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Class Notes 3.12

5.3 Cont.
Ackermans Function

It is a two variable recursive function. Variables used are m and n. It is piece wise defined.

$A : NxN \rightarrow N$
Takes a natural number of pairs and produces one natural number as output
Defined by $A(m, n)$

$$1)2n, if m = 0$$
$$2)0, if m \geq 1 and n = 0$$
$$3)2, if m \geq 1 and n = 1$$
$$4)A(m-1, A(m, n-1), if m \geq 1, and n \geq 2$$

Compute ackermans function A(m,n):
a) a(0,6)
Since $m = 0$, then 2(6) thus 12.
b) a(6,0)
Since $n = 0$ then it equals 0
c) a(6,1)
Since m is greater then one and n is equal to 1 then the answer is 2
d) a(1, 2)
Since m is greater then or equal to 1 and n is greater then or equal to 2.
A(1-1, A(1, 2,1))
2(1)
Ackermans function is interesting because of how fast it grows.
Claim $A(2, 5) = 2^6 5536$
$A(3, 6)$ is even bigger
Used to test compilers for recursion handling because its so big.
Proof involving Ackermans function and induction.
Fact(to be proved in HW 5.3)
A(1,n) is always 2^n. When n is any natural number.

Prove: $A(2, n) = 2^n$ for any integer n $\geq$ 1.