HW 4 **Prof. Caldwell**
Due: 6 September 2011 **COSC 3015**

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In class we discussed the following code.

plus (x,y) = x + y
plusc x y = x + y
curry f x y = f(x,y)
uncurry f (x,y) = f x y
(f . g) x = f (g x)
id x = x

Recall the extensionality rule for proving functions $f, g : a \to b$ are equal.

$$f = g \stackrel{\text{def}}{=} \forall x : a.\ fx = gx$$

Heree's a proof that the identity function is a right identory for function composition.

Lemma 1.1. For every function f where $f :: a \to b$, the following equality holds

$$f.\, id = f$$

Proof: To show these functions are equal, by extensionality, we must show that

$$\forall x : a.(f\ .\ id)x = f\ x$$

Choose an arbitrary x of type a and show $(f\ .\ id)x = f\ x$. Starting on the left side:

$$(f.\ id)\ x \stackrel{\langle\langle def.\,of\,(.)\rangle\rangle}{=} f(id\ x) \stackrel{\langle\langle def.\,of\,id\rangle\rangle}{=} f\ x$$

So the identity holds.
□

We also proved in class that the compostion $(uncurry\,.\,curry) = id$.

Theorem 1.1. $(uncurry\,.\,curry) = id$.
Proof: The type of $(uncurry\,.\,curry)$ is $((a, b) \to c) \to ((a, b) \to c)$. The type of id is $a \to a$ where a can be any type, so letting $a = ((a, b) \to c)$ gives $id :: ((a, b) \to c) \to ((a, b) \to c)$. Since the left and right sides have the same function type, we apply extensionality to prove the following:

$$\forall f :: ((a, b) \to c).\ (uncurry\,.\,curry)f = idf$$

Choose arbitrary $f :: ((a, b) \to c)$ and show

$$(uncurry\,.\,curry)\,f = id$$

Notice that $((uncurry\,.\,curry)f)$ has type $((a, b) \to c)$ so we apply extensionality again and prove the following:

$$\forall(x, y) :: (a, b).\ ((uncurry\,.\,curry)f)(x, y) = id\ f\ (x, y)$$

Choose an arbitrary pair (x, y) of type (a, b) and show that the equality holds. Starting on the left we get the following sequence of equalities:

$$\begin{aligned}
&((uncurry\ .\ curry) f)(x, y)\\
&\langle\langle by.\ def.\ of\ compose\rangle\rangle\\
&= uncurry\ (curry\, f)\ (x, y)\\
&\langle\langle by.\ def.\ of\ uncurry\rangle\rangle\\
&= (curry\, f)\ x\ y\\
&\langle\langle by.\ def.\ of\ curry\rangle\rangle\\
&= f(x, y)
\end{aligned}$$

On the right side of we get the following:

$$id\ f\ (x, y) \overset{\langle\langle \text{def. of } id\rangle\rangle}{=} f(x, y)$$

□

Problem 1.1. Following the proofs given in class, prove the following equalities using extensionality.

i.) $curry\ .\ uncurry = id$

ii.) $id\ .\ f = f$

Problem 1.2. Haskell has a built-in function called `flip` having the following type:

```
(a -> b -> c) -> b -> a -> c
```

Write a bit of Haskell code that has this type.