**Steven Murr**
*HW 6.4*
*Problems = { 1,4,7,8,12,13,15,19 }*

1) Find the expansion of $(x+4)^4$
    a) Using combinatorial reasoning, as in Example 1.

$$\begin{aligned}(x+y)^4 &= (x+y)(x+y)(x+y)(x+y) = (xx+xy+yx+yy)(xx+xy+yx+yy)\\ &= (xxxx+xxxy+xxyx+xxyy+xyxx+xyxy+xyyx+xyyy+yxxx+yxxy+yxyx+yxyy+yyxx+yyx\\ &= x^4+4x^3y+6x^2y^2+4xy^3+y^4\end{aligned}$$

b) Using the Binomial Theorem

$$\begin{aligned}&= \sum_{j=0}^{4}\binom{4}{j}x^{4-j}y^j\\ &= \binom{4}{0}x^4+\binom{4}{1}x^3y+\binom{4}{2}x^2y^2+\binom{4}{3}xy^3+\binom{4}{4}y^4\\ &= x^4+4x^3y+6x^2y^2+4xy^3+y^4\end{aligned}$$

4) Find the coefficient of $x^5y^8$ in $(x+y)^13$
    $\frac{13!}{5!8!} = 1287$

7) What is the coefficient of $x^9$ in $(2-x)^{19}$
    $-1^9 2^{10}\binom{19}{9} = -94595072$

8) What is the coefficient of $x^8y^9$in the expansion of $(3x+2y)^{17}$
    $3^8 2^9\binom{17}{9} = 8.1666e10$

12) The row of Pascal's triangle containing the binomial coefficients $\binom{10}{k}, 0 \le k \le 10$ is:
1 10 45 120 210 252 210 120 45 10 1
Use Pascal's identity to produce the row immediately following this row in Pascal's triangle.

*See attached sheet.

13) What is the row of Pascal's triangle containing the binomial coefficients $\binom{9}{k} 0 \le k \le 9$
    *See attached sheet

15) Show that $\binom{n}{k} \le 2^n$ for all positive integers n and all integers k with $0 \le k \le n$
    A set with n elements has a total of $2^n$ different subsets. Each subset has zero elements, one element to n elements in it. There are $\binom{n}{0}$subsets with zero elements, $\binom{n}{1}$ subsets with one element, $\binom{n}{2}$ subsets with two elements and $\binom{n}{n}$ subsets with n elements. Therefore, $\binom{n}{k \le 2^n}$.

19) Prove Pascal's identity, using the formula $\binom{n}{r}$.