HW 11 **Prof. Caldwell**
Due: 9 October 2008 **COSC 3015**

Recall the list induction principle.

$$
\begin{aligned}
&[P([\,])\wedge \\
&\ \ \forall a:\alpha.\forall m:[\alpha].\,P(m)\Rightarrow P(a:m)] \\
&\ \ \ \Rightarrow \forall m:[\alpha].P(m)
\end{aligned}
$$

Thus, for a property P of lists, to show that $\forall m:[\alpha].\ P(m)$ it is enough to show two things:

i.) $P([\,])$
ii.) $\forall a:\alpha.\forall m:[\alpha].\,P(m)\Rightarrow P(a:m)$

Here are some definitions.

$$
\begin{aligned}
&head(h:t) = h \\
&head[\,] = \perp \\
\\
&last[x] = x \\
&last(h:t) = last\ t \\
&last[\,] = \perp \\
\\
&reverse[\,] = [\,] \\
&reverse(h:t) = (reverse\ t) ++ [h] \\
\\
&map\ f\ [\,] = [\,] \\
&map\ f\ (h:t) = (f\ h)\ :\ map\ f\ t \\
\\
&(f\ .\ g)\ x = f\ (g\ x)
\end{aligned}
$$

Prove the following by list induction.

1.) $\forall m{:}[\alpha].\ map\,(f\ .\ g)\,m = ((map\ f)\,.\,(map\ g))\ m$
2.) $\forall m{:}[\alpha].\ head\,(reverse\ m) = last\ m$