

## 1 Modelling Finite Functions as Lists

**Exercise 1.1.** Based on the code provided on the HW web-page you must write the following functions.

```
update :: Eq a => (a,b) -> FinFun a b -> FinFun a b
functional :: Eq a => [(a,b)] -> Bool
domain :: (Eq a, Eq b) => FinFun a b -> [a]
range :: (Eq a, Eq b) => FinFun a b -> [b]
apply :: (Eq a, Eq b) => FinFun a b -> a -> Maybe b
```

**1.1** `update :: Eq a => (a,b) -> FinFun a b -> FinFun a b`

This function should be called as `update(i,v) f` where `f` is a finite function with domain `a` and codomain `b`. If `i` is the first element of any pair in `f` return the finite function that is just like `f` except that it maps `i` to `v`. If `i` is not the first element of any pair in `f`, return the finite function that behaves just like `f` but also contains the pair `(i,v)`.

**1.2** `functional :: Eq a => [(a,b)] -> Bool`

Call this predicate as `functional m` where `m` is a list of pairs. The predicate returns `True` iff for every pair `(i,v) ∈ m`, there is no pair `(j,v') ∈ m` with `i=j`.

**1.3** `domain :: (Eq a, Eq b) => FinFun a b -> [a]`

This function returns a list of values in the domain `a` that the function is actually defined for.

**1.4** `range :: (Eq a, Eq b) => FinFun a b -> [b]`

This function returns a list of values in the codomain `b` that is the range of the finite function.

**1.5** `apply :: (Eq a, Eq b) => FinFun a b -> a -> Maybe b`

`apply f x ::` apply the finite function `f` to argument `x` and return `Nothing` if `x ∉ (domain f)` and return `Just y` when the pair `(x,y) ∈ f`.

**Exercise 1.2.** Instantiate the type of finite functions `FinFun` as an instance of the `Eq` type class by defining `==` to be the extensional equality on functions. You can use the equality in the module `Sets` as a model, or you may even use it in your definition.

### 1.6

Recall that extensional equality for finite functions means the list representing the function is functional *and* they have the same elements.