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Class notes

$\sum_{j=1}^{k} j = \frac{k(k+1)}{2}$

Show $P(k+1)$ is true for:
$\sum_{j=1}^{k+1} j = \frac{(k+1)((k+1)+1)}{2}$

Start with the left side only of the $p(k+1)$ equation and write down something that you know it's equal to.
$\sum_{j=1}^{k+1} j = 1 + 2 + 3 + ... + k + (k+1)$
$\sum_{j=1}^{k+1} j = \frac{k(k+1)}{2} + (k+1)$
Common denominators - algebra

$\frac{k(k+1)}{2} + \frac{2(k+1)}{2} = \frac{k^2+k}{2} + \frac{2k+2}{2}$

Multiply together and factor

$\frac{(k+1)(k+2)}{2}$ Which is the right hand of p(k+1) thus it is proved.

Why is it important to show the basis step?

Prove: $3^n - 2$ is even for all(integers) $n \geq 1$

Assume $3^k - 2$ is even (Show $3^{k+1} - 2$ is even

We know $3^k - 2 = 2j$ for some integer j

$3^k = 2j + 2$

$3x3^k = 3(2j + 2)$

Inductive inequality proofs

Prove: $n^2 \leq n!$ for all integers $n \geq ?$

For example $6! = 6.5.4.3.2.1 = 720$

$0! = 1$ for some reason.

Base Case: $1^2 \leq 1!$ is true and $2^2 \leq 2!$ is false

Maybe $n = 4$ then $4^2 \leq 4!$ which is $16 \leq 24$

Beyond n=4 it starts to diverge even further so 4 as our base case makes sense.

$n^2 \leq n!$ for all integers $n \geq 4$

Now we most likely want to prove n+1 for all integers $k \geq 4$
$(k+1)^2 \leq (k+1)!$

Start with left hand side:

$k^2 + 2k + 1 \leq k! + 2k + 1$ This can potentially work however, use the following:

$k^2 + 2k + 1 \leq k^2 + 2k^2 + 1k^2 = 4k^2$ Use the big O approach where you sub in a k^2 for all values and sum them together.

$k^2 + 2k + 1 \leq 4k^2$ now we move to:

$4k^2 \leq 4 \cdot k!$

$(n+1)! = (n+1) \cdot n!$

$4k^2 \leq (k+1) \cdot k!$

Since $k \geq 4$ then $k + 1 \geq 5$

So $4k! \leq 4k! \leq (k+1)k!$

Show $2^n \leq n!$ for all $n \geq 4$

Basis Step: 4 so $2^4 \leq 4!$ which is $16 \leq 24$ so base case has been proved.

Inductive Step: $2^{k+1} \leq (k+1)!$ We want to prove the left side is equal to the right side

$2^{k+1} = 2(2^k) \leq 2 \cdot k! \leq (k+1)k! = (k+1)!$

Since $k \geq 4$ and $k + 1 \geq 4 > 2$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{2^n} =?$ for all $n \geq 1$

Fill in a formula and prove it by induction.

Examples of small values of n:
n=1
n=2
n=3

$\frac{1}{2^1} = \frac{1}{2}$
$\frac{1}{2^1} + \frac{1}{2^2} = \frac{3}{4}$
$\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} = \frac{15}{16}$

This looks like the equation is $\frac{2^n - 1}{2^n}$

$1 - \frac{1}{2^n}$ for $n \geq 1$

Basis Step: $1 - \frac{1}{2^{(1)}} = \frac{1}{2} = 1 - \frac{1}{2}$

Inductive Step: Assume $\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^{k+1}} = 1 - \frac{1}{2^k+1}$
$\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^k} + \frac{1}{2^{k+1}}$

$1 - \frac{1}{2^k} + \frac{1}{2^{k+1}} = \frac{1}{2^k \cdot 2} + \frac{1}{2^k+1}$

Define $a_1 = 5$ then

$a_{n+1} = 2 \cdot a_n$ for all $n \geq 1$

A non-recursive sequence would be $a_n = \frac{n^2}{4}$

2,6,10,14,18 . . . - Pattern seems to be $a_n + 4 = a_{n+1}$

$a_{n+1} = 2 \cdot a_n$
Find a_1, a_2, a_3, a_4
$a_1 = 5$
$a_2 = 2 \cdot 5$

Non-recursive formula:

5,10,20,40,80,160,320

$a_n = 10 \cdot 2^n$ for $n \geq -1$

We say f is a recursive function if $f : N \to S$ and its defined in two parts.

1) $f(0)$
2) For $n \geq 0, f(n+1)$ is defined in terms of $f(n)$

Specific example:

Define $f : N \to N$
by $f(0) = 1$
and $f(n+1) = (n+1) \cdot f(n)$ for $n \geq 0$

find $f(1), f(2), f(3)$
$f(0)$ occurs when $n = 0$

Define $f : N \to Z$
by $f(0) = -2$
$f(n+1) = f(n)^2 + 4f(n), forn \geq 0 = [f(n)]^2 + 4f(n)$
$f(1) = [f(0)]^2 + 4f(0)$
$f(2) = [f(1)]^2 + 4f(1)$