vdash \Delta_1, \psi, \Delta_2}{\Gamma \vdash \Delta_1, (\phi \Rightarrow \psi), \Delta_2} \text{ ($\Rightarrow$R)}$$

$$\frac{\Gamma_1, \Gamma_2 \vdash \phi, \Delta}{\Gamma_1, \neg\phi, \Gamma_2 \vdash \Delta} \text{ ($\neg$L)}$$

$$\frac{\Gamma, \phi \vdash \Delta_1, \Delta_2}{\Gamma \vdash \Delta_1, \neg\phi, \Delta_2} \text{ ($\neg$R)}$$

$$\frac{\Gamma_1, \phi[x := t], \Gamma_2 \vdash \Delta}{\Gamma_1, \forall x.\phi, \Gamma_2 \vdash \Delta} \text{ ($\forall$L)}$$

where $t \in \mathcal{T}$ is any term.

$$\frac{\Gamma \vdash \Delta_1, \phi[x := y], \Delta_2}{\Gamma \vdash \Delta_1, \forall x.\phi, \Delta_2} \text{ ($\forall$R)}$$

where variable y is not free in any formula of $(\Gamma \cup \Delta_1 \cup \{\forall x.\phi\} \cup \Delta_2)$.

$$\frac{\Gamma_1, \phi[x := y], \Gamma_2 \vdash \Delta}{\Gamma_1, \exists x.\phi, \Gamma_2 \vdash \Delta} \text{ ($\exists$L)}$$

where variable y is not free in any formula of $(\Gamma_1 \cup \Gamma_2 \cup \{\exists x.\phi\} \cup \Delta)$.

$$\frac{\Gamma \vdash \Delta_1, \phi[x := t], \Delta_2}{\Gamma \vdash \Delta_1, \exists x.\phi, \Delta_2} \text{ ($\exists$R)}$$

where $t \in \mathcal{T}$ is any term.