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HW 3.2

Problems = { 1abc, 2abc, 3, 10, 21, 22, 23, 24, 25ab, 26ab }

1) Determine whether each of these functions is $O(x)$

a) $f(x) = 10$

$x > 10$ whenever $x > 10$

We then replace 10 with x and get $x \leq c * x$

Thus, f(x) is O(x) for $C = 10, k = 1$

b) $3x + 7$

Since $x > 7$ whenever $x > 7$ we can replace 7 with x.

$3x + x \leq 4x$

This yields $4x \leq c * x$

Thus, f(x) is O(x) when C = 4, k = 7

c) We know that a function is O(x) of it's largest exponent. Since $x^2 + x + 1$ has an x^2 we know that this is not O(x). It's in fact $O(x^2)$.

2) Determine whether each of these functions is $O(x^2)$

a) $f(x) = 17x + 11$

Since $x^2 > 11$ whenever $x^2 > 11$ we can write the equation as $17x^2 + x^2$ which becomes $18x^2$.

Thus, f(x) is $O(x^2)$ when C = 18, k = 11

b) $f(x) = x^2 + 1000$

Since we know that $x^2 > 1000$ whenever $x^2 > 1000$ thus we can replace 1000 with x^2.

$x^2 + x^2 \leq 2x^2$

Thus, f(x) is O(x) whenever $C = 2, k = \sqrt{1000}$

c) $f(x) = xlogx$

Since $xlogx \leq x^2$ for all values of x C = 1, k = 0.

3) Use the definition of "f(x) is O(g(x))" to show that $x^4 + 9x^3 + 4x + 7 is O(x^4)$

We first append x^4 to each of the exponents like:

$x^4 + x^4 + x^4 + x^4 = 4x^4$. C = 4, k = 9

10) Show that $x^3 is O(x^4)$ but that x^4 is not $O(x^3)$

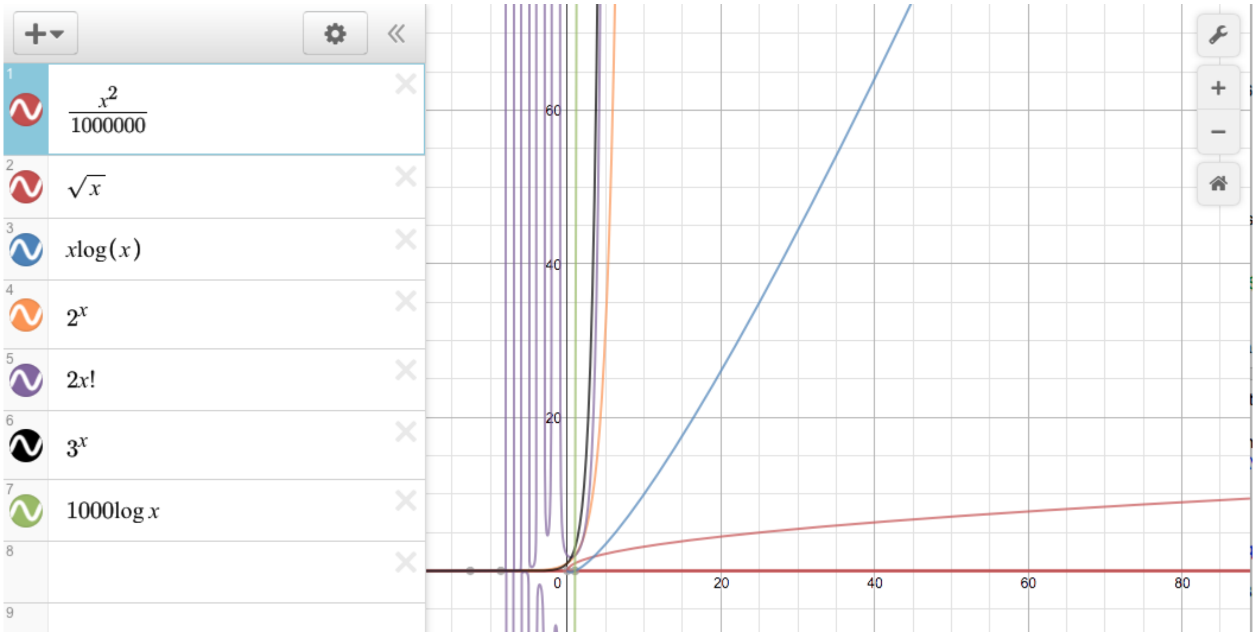
The definition for Big-O notation is $|f(x)| \leq C|g(x)|$

x^3 is $O(x^4)$

$x^3 \leq x^4$ however x^4 is not $\leq x^3$ therefore a larger exponent cannot be Big-O of a smaller exponent.

