HW 12 **Prof. Caldwell**
Due: 7October 2010 **COSC 3015**

Recall the list induction principle to prove a proeprty for finite lists of type a.

$$[P([\,])\wedge$$
$$\forall x :: a.\, \forall xs : [a].\, P(xs) \Rightarrow P(x : xs)$$
$$\Rightarrow \forall ys : [a].P(ys)$$

Thus, for a property P of lists, to show that $\forall ys : [a].\ P(ys)$ it is enough to show two things:

i.) $P([\,])$
ii.) $\forall x :: a.\, \forall xs :: [a].\, P)(xs) \Rightarrow P(x : xs)$

Here are some definitions.

$$head(h : t) = h$$
$$head[\,] = \bot$$

$$last[x] = x$$
$$last(h : t) = last\ t$$
$$last[\,] = \bot$$

$$reverse[\,] = [\,]$$
$$reverse(h : t) = (reverse\ t) + +[h]$$

$$map\ f\ [\,] = [\,]$$
$$map\ f\ (h : t) = (f\ h)\ :\ map\ f\ t$$

$$(f\ .\ g)\ x = f\ (g\ x)$$

Prove the following by list induction.

1.) $\forall m : [a].\ map\,(f\ .\ g)\,m = ((map\ f)\,.\,(map\ g))\ m$
2.) $\forall m : [a].\ head\,(reverse\ m) = last\ m$
3.) $\forall m : [a].\ last\,(reverse\ m) = head\ m$