HW 19 **Prof. Caldwell**
Due: 20 November 2008 **COSC 3015**

Exercise 0.1. Read chapter 10 in Bird.

1 Type Derivations

1.1 Proof Rules

Sequents in the system (which represent the state of a type derivation) are of the form:

$$\Gamma, E \vdash M : T$$

In this structure, Γ is a *context* representing a state of knowledge about the types of some variables. Contexts have the form:

$$\Gamma = [x_1 : \tau_1, \cdots, x_k : \tau_k]$$

where the x_i's are variables and τ_i's are types.

E is a list of constraints between pairs of types and in the rules is presented as follows:

$$E = \{\tau_{(1,1)} = \tau_{(1,2)}, \cdots, \tau_{(k,1)} = \tau_{(k,2)}\}$$

wher $\tau_{i,j}$'s are types.

We write $\Gamma \backslash x$ to denote the list obtained from Γ by deleting all pairs whose first element is x.

As presented in the last homework, The proof rules for Wand's type inference system including product types are given as follows:

$$\frac{}{\Gamma, \{\alpha = \tau\} \quad \vdash \quad x : \tau}(\text{Ax}) \qquad \text{if } (x, \alpha) \in \Gamma.$$

$$\frac{[x : \alpha] +\!\!+ (\Gamma \backslash x),\ E \quad \vdash \quad M : \beta}{\Gamma, E \cup \{\tau = \alpha \to \beta\} \quad \vdash \quad \lambda x.M : \tau}(\text{Abs}) \qquad \text{where } \alpha \text{ and } \beta \text{ are fresh.}$$

$$\frac{\Gamma, E_1 \vdash M : \alpha \to \tau \qquad \Gamma, E_2 \vdash N : \alpha}{\Gamma, E_1 \cup E_2 \vdash MN : \tau}(\text{App}) \qquad \text{where } \alpha \text{ is fresh.}$$

$$\frac{\Gamma, E_1 \vdash M : \alpha \qquad\qquad \Gamma, E_2 \vdash N : \beta}{\Gamma, E_1 \cup E_2 \cup \{\tau = \alpha \times \beta\} \vdash \langle M, N\rangle : \tau}(\text{Pair}) \qquad \text{where } \alpha \text{ and } \beta \text{ are fresh.}$$

$$\frac{\Gamma, E_1 \vdash M : \alpha \times \beta \qquad \{x : \alpha, y : \beta\} \cup ((\Gamma \backslash x) \backslash y), E_2 \vdash N : \tau}{\Gamma, E_1 \cup E_2 \vdash \mathit{spread}(M; x, y.N) : \tau}(\text{Spread}) \qquad \text{where } \alpha \text{ and } \beta \text{ are fresh.}$$

In the last homework, some sample deriviations were given. Recall that to prove a sequent of the form $\Gamma, E \vdash M : \tau$ you leave E blank and work backwards (up) until you get to tghe leaves and then compute the constraint sets E coming back down through the derivation tree. I strongly urge you to study the examples in HW 18.

Exercise 1.1. Complete the following devivation trees and compute the constraint sets E.

i.) $[z : (\alpha \to \alpha) \to \beta], E \vdash z(\lambda w.w) : \tau$
ii.) $[y : \beta], E \vdash (\lambda x.x)y : \tau$
iii.) $[x : \alpha, y : \beta], E \vdash \mathit{spread}(\langle x, y\rangle; z, w.\langle w, z\rangle) : \tau$