HW 12 **Prof. Caldwell**
Due: 9 October 2008 **COSC 3015**

1 Trees

Exercise 1.1. Read chapter 6 of the Bird text.

Recall the `BTree` datatype and it's structural induction principle for finite `BTree`s.

```
data  BTree a = Leaf a | Fork  (BTree a) (BTree a)
 deriving (Show,Eq)
```

$$\begin{array}{l}(\forall x : a.\ P[\texttt{Leaf}\ x]\ \ \wedge \\ \quad \forall x : a.\ \forall t_1, t_2 : (\texttt{BTree}\, a).\ \ (P[t_1]\ \wedge\ P[t_2]) \Rightarrow P[\texttt{Fork}(t_1, yt_2)]) \\ \ \Rightarrow \\ \forall t : (\texttt{BTree}\ a).\ P[t]\end{array}$$

Exercise 1.2. Here is a data-type declaration for a type of tree that can store two different types of values in it's nodes.

```
data  ABTree a b =  Leaf |
                    ANode a (ABTree a b) (ABTree a b) |
                    BNode b (ABTree a b) (ABTree a b)
 deriving (Show,Eq)
```

Write the structural induction principle for this data-type.

Exercise 1.3.

i.) Write an recursive datatype declaration (of your choice) for another tree-like data type and,

ii.) write the structural induction principle for properties of finite instanes of your data-type.

Exercise 1.4. consider the following:

```
size (Leaf x) = 1
size (Fork xt yt) = (size xt) + (size yt)

height (Leaf x) = 0
height (Fork xt yt) = 1 + (height xt ‘max‘ height yt)

perfect (Leaf x)  = True
perfect (Fork xt yt) = (height xt == height yt) && perfect xt && perfect yt
```

Prove[1] the following theorem hold for all finite `BTree`s by structural induction.

Theorem 1.1. $\forall t : \texttt{BTree}\ a.\ (\texttt{perfect}\ t) \Rightarrow \texttt{size}\ t = 2^{(\texttt{height}\ t)}$

[1]Hints will be given in class on Thursday.