Steven Murr
HW 1.3

2) Show that p and $\neg(\neg p)$ are logically equivalent.

p	$\neg p$	$\neg(\neg p)$
T	F	T
F	T	F

p and $\neg(\neg p)$ have the same truth values in the above truth table. Therefore, they are logically equivalent.

5) Use a truth table to verify the Distributive Law.

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

p	q	r	$(q \vee r)$	$(p \wedge (q \vee r))$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

p	q	r	$(p \wedge q)$	$(p \wedge r)$	$((p \wedge q) \vee (p \wedge r))$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	F	F
F	T	T	F	F	F
F	T	F	F	F	F
F	F	T	F	F	F
F	F	F	F	F	F

The final columns are identical, thus the distributive law is verified and both sides of the statement are logical equivalent.

10) Show that each of these conditional statements is a tautology by using truth tables.

a) $[\neg p \wedge (p \vee q)] \rightarrow q$

p	q	$\neg p$	$(p \vee q)$	$(\neg p \wedge (p \vee q))$	$((\neg p \wedge (p \vee q)) \rightarrow q)$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

The final row is all true thus the conditional statement is a tautology.

b) $[(p \to q) \wedge (q \iff r)] \to (p \to r)$

p	q	r	$(p \to q)$	$(q \to r)$	$((p \to q) \wedge (q \to r))$	$(p \to r)$	$(((p \to q) \wedge (q \to r)) \to (p \to r))$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

The final row is all true thus the conditional statement is a tautology.

c) $((p \wedge (p \to q)) \to q)$

p	q	$(p \to q)$	$(p \wedge (p \to q))$	$((p \wedge (p \to q)) \to q)$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

The final row is all true thus the conditional statement is a tautology.

d) $((p \wedge q) \wedge (p \to r) \wedge (q \to r)) \to r)$

p	q	r	$(p \vee q)$	$(p \to r)$	$(q \to r)$	$((p \to r) \wedge (q \to r))$	$((p \vee q) \wedge ((p \to r) \wedge (q \to r)))$	$(((p \vee q) \wedge ((p \to r) \wedge (q$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	T	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	F	F	T
F	F	T	F	T	T	T	F	T
F	F	F	F	T	T	T	F	T

The final row is all true thus the conditional statement is a tautology.

14) Determine whether $(\neg p \wedge (p \to q)) \to \neg q)$ is a tautology.

p	q	$\neg p$	$(p \to q)$	$(\neg p \wedge (p \to q))$	$\neg q$	$((\neg p \wedge (p \to q)) \to \neg q)$
T	T	F	T	F	F	T
T	F	F	F	F	T	T
F	T	T	T	T	F	F
F	F	T	T	T	T	T

The last column in the above truth table has one false value, thus it is <u>not</u> a tautology.

18) Show that $p \to q$ and $\neg q \to \neg p$ are logically equivalent.

p	q	$(p \to q)$
T	T	T
T	F	F
F	T	T
F	F	T

q	p	$\neg q$	$\neg p$	$(\neg q \to \neg p)$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

The above truth tables have the same truth values in the final column, thus $p \to q$ and $\neg q \to \neg p$ are logically equivalent.

31) Show that $(p \to q) \to r$ and $p \to (q \to r)$ are not logically equivalent.

p	q	r	$(p \to q)$	$((p \to q) \to r)$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	F
F	F	T	T	T
F	F	F	T	F

p	q	r	$(q \to r)$	$(p \to (q \to r))$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	T

The above truth tables have differing values in the final column, thus $(p \to q) \to r$ and $p \to (q \to r)$ are not logically equivalent.

The following exercises involve the logical operators NAND and NOR. The proposition p NAND q is true when either p and q, or both, are false; and it is false when both p and q are true. The proposition p NOR q is true when both p and q are false, and it is false otherwise. The propositions p NAND q and p NOR q are denoted by p — q and $p \downarrow q$ respectively.

46) Construct the truth table for the logical operator $NAND$

p	q	$(pNANDq)$
T	T	F
T	F	T
F	T	T
F	F	T

** The NAND gate is an inverted AND. It is false only when both p and q are True.

47) Show that $p|q$ is logically equivalent to $\neg(p \wedge q)$
** I will reuse the table used in problem 46 to represent the p NAND q table.

p	q	$(p \wedge q)$	$\neg(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

The final column in the p NAND q table used for problem 46 is identical to the final column of the $\neg(p \wedge q)$ truth table. Thus, p NAND q and $\neg(p \wedge q)$ are logically equivalent.

48) Construct a truth table for the logical operator NOR.

p	q	$(pNORq)$
T	T	F
T	F	F
F	T	F
F	F	T

** The is the truth table for the NOR operator. It is an inverted OR table. The table is only True when both p and q are false.

49) Show that $p \downarrow q$ is logically equivalent to $\neg(p \vee q)$
** I will reuse my NOR table from problem 48 for brevity.

p	q	$(p \vee q)$	$\neg(p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

The final column in the truth table for NOR matches the final column in the truth table for $\neg(p \vee q)$ thus, p NOR q and $\neg(p \vee q)$ are logically equivalent.