Solving Optimal Control Problems with ACADO Toolkit

Boris Houska, Hans Joachim Ferreau, Moritz Diehl

Electrical Engineering Department
K.U. Leuven

OPTEC Seminar, 2/9/2009

Overview

Part 1:

- Scope of ACADO Toolkit
- An Optimal Control Tutorial Example (with Software Demo)
- Algorithms and Modules in ACADO

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Motivation: Optimal Control and Engineering Applications

Optimal Control Applications in OPTEC:

- Optimal Robot Control, Kite Control, Solar Power Plants
- Batch Distallation Processes, Bio-chemical reactions...

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Umbiquos Need for Nonlinear Optimal Control Software

Existing Packages:

- IPOPT (C++,open source, collocation, interior point method)
- MUSCOD (Fortran/C, proprietary, Multiple Shooting, SQP)
- PROPT (commercial Matlab software, collocation, SQP)
- DSOA (C/C++, open-source, single shooting, SQP)
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All packages have their particular strengths in a specific range of applications, but ...

Motivation for ACADO Toolkit

Most of the existing Packages are ...

- either not open-source or limited in their user-friendliness
- difficult to install - especially on embedded hardware
- not designed for closed loop MPC applications
- hard to extend with specialized algorithms

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Key Propeties of ACADO Toolkit

- Open Source (LGPL) `www.acadotoolkit.org`
- user friendly interfaces close to mathematical syntax
- Code extensibility: use `C++` capabilities
- Self-containedness: only need `C++` compiler

Problem Classes and the Scope of ACADO

Optimal Control of Dynamic Systems

- Objectives: Mayer and/or Lagrange terms.
- Differential and algebraic equations.
- Initial value-, terminal-, path- and boundary constraints.

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- A posteriori analysis: Computation of variance-covariances.

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Feedback control based on real-time optimization (MPC/MHE)

- Coputation of current process state using measurements.
- Comptation of optimal control action in real-time.

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A simple rocket model

- Three differential states: s, v, and m.
- Control input: u
- Dynamic equations (model):

$$
\begin{aligned}
\dot{s}(t) &= v(t) \\
\dot{v}(t) &= \left[u(t) - 0.2\, v(t)^2\right] / m(t) \\
\dot{m}(t) &= -0.01\, u(t)^2 \; .
\end{aligned}
$$

A tutorial example: time optimal control of a rocket

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Aim:

- Fly in mimimum time T from $s(0) = 0$ to $s(T) = 10$.
- Start/land at rest: $v(0) = 0, \quad v(T) = 0$.
- Start with $m(0) = 1$ and satisfy $v(t) \leq 1.7$.
- Satisfy control constraints: $-1.1 \leq u(t) \leq 1.1$.

Mathematical Formulation:

$$\underset{s(\cdot),v(\cdot),m(\cdot),u(\cdot)}{\text{minimize}} \qquad T$$

subject to

$$\begin{aligned}
\dot{s}(t) &= v(t) \\
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\end{aligned}$$

$$\begin{aligned}
s(0) &= 0 & s(T) &= 10 \\
v(0) &= 0 & v(T) &= 0 \\
m(0) &= 1
\end{aligned}$$

$$\begin{aligned}
-0.1 &\leq v(t) \leq 1.7 \\
-1.1 &\leq u(t) \leq 1.1 \\
5 &\leq T \leq 15
\end{aligned}$$

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```
DifferentialState                         s,v,m;
Control                                      u;
Parameter                                    T;
DifferentialEquation          f( 0.0, T );
OCP ocp( 0.0, T );
ocp.minimizeMayerTerm( T );
```

A tutorial example: time optimal control of a rocket

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\end{array}$$

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f << dot(s) == v;
f << dot(v) == (u-0.2*v*v)/m;
f << dot(m) == -0.01*u*u;
ocp.subjectTo( f                                  );
```

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ocp.subjectTo( AT_END  , s == 10.0 );
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ocp.subjectTo( -0.1 <= v <=  1.7   );
ocp.subjectTo( -1.1 <= u <=  1.1   );
ocp.subjectTo(  5.0 <= T <= 15.0   );
OptimizationAlgorithm algorithm(ocp);
algorithm.solve();
```
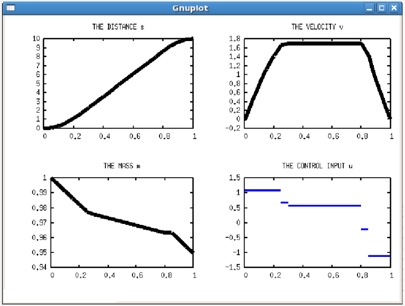

Graphical Output:



On the terminal:

#:	KKT tol.	Obj. Value
1:	1.001e+03	1.000e+01
2:	5.766e+00	9.950e+00
3:	2.946e−02	9.932e+00
4:	7.481e−02	9.906e+00
.	.	.
.	.	.
.	.	.
12:	8.740e−04	7.442e+00
13:	3.308e−07	7.442e+00

convergence achieved.

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The Power of Symbolic Functions

Symbolic Functions allow:

- Dependency/Sparsity Detection
- Automatic Differentiation
- Symbolic Differentiation
- Convexity Detection
- Code Optimization
- C-code Generation

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Example 1:

