Steven Murr

3.2 REDO - 2

10) Show that $x^3 is O(x^4)$ but that x^4 is not $O(x^3)$

The definition for Big-O notation is $|f(x)| \leq C|g(x)|$

x^3 is $O(x^4)$

As x grows without bound, x^3 will always be less than or equal to x^4.

However, x^4 is not $O(x^3)$ because:

$$x^4 \leq Cx^3$$
$$\frac{x^4}{x^3} \leq C\frac{x^3}{x^3} - divide$$
$$\frac{x^1}{1} \leq C$$

Since x grows without bound and C is a constant x will not always be less than or equal to C.

21) Arrange the functions $\sqrt{n}, 1000logn, nlogn, 2n!, 2^n, 3^n$ and $n^2/1,000,000$ in a list so that each function is big-O of the next function.

$\frac{n^2}{1,000,000} < \sqrt{n} < xlog(x) < 2^x < 2x! < 3^x < 1000log(x)$

22) Arrange the functions $(1.5)^n, n^{100}, (long)^3, \sqrt{n}logn, 10^n, (n!)^2$, and $n^{99} + n^{98}$ in a list so that each function is big-O of the next function.

$(logx)^3 < \sqrt{x}logx < x^{99} + x^{98} < x^{100} < (1.5)^x < 10^x < (x!)^2$

26) Give a big-O estimate for each of these functions. For the function g in your estimate $f(x)$ is $O(g(x))$, use a simple function g of smallest order.

a) $(n^3 + n^2logn)(logn + 1) + (17logn + 19)(n^3 + 2)$

$O(n^3\ logn)$

b) $(2^n + n^2)(n^3 + 3^n)$

Since 2^n is the maximum Big O for $2^n + n^2$ and 3^n is the maximum Big O for $n^3 + 3^n$ we get 3^n2^n which simplifies to 6^n.