

1

In class we discussed the following code.

```
plus (x,y) = x + y
plusc x y = x + y
curry f x y = f(x,y)
uncurry f (x,y) = f x y
(f . g) x = f (g x)
id x = x
```

Recall the extensionality rule for proving functions $f, g : A \rightarrow B$ are equal.

$$f = g \stackrel{\text{def}}{=} \forall x : A. f x = g x$$

We gave a proof in class of the following identity.

Lemma 1.1. For every function f where $f :: A \rightarrow B$, the following equality holds

$$f . id = f$$

Proof: To show these functions are equal, by extensionality, we must show that

$$\forall x : A. (f . id) x = f x$$

Choose an arbitrary $x \in A$ and show $(f . id)x = f x$. Starting on the left side:

$$(f . id) x \stackrel{\langle\langle \text{def. of } (.)\rangle\rangle}{=} f(id x) \stackrel{\langle\langle \text{def. of } id\rangle\rangle}{=} f x$$

So the identity holds.

□

Following the proofs given in class, prove the following equalities using extensionality.

Problem 1.1.

- i.) $curry\ plus = plusc$
- ii.) $curry\ (uncurry\ plusc) = plusc$
- iii.) $uncurry\ (curry\ plus) = plus$

Problem 1.2. Haskell has a built-in function called `flip` having the following type:

$$(a \rightarrow b \rightarrow c) \rightarrow b \rightarrow a \rightarrow c$$

Write a bit of Haskell code that has this type.