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HW 6.3

Problems = { 2, 3, 4b, 10, 13, 14, 16, 17, 27, 30, 31 }

2) How many different permutations are there of the set $\{a, b, c, d, e, f, g\}$

7! or $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$

3) How many permutations of $\{a, b, c, d, e, f, g\}$ end with a.

6! or $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

4b) Les S = $\{1, 2, 3, 4, 5\}$

b) List all the 3 combinations of S.

P(5,3) = 60

10) There are six different candidates for the governor of a state. In how many different orders can the names of the candidates be printed on a ballot?

6! or $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$ ways.

13) A group contains n men and n women. How many ways are there to arrange these people in a row if the men and women alternate?

It was useful for me to think about the lists separately for a moment. If I just wanted to find all the possible arrangements of n men it would be $n!$ and similarly for women. The combinations of those two lists would be $n!^2$. Additionally there are 2 possible ways to start a row so we multiply $n!^2$ by 2 yielding $2(n!)^2$.

14) In how many ways can a set of two positive integers less than 100 be chosen?

C(99,2) = 4851.

16) How many subsets with an odd number of elements does a set of 10 elements have?

C(10, 1) + C(10,3) + C(10,5) + C(10,7) + C(10,9) = 512 subsets.

17) How many subsets with more than two elements does a set with 100 elements have?

We want to subtract the subsets that have 0,1 or 2 elements:

$2^100 - C(100, 0) - C(100, 1) - C(100, 2) = 2^100 - 1 - 100 - 4950 = 1.2676506e30$

27) A club has 25 members.

a) How many ways are there to choose four members of the club to serve on an executive committee?

C(25, 4) = 12650

b) How many ways are there to choose a president, vice president, secretary and treasurer of the club, where no person can hold more than one office?

P(25, 4) = 303600

30) Seven women and nine men are on the faculty in the mathematics department at a school.

a) How many ways are there to select a committee of five members of the department if at least one woman must be on the committee?

It's helpful to break this down into two parts. C(16,5) would be the number of ways to select a committee from all the men and women. If we subtract the men from this we will have the number of committees with at least one women. C(16,5) - C(9,5) = 4368 - 126 = 4242.

b) How many ways are there to select a committee of five members of the department if at least one woman and and at least one man must be on the committee?

Again we break this down into C(9,5) ways for a committee of men, C(7,5) ways for a committee of woman. C(16,5) ways including men and women. Since C(16,5) - C(9,5) finds a committee with at least one woman, if we subtract C(7,5) from that we will have a committee with at least one man and one woman. C(16,5)-C(9,5)-C(7,5) = 4221 different committees.

31) The English alphabet contains 21 consonants and five vowels. How many strings of six lowercase letters of the English alphabet contain?

a) Exactly one vowel?

We have 5 vowels in 6 possible positions and consonants in every other position. This becomes $5 \cdot 6 \cdot 21^5 = 122523030$ ways.

b) exactly two vowels?

Same approach. We can choose the vowels in 5^2 ways, choose the position in C(6,2) ways and choose the remaining 4 slots with consonants in 21^4 ways. Multiply them together for
$5^2 \cdot C(6,2) \cdot 21^4 = 72930375$

c) At least one vowel?

All the letters is 26^6 and all the consonants is 21^6. $26^6 - 21^6 = 223149655$ to exclude combinations with no vowels.

d) At least two vowels?

We know that exactly one vowel can be found in 122523030 ways and we know that at least one vowel can be found in 223149655 ways. 223149655 - 122523030 = 100626625.