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HW 1.8 REDO

6) Prove using proof by cases that $5x + 5y$ is an odd integer when x and y are integers of opposite parity.

We know that for any integer k and j, $2k+1$ will yield an odd integer and $2j$ will yield an even integer. We have two cases to prove. One in which x is odd and y is even and one in which x is even and y is odd.

case i: If x is odd and y is even:

If we substitute x and y for $2k + 1$ and $2j$ and multiply both out we get: $10k + 10j + 5$.
We are looking to put the equation into the form $2k + 1$ to yield an odd integer.
If we make the equation $10k + 10j + 5$ look like $10k + 10j + 4 + 1$ we are now able to factor out a 2 and leave a one outside the parenthesis. The equation then becomes:
$2(5k + 5j + 2) + 1$
Using the products of integers and sum of integers rule we know that $5k + 5j + 2$ will yield an integer. We are now able to rewrite the equation in the form.
$2(integer) + 1$ which satisfies $5x + 5y =$ an odd integer.

case ii: If x is even and y is odd:

$5(2j) + 2(2k + 1) = 10j + 10k + 5$
Similarly we want the equation to be of the form $2k + 1$
If we rewrite the equation to look like $10j + 10k + 4 + 1$ we are now able to factor out a two while still leaving a one on the outside.
After factoring it becomes $2(5j + 5y + 2) + 1$.
Due to the products of integers being integers and the sum of integers being integers we can rewrite the equation of the form: $2(integer) + 1$

We have proven our two cases.