```
DifferentialState x;
IntermediateState z;
TIME              t;
Function          f;

z = 0.5*x + 1.0    ;
f << exp(x) + t    ;
f << exp(z+exp(z)) ;

if( f.isConvex() == BT_TRUE )
printf("f is convex.    ");
```

Example 2 (code optimization):

```
Matrix A(3,3);
Vector b(3);
DifferentialStateVector x(3);
Function f;
A.setZero();
A(0,0) = 1.0; A(1,1) = 2.0; A(2,2) = 3.0;
b(0)   = 1.0; b(1)   = 1.0; b(2)   = 1.0;
f << A*x + b;
```

- We would expect 12 flops to evaluate `f`.
- ACADO Toolkit needs only 6 flops.

Integration Algorithms

DAE simulation and sensitivity generation

- ACADO provides several Runge Kutta and a BDF integrator.
- All integrators provide first and second order numeric and automatic internal numerical differentiation.

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- BDF integrator uses diagonal implicit Runge Kutta starter
- The BDF routine can deal with fully implicit index 1 DAE's:

$$\forall t \in [0, T] : \quad F(\dot{y}(t), y(t), u(t), p, T) = 0 \ .$$

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$$\forall t \in [0, T] : \quad F(\dot{y}(t), y(t), u(t), p, T) = 0 \, .$$

- The Integrators are also available as a stand alone package.
- Sparse LA solvers can be linked.

Nonlinear Optimal Control Problem

ACADO solves problem of the general form:

$$\boxed{\begin{array}{lrcl} \underset{y(\cdot),u(\cdot),p,T}{\text{minimize}} & & & \int_0^T L(\tau, y(\tau), u(\tau), p)\,\mathrm{d}\tau \;+\; M(y(T), p) \\ \\ \text{subject to:} & & & \\ \\ \forall t \in [0, T]: & 0 & = & f(t, \dot{y}(t), y(t), u(t), p) \\ & 0 & = & r(y(0), y(T), p) \\ \forall t \in [0, T]: & 0 & \geq & s(t, y(t), u(t), p) \end{array}}$$

Implemented Soultion Methods

- Discretization: Single- or Multiple Shooting
- NLP solution: several SQP type methods e.g. with
  - BFGS Hessian approximations or
  - Gauss-Newton Hessian approximations
- Globalization: based on line search
- QP solution: active set methods (`qpOASES`)

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Currently under development

- Collocation methods
- Interior point methods
- Sequential convex optimization techniques
- Lifted Newton methods
- ...

Using ACADO Toolkit for Parameter Estimation and Model Predictive Control

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Parameter Estimation with ACADO

- `ACADO Toolkit` can solve **parameter estimation problems** of the following form:

$$
\begin{array}{lrcl}
\displaystyle\min_{x(\cdot),z(\cdot),u(\cdot),p} & \multicolumn{3}{l}{\sum_{i=1}^{N} \| \, h(t_i, x(t_i), z(t_i), u(t_i), p) - \eta_i \|_{S_i}^2} \\
\text{subject to:} & & & \\
\forall t \in [0, T]: & \dot{x}(t) & = & f(t, x(t), z(t), u(t), p) \\
\forall t \in [0, T]: & 0 & = & g(t, x(t), z(t), u(t), p) \\
 & 0 & = & r(x(0), z(0), x(T), z(T), p) \\
\forall t \in [0, T]: & 0 & \geq & s(t, x(t), z(t), u(t), p)
\end{array}
$$

- **Simple pendulum** model describing the exctitation angle ϕ governed by the following ODE:

$$\ddot{\phi}(t) \; = \; -\frac{g}{l}\phi(t) - \alpha\dot{\phi}(t)$$

where l is the length of the line, α the friction coefficient and g the gravitational constant

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  where l is the length of the line, α the friction coefficient and g the gravitational constant

- **Aim is to estimate l and** α from ten measurements of the state ϕ

Mathematical Formulation:

$$\underset{\phi(\cdot),\alpha,l}{\text{minimize}} \quad \sum_{i=1}^{10} \left(\phi(t_i) - \eta_i\right)^2$$

subject to:

$$\forall t \in [0,2]: \quad \ddot{\phi}(t) = -\frac{g}{l}\phi(t) - \alpha\dot{\phi}(t)$$

$$0 \leq \alpha \leq 4$$

$$0 \leq l \leq 2$$

Tutorial Example: A Simple Pendulum

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C++ Implementation:

