

$$\text{Problems} = \{ 1, 2, 3, 4, 7, 8, 12, 20, 24, 25, 35, 40, 42abc \}$$

In exercises 1-3 find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Identify all isolated and pendant vertices.

**See attached paper.

4) Find the sum of the degrees of the vertices of each graph in Exercises 1-3 and verify that it equals twice the number of edges in the graph.

Graph 1) $2 + 4 + 1 + 0 + 2 + 3 = 12$ and the $6 \cdot 2 = 12$ therefore the sum of the degrees has been verified.

Graph 2) $6 + 6 + 6 + 5 + 3 = 26$ and $13 \cdot 2 = 26$

Graph 3) $3 + 2 + 4 + 0 + 6 + 0 + 4 + 2 + 3 = 24$ and $12 \cdot 2 = 24$

In Exercises 7-9 determine the number of vertices and edges and find the in-degree and out-degree of each vertex for the given directed multigraph.

**See attached paper.

12) What does the degree of a vertex represent in the acquaintanceship graph, where vertices represent all the people in the world? What does the neighborhood a vertex in this graph represent? What do isolated and pendant vertices in this graph represent? In one study it was estimated that the average degree of a vertex in this graph is 1000. What does this mean in terms of the model?

The degree of the vertex represents the number of acquaintances in relation to that specific vertex or node.

The neighborhood of a vertex in this graph represents a subset of vertex's connected to any vertex. (i.e. $N(a) =$ subset of other vertex's connected to a by an edge)

Isolated vertices represent a person who has no acquaintances.

A pendant vertex represents someone who has only one acquaintance.

If the average degree of a vertex is 1000 then that means, the average number of acquaintances in the graph is 1000 (i.e. there are on average 1000 edges connecting each vertex).

20) Draw these graphs:

**See attached paper.

In exercises 21 to 25 determine whether the graph is bipartite. You may find it useful to apply Theorem 4.

**See attached paper.

35) How many vertices and how many edges do these graphs have?

a) K_n

Vertices: n , edges: $n(n-1) / 2$

b) C_n

Vertices: c , edges: c

c) W_n

Vertices: $n+1$, edges: $2(n-1)$

d) $K_{m,n}$

Vertices: $n+m$, edges: mn

e) Q_n

Vertices: 2^n , edges: $2^{n-1}n$

40) How many edges does a graph have if its degree sequence is 4,3,3,2,2? Draw such a graph.

**See attached paper.