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HW 2.2

Problems = { 1, 2, 3, 14, 15, 16, 19, 25, 32, 33, 34, 52, 53, 54 }

1) Let A be the set of students who live within one mile of school and let B be the set of students who walk to classes. Describe the students in each of these sets.

a) $A \cap B$

The intersection of A and B is the set of students who live within one mile of school AND students who walk to classes.

b) $A \cup B$

The union of A and B is the set of students who live within one mile of school OR students who walk to classes.

c) $A - B$

The difference of A and B is the set of students who live within one mile of school who are NOT students who walk to classes.

d) $B - A$

The difference between B and A is the set of students who walk to classes who are NOT students who live within one mile of school.

2) Suppose that A is the set of sophomores at your school and B is the set of students in discrete mathematics at your school. Express each of these sets in terms of A and B.

a) The set of sophomores taking discrete mathematics in your school.

$A \cap B$

b) The set of sophomores at your school who are not taking discrete mathematics.

$A - (A \cap B)$

c) The set of students at your school who either are sophomores or are taking discrete mathematics.

$A \cup B$

d) The set of students at your school who either are not sophomores or are not taking discrete mathematics.

$\overline{A} \cup \overline{B}$

3) Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{0, 3, 6\}$. Find the following:

a) $A \cup B$

$A \cup B = \{0, 1, 2, 3, 4, 5, 6\}$

b) $A \cap B$

$A \cap B = \{3\}$

c) $A - B$

$A - B = \{1, 2, 4, 5\}$

d) $B - A$

$B - A = \{0, 6\}$

14) Find the sets A and B if $A - B = \{1, 5, 7, 8\}, B - A = \{2, 10\}$ and $A \cap B = \{3, 6, 9\}$

$A = \{1, 3, 5, 6, 7, 8, 9\}$ and $B = \{2, 3, 6, 9, 10\}$

Starting with $A \cap B$ we know those values are in both A and B.

We then know that the set A - B will be all the values not in B. So we find A by:

$(A \cap B) \cup (A - B) = A$

$(A \cap B) \cup (B - A) = B$

15) Prove the second Demorgan Law in Table 1 by showing that if A and B are sets then $\overline{A \cup B} = \overline{A} \cap \overline{B}$

a) By showing each side is a subset of the other side.

$\overline{A \cup B} = \overline{A} \cap \overline{B}$

$x \in \overline{A \cup B}$ then $x \notin A \cup B$

Meaning $x \notin A$ AND $x \notin B$ since x is not in the union it will not be in the sets.

If $x \notin A$ or $x \notin B$ it will be in the intersection of $\overline{A} \cap \overline{B}$

Since both sides are equivalent they will be subsets of the other side because all elements x are present in both sets.

b) Using a membership table.

See handwritten figure attached to the back of this homework.

16) Let A and B be sets. Show that:

a) $(A \cap B) \subseteq A$

If $x \in A$ and $x \in B$ then the intersection of A and B will be a subset of set A.

b) $A \subseteq (A \cup B)$

A will be a subset of $A \cup B$ because if $x \in A$ then it will be present in the union of A with B.

c) $A - B \subseteq A$

If $x \notin B$ on the left side, and $x \in A$ then we can simplify by showing $A \subseteq A$. All subsets are subsets of themselves.

d) $A \cap (B - A) = \emptyset$

If $x \in A$ on the left side and $x \not{i}nB$ on the right due to the subtraction of A from B, then the two sets will have no shared values and thus will only have the empty set in the union.

e) $A \cup (B - A) = A \cup B$

If $x \in A$ and $x \notin A$ on the right, then $x \in A$ and $x \in B$ in the union of A and B.

19) Show that if A and B are sets, then:

a) $A - B = A \cap \overline{B}$

If $A - B$ then $B \notin A$

If $B \notin A$ then $\overline{B} \in A$

Therefore, $A \cap \overline{B}$ is equivalent to $A - B$

b) $(A \cap B) \cup (A \cap \overline{B}) = A$

See handwritten membership table.

25) Let $A = \{0, 2, 4, 6, 8, 10\}, B = \{0, 1, 2, 3, 4, 5, 6\}$, and $C = \{4, 5, 6, 7, 8, 9, 10\}$ Find:

a) $A \cap B \cap C$

$\{2, 4, 6\}$

b) $A \cup B \cup C$

$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

c) $(A \cup B) \cap C$

$\{4, 5, 6, 8, 9, 10\}$

d) $(A \cap B) \cup C$

$\{0, 2, 4, 5, 6, 7, 8, 9, 10\}$

32) Find the symmetric difference of { 1, 3, 5 } and { 1, 2, 3 }

{ 2, 5 }

33) Find the symmetric difference of the set of computer science majors at a school and the set of mathematics majors at this school.

The symmetric difference are the students who are either computer science majors or mathematics majors.

34) Draw a Venn diagram for the symmetric difference of the sets A and B.

52) Suppose that the universal set is $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Express each of these sets with bit strings where the with bit in the string is 1 if i is in the set and 0 otherwise.

a) { 3,4,5 }

0011100000

b) {1,3,6,10}

1010010001

c) {2,3,4,7,8,9}

0111001110

53) Using the same universal set as the last problem, find the set specified by each of these bit strings.

a) 11 1100 1111

{ 1,2,3,4,7,8,9,10 }

b) 01 0111 1000

{ 2,4,5,6,7 }

c) 10 0000 0001

{ 1, 10 }

54) What subsets of a finite universal set do these bit strings represent?

a) The string with all zeros

The null set.

b) The string with all ones.

It represents the universal set itself.