```
DifferentialState      phi, dphi;
Parameter              l, alpha;
const double           g = 9.81;
DifferentialEquation   f;
Function               h;

OCP ocp( 0.0, 2.0 );
h << phi;
ocp.minimizeLSQ( h, "data.txt" );

f << dot(phi ) == dphi;
f << dot(dphi) == -(g/l) * sin(phi)
                  -alpha * dphi;

ocp.subjectTo( f );
ocp.subjectTo( 0.0 <= alpha <= 4.0 );
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ParameterEstimationAlgorithm alg(ocp);
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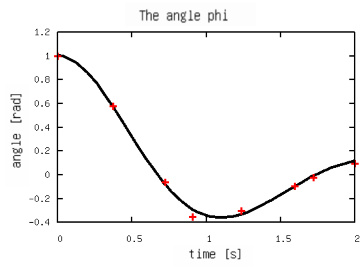
- Parameter estimation problems are (nonlinear) least-square problems with objective function $\frac{1}{2}\left\|F(x)\right\|_2^2$
- Parameter estimation problems are solved using the **constrained Gauss-Newton method**
- Newton-type method where the Hessian matrix is approximated by $\left(\frac{\partial F(x)}{\partial x}\right)^T \left(\frac{\partial F(x)}{\partial x}\right)$
- The constrained Gauss-Newton method works well for:
  - small residual problems
  - almost linear problems

Tutorial Example: A Simple Pendulum

Data file `data.txt`:

```
TIME POINTS MEASUREMENTS
----------- ------------
0.00000e+00 1.00000e+00
2.72321e-01 nan
3.72821e-01 5.75146e-01
7.25752e-01 -5.91794e-02
9.06107e-01 -3.54347e-01
1.23651e+00 -3.03056e-01
1.42619e+00 nan
1.59469e+00 -9.64208e-02
1.72029e+00 -1.97671e-02
2.00000e+00 9.35138e-02
```

Fitting Results:




```
l     = 1.001e+00 +/- 1.734e-01
alpha = 1.847e+00 +/- 4.059e-01
```

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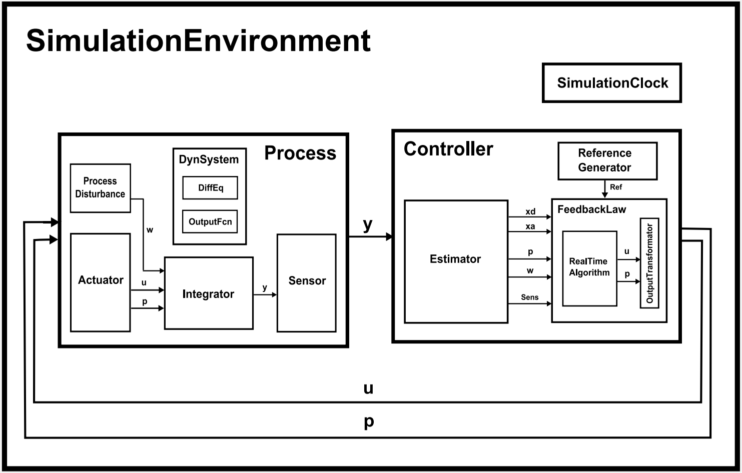
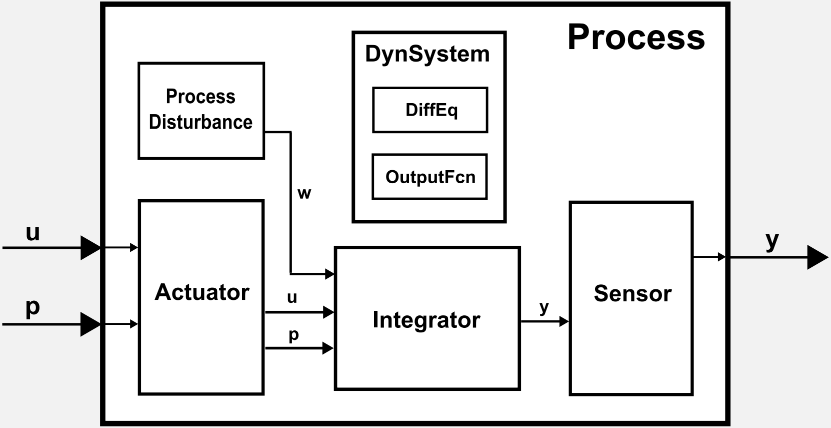
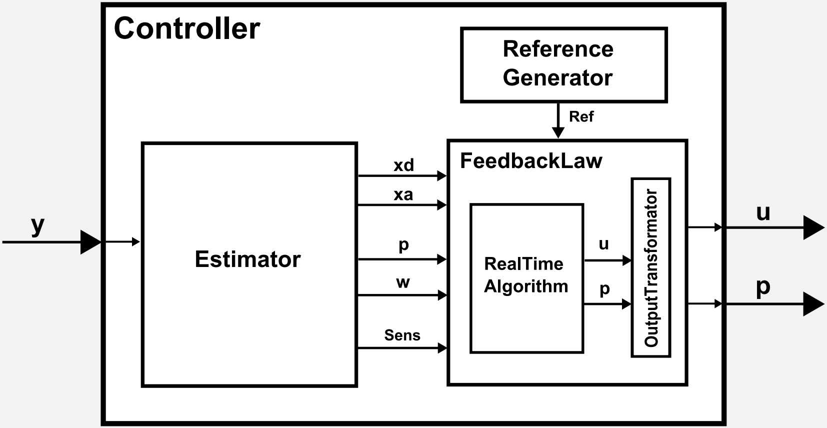
Simulation Environment of ACADO Toolkit




