

Here is the code for an implementation of sets as lists.

```
module Set where

import Lists

data (Eq a, Show a) => Set a = MkSet [a]

instance (Eq a, Show a) => Show (Set a) where
  show m = "{" ++ contents ++ "}"
    where MkSet m' = m
          contents = reverse (drop 1 (reverse ( drop 1 (show (unique m'))))))

instance (Eq a, Show a) => Eq (Set a) where
  s == t = all ((flip elem) s') t' && all ((flip elem) t') s'
    where MkSet s' = s
          MkSet t' = t

insert x (MkSet m) = MkSet (x:m)
delete x (MkSet m) = MkSet (remove_all x m)
union (MkSet m) (MkSet n) = MkSet (m ++ n)
intersection (MkSet m) (MkSet n) = MkSet (filter ((flip elem) n) m)
ismem x (MkSet m) = elem x m
```

Here is a module containing the supporting definitions for operations on lists.

```
module Lists where

unique [] = []
unique (h:t) = if (elem h t) then (unique t ) else h:(unique t)

remove_all x = filter (/=x)
```

0.1 Multisets

Multisets (sometimes called bags) are like sets except that multiplicity does count *e.g.*

```
[1,2,2] /= [1,2]
[1,2,2] == [2,1,2] == [2,2,1]
```

So the order of elements does not count for determining equality but the number of occurrences of each element does.

Definition 0.1 (multiset intersection) For multisets M and N , if there are k occurrences of x in M and there are j occurrences of x in N then there are $\min(j, k)$ occurrences of x in the multiset $(m \text{ 'intersection' } n)$.

Definition 0.2 (multiset union) For multisets m and n , if there are k occurrences of x in m and there are j occurrences of x in n then there are $\max(j, k)$ occurrences of x in the multiset $(m \text{ 'union' } n)$.

Definition 0.3 (multiset delete) Given a multiset m , if x occurs k times in m then there are $\max(0, k - 1)$ occurrences of x in the multiset $(\text{delete } x \ m)$.

Definition 0.4 (multiset insert) Given a multiset m , if x occurs k times in m then there are $k + 1$ occurrences of x in the multiset $(\text{insert } x \ m)$.

Definition 0.5 (multiset ismem) Given a multiset m , if x occurs 1 or more times in m then $(\text{ismem } x \ m)$ is true and is false otherwise.

Exercise 0.1. Write Haskell code to implement Multisets as lists by modifying the Set module presented above. You'll have to reimplement `show`, `==`, `insert`, `delete`, `union`, `intersection`, and `ismem` so they give the proper answers for multisets.