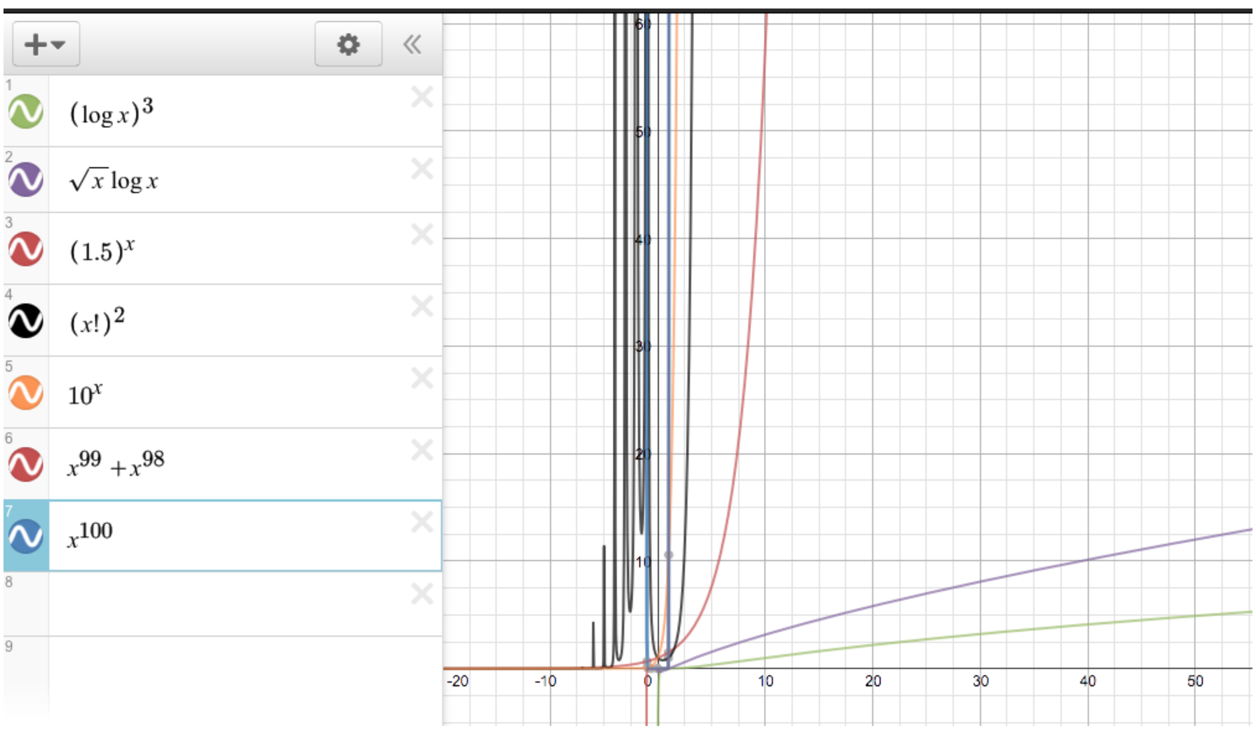
21) Arrange the functions $\sqrt{n}, 1000logn, nlogn, 2n!, 2^n, 3^n$ and $n^2/1,000,000$ in a list so that each function is big-O of the next function.

$\frac{n^2}{1,000,000} > \sqrt{n} > xlog(x) > 2^x > 2x! > 3^x > 1000log(x)$



22) Arrange the functions $(1.5)^n, n^{100}, (long)^3, \sqrt{n}logn, 10^n, (n!)^2$, and $n^{99}+n^{98}$ in a list so that each function is big-O of the next function.

$(logx)^3 > \sqrt{x}logx > (1.5)^x > (x!)^2 > 10^x > x^{99}+x^{98} > x^{100}$



23) Suppose that you have two different algorithms for solving a problem. To solve a problem of size n, the first algorithm uses exactly n(log n) operations and the second algorithm uses exact $n^{\frac{3}{2}}$ operations. As n grows, which algorithm uses fewer operations?

Exponential operations require more operations than logarithmic ones. n(log n) uses fewer operations.

25)Give as good a big-O estimate as possible for each of these functions.

a) $(n^2+8)(n+1)$

n^3+n^2+8n+8

Algorithms are big-O of it's highest exponent. This algorithm is $O(x^3)$

b) $(nlogn+n^2)(n^3+2)$

$n^3logn+2logn+n^5+2n^2$

n^5 is the largest value so, $O(n^5)$

c) $(n!+2^n)(n^3+log(n^2+1))$

$n!n^3$

26) Give a big-O estimate for each of these functions. For the function g in your estimate $f(x)$ is $O(g(x))$, use a simple function g of smallest order.

a) $(n^3+n^2logn)(logn+1)+(17logn+19)(n^3+2)$

$O(n^3)$

b) $(2^n+n^2)(n^3+3^n)$

$3^n n^2$