Process
DynSystem
DiffEq
OutputFcn
Process
Disturbance
w
u
p
Actuator
u
p
Integrator
y
Sensor
y


Controller
Reference Generator
Ref
FeedbackLaw
xd
xa
p
w
Sens
y
Estimator
RealTime Algorithm
u
p
OutputTransformator
u
p

Model Based Feedback Control

- ACADO Toolkit can solve **model predictive control problems** of the following form:

$$
\begin{array}{rrcl}
\underset{x(\cdot),z(\cdot),u(\cdot),p}{\text{minimize}} & \multicolumn{3}{l}{\int_0^T \|y(t)-y_{\mathrm{ref}}(t)\|_Q^2 + \|u(t)-u_{\mathrm{ref}}(t)\|_R^2 \,\mathrm{d}\tau + \|y(T)-y_{\mathrm{ref}}(T)\|_P^2} \\
\text{subject to:} & & & \\
 & x(0) & = & x_0 \\
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\forall t \in [0,T]: & 0 & = & g(t,x(t),z(t),u(t),p) \\
\forall t \in [0,T]: & y(t) & = & h(t,x(t),z(t),u(t),p) \\
\forall t \in [0,T]: & 0 & \geq & s(t,x(t),z(t),u(t),p)
\end{array}
$$

Model Based Feedback Control

- Each MPC problem might be solved till convergence
- Preferably, the **real-time iteration scheme** is employed:
  - Only one real-time SQP step per MPC loop
  - Initial value embedding
  - Division into feedback and preparation phase

Model Based Feedback Control

- Each MPC problem might be solved till convergence
- Preferably, the **real-time iteration scheme** is employed:
  - Only one real-time SQP step per MPC loop
  - Initial value embedding
  - Division into feedback and preparation phase
- Model based feedback control often requires an online **state estimator**
- Moving Horizon Estimation (MHE) and Kalman filters will be implemented

A Simple MPC Simulation

- First principle **quarter car** model **with active suspension**, four states x_b, x_w, v_b, v_w describing vertical position/velocity of body/wheel
- Control input: limited damping force F to act between body and wheel
- External disturbance: road excitation R

A Simple MPC Simulation

- First principle **quarter car** model **with active suspension**, four states x_b, x_w, v_b, v_w describing vertical position/velocity of body/wheel
- Control input: limited damping force F to act between body and wheel
- External disturbance: road excitation R
- **Simulation scenario:** road excitation set to zero, body has initial displacement of 1 cm
- **Aim:** Bring body and wheel back to rest with zero displacement

Mathematical Formulation:

$$\underset{x_b, x_w, v_b, v_w, F}{\text{minimize}} \quad \int_0^1 \|y(t)\|_Q^2 \, \mathrm{d}\tau$$

subject to:

$$\forall t \in [0,1]: \quad \dot{x}_b(t) = v_b(t)$$

$$\forall t \in [0,1]: \quad \dot{x}_w(t) = v_w(t)$$

$$\forall t \in [0,1]: \quad \dot{v}_b(t) = f_1(x_b(t), x_w(t), F(t))$$

$$\forall t \in [0,1]: \quad \dot{v}_w(t) = f_2(x_b(t), x_w(t), F(t), R(t))$$

$$\forall t \in [0,1]: \quad y(t) = (x_b(t), x_w(t), v_b(t), v_w(t))^T$$

$$\forall t \in [0,1]: \quad -500 \ \leq \ u(t) \ \leq \ 500$$

A Simple MPC Simulation

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$$\forall t \in [0,1]: \quad -500 \leq u(t) \leq 500$$

