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HW 5.3

Problems = { 1,4,7,9,10,15,18,20,49,57 }

1) Find $f(1), f(2), f(3)$, and $f(4)$ if $f(n)$ is defined recursively by $f(0) = 1$ and for $n = 0, 1, 2, ...$

a) $f(n+1) = f(n) + 2$

$$\begin{aligned} f(0) &= 1 \\ f(0+1) &= 1 + 2 = 3 \\ f(1+1) &= 3 + 2 = 5 \\ f(2+1) &= 5 + 2 = 7 \\ f(3+1) &= 7 + 2 = 9 \end{aligned}$$

b)$f(n+1) = 3f(n)$

$$\begin{aligned} f(0) &= 1 \\ f(0+1) &= 3(1) = 3 \\ f(1+1) &= 3(3) = 9 \\ f(2+1) &= 3(9) = 27 \\ f(3+1) &= 3(27) = 81 \end{aligned}$$

c) $f(n+1) = 2^{f(n)}$

$$\begin{aligned} f(0) &= 1 \\ f(0+1) &= 2^1 = 2 \\ f(1+1) &= 2^2 = 4 \\ f(2+1) &= 2^4 = 16 \\ f(3+1) &= 2^{16} = 65536 \end{aligned}$$

d) $f(n+1) = f(n)^2 + f(n) + 1$

$$\begin{aligned} f(0) &= 1 \\ f(0+1) &= 1^2 + 1 + 1 = 3 \\ f(1+1) &= 3^2 + 3 + 1 = 13 \\ f(2+1) &= 13^2 + 13 + 1 = 183 \\ f(3+1) &= 183^2 + 183 + 1 = 33673 \end{aligned}$$

Find $f(2), f(3), f(4)$, and $f(5)$ if f is defined recursively by $f(0) = f(1) = 1$and for $n = 1, 2, ...$

a) $f(n+1) = f(n) - f(n-1)$

$$\begin{aligned} f(0) &= f(1) = 1 \\ f(0+1) &= 1 - 0 = 1 \\ f(1+1) &= 1 - 0 = 1 \\ f(2+1) &= 1 - 0 = 1 \\ f(3+1) &= 1 - 0 = 1 \\ f(4+1) &= 1 - 0 = 1 \end{aligned}$$

b) $f(n+1) = f(n)f(n-1)$

$$\begin{aligned}
f(0) &= f(1) = 1 \\
f(0+1) &= 1 \cdot 0 = 0 \\
f(1+1) &= 0 \cdot 0 = 0 \\
f(2+1) &= 0 \cdot 0 = 0 \\
f(3+1) &= 0 \cdot 0 = 0 \\
f(4+1) &= 0 \cdot 0 = 0
\end{aligned}$$

c) $f(n+1) = f(n)^2 + f(n-1)^3$

$$\begin{aligned}
f(0) &= f(1) = 1 \\
f(0+1) &= 1^2 + 0 = 1 \\
f(1+1) &= 1^2 + 0 = 1 \\
f(2+1) &= 1^2 + 0 = 1 \\
f(3+1) &= 1^2 + 0 = 1 \\
f(4+1) &= 1^2 + 0 = 1
\end{aligned}$$

d) $f(n+1) = f(n)/f(n-1)$

$$\begin{aligned}
f(0) &= f(1) = 1 \\
f(0+1) &= 1/0 = undefined \\
f(1+1) &= und/und \\
f(2+1) &= und/und \\
f(3+1) &= und/und \\
f(4+1) &= und/und
\end{aligned}$$

7) Give a recursive definition of the sequence $a_n, n = 1, 2, 3$ if:

a) $a_n = 6n$

$$\begin{aligned}
a_n &= 6n \\
a_1 &= 6 \\
a_{n+1} &= 6(n+1) \\
a_{n+1} &= 6n + 6 \\
&= a_n + 6
\end{aligned}$$

b) $a_n = 2n + 1$

$$\begin{aligned}
a_n &= 2n + 1 \\
a_1 &= 3 - 2(1) + 1 \\
a_{n+1} &= 2(n+1) + 1 = 2n + 2 + 1 \\
&= 2n + 1 + 2 = a_n + 2
\end{aligned}$$

c) $a_n = 10^n$

$$\begin{aligned}
a_n &= 10^n \\
a_1 &= 10 \\
a_{n+1} &= 10^{n+1} \\
&= 10 \cdot 10^n \\
&= a_n \cdot 10
\end{aligned}$$

d) $a_n = 5$

$$\begin{aligned} a_n &= 5 \\ a_1 &= 5 \\ a_{n+1} &= 5 \\ &= a_n \end{aligned}$$

**(Since a_n is referenced as a constant all values will point to 5)

9) Let F be the function such that $F(n)$ is the sum of the first n positive integers. Give a recursive definition of $F(n)$

$$\begin{aligned} a_0 &= 0 \\ f(n) &= n + a_{n-1} : n > 1 \end{aligned}$$

10) Give a recursive definition of $P_m(n)$, the sum of the integer m an the nonnegative integer n.

$$\begin{aligned} P_0 &= 0 \\ P_m(n) &= p_m + n \end{aligned}$$

15) Show that $f_0 f_1 + f_1 + f_2 + ... + f_{2n-1} f2n = f\frac{2}{2n}$
Basis Step: $f_0 f_1 + f_1 f_2 = 0 \cdot 1 + 1 \cdot 1 = 1^2 = f\frac{2}{2}$
Inductive Step: $f_0 f_1 + f_1 f_2 + ... + f_{2k-1} f_{2k} = f\frac{2}{2k}$ Then
$f_0 f_1 + f_1 f_2 + ... + f_{2k-1} f_{2k} + f2kf2k + 1 + f2k + 1 + f_{2k+2} = f\frac{2}{2k} + f_{2k} f2k + 1 + f_{2k+1} f_{2k+2} =$
$f_{2k}(f_{2k} + f_{2k+1} + f_{2k+1} f_{2k+2} = f_{2k} f_{2k+2} + f_{2k+1} f_{2k+2} = (f_{2k} + f_{2k+1}) f_{2k+2} = f\frac{2}{2k+2}$
18) Let

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

Show that $A^n =$

$$\begin{pmatrix} f_{n+1} & f_n \\ f_n & f_{n-1} \end{pmatrix}$$

**I'll be coming to your office hours on thursday for help with this problem.
49) Show that $A(m, 2) = 4$ whenever $m \geq 1$
Basis Case: Assume P(1) or m = 1.

$$\begin{aligned} A(1,2) &- 4thcase \\ A(1-1, A(1, 2-1)) &- 3rdcase \\ A(0,2) &- 1stcase, 2(2) \\ &= 4 \end{aligned}$$

Inductive Step:

$$\begin{aligned} &= A(m, 2) \\ &= A(m+1, 2) \\ &= A(m, A(m+1, 1)) \\ &= A(m, 2) = 4 \end{aligned}$$

**Whenever m is greater then or equal to one and n is greater then or equal to two, it will always trigger the fourth step causing m to decrement until m = 0 is reached.

57) Use strong induction to prove that a function F defined by specifying F(0) and a rule for obtaining F(n+1) from the values F(k) for k = 0,1,2,..., n is well defined.
Basis Step: F(0) = 0 is true
Inductive Step: F(k) is true since f(0) = 0 then f(0+1) or f(1) = 1 when k ¡ n(any integer).
This is a well defined function because recursively defined functions are well defined.