C++ Implementation:

```
DifferentialState xB, xW, vB, vW;
Control F;
Disturbance R;

DifferentialEquation f;
//...

Function y;
y << xB;
y << xW;
y << vB;
y << vW;

Matrix Q(4,4);
Q.setIdentity();

OCP ocp( 0.0,1.0, 20 );
ocp.minimizeLSQ( Q, y );

ocp.subjectTo( f );
ocp.subjectTo( -500 <= F <= 500 );
ocp.subjectTo( R == 0.0 );
```

A Simple MPC Simulation

Mathematical Formulation:

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A Simple MPC Simulation

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$$\begin{aligned}
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\forall t \in [0,1]: &\quad \dot{x}_w(t) = v_w(t) \\
\forall t \in [0,1]: &\quad \dot{v}_b(t) = f_1(x_b(t), x_w(t), F(t)) \\
\forall t \in [0,1]: &\quad \dot{v}_w(t) = f_2(x_b(t), x_w(t), F(t), R(t)) \\
\forall t \in [0,1]: &\quad y(t) = (x_b(t), x_w(t), v_b(t), v_w(t))^T \\
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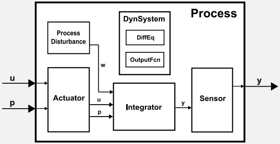
A Simple MPC Simulation

Simulation Setup:

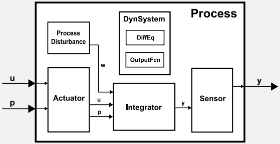


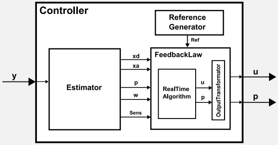
C++ Implementation:

```
OutputFcn identity;
DynamicSystem dynamicSystem( f,identity );

Process process( dynamicSystem,INT_RK45 );
```

Simulation Setup:





C++ Implementation:

```
OutputFcn identity;
DynamicSystem dynamicSystem( f,identity );

Process process( dynamicSystem,INT_RK45 );

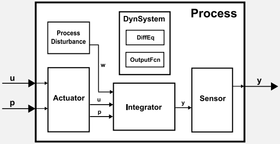

RealTimeAlgorithm alg( ocp );
DynamicFeedbackLaw feedbackLaw( alg,0.05 );

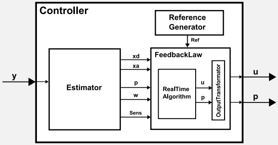
Estimator trivialEstimator;
StaticReferenceTrajectory zeroReference;

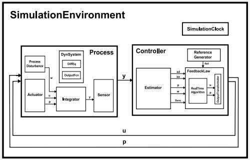
Controller controller(
feedbackLaw,trivialEstimator,zeroReference );
```

A Simple MPC Simulation

Simulation Setup:







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DynamicFeedbackLaw feedbackLaw( alg,0.05 );

Estimator trivialEstimator;
StaticReferenceTrajectory zeroReference;

Controller controller(
feedbackLaw,trivialEstimator,zeroReference );


SimulationEnvironment sim(
0.0,3.0,process,controller );

Vector x0(4);
x0(0) = 0.01;

sim.init( x0 );
sim.run( );
```

Part 1:

- Scope of ACADO Toolkit
- An Optimal Control Tutorial Example (with Software Demo)
- Algorithms and Modules in ACADO

Part 2:

- A Parameter Estimation Tutorial Example
- A Simple Model Predictive Control Simulation (with Software Demo)
- **Outlook**

Outlook

- **Algorithmic extensions** currently under development:
  - Collocation schemes
  - Convex optimization algorithms
  - Nonlinear interior point solver for solving NLPs
  - Sequential convex programming algorithms
  - State estimators for feedback control (MHE/Kalman)

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  - Collocation schemes
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  - Sequential convex programming algorithms
  - State estimators for feedback control (MHE/Kalman)
- **Matlab interfaces** for Integrators and Optimal Control Problems

Outlook

- Additional **problem classes**:
  - Multi-stage formulations
  - Robust optimization
  - Multi-objective problems
  - Optimum experimental design
- Modular design of `ACADO Toolkit` allows for easy combination of different algorithmic features