Steven Murr
HW 1.4

2) Let $P(x)$ be the statement "the word x contains the letter a." What are the truth values?

a) P(orange)

True, because the letter a appears in the word "orange."

b) P(lemon)

False, because the letter a does not appear in the word lemon.

c) P(true)

False, because the letter a does not appear in the world true.

d) P(false)

True, because the letter a appears in the word false.

5) Let $P(x)$ be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English.

a) $\exists x P(x)$

"There exists more than one student who spends more than five hours every weekday in class."

b) $\forall x P(x)$

"Every student spends more than five hours every weekday in class."

c) $\exists \neg P(x)$

"Some of the students don't spend more than five hours every weekday in class."

d) $\forall x \neg P(x)$

"Every student doesn't spend more than five hours every weekday in class."

9) Let $P(x)$ be the statement "x can speak Russian" and let $Q(x)$ be the statement "x knows the computer language C++." Express each of these sentences in terms of $P(x), Q(x)$,quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.

a) There is a student at your school who can speak Russian and who knows C++.

$\exists x(P(x) \land Q(x))$

b) There is a student at your school who can speak Russian but who doesn't know C++.

$\exists x(P(x) \land \neg(Q(x))$

c) Every student at your school either can speak Russian or knows C++.

$\forall x(P(x) \lor Q(x))$

d) No student at your school can speak Russian or knows C++.

$\forall x \neg(P(x) \lor Q(x))$

10) Let $C(x)$ be the statement "x has a cat," let $D(x)$ be the statement "x has a dog," and let $F(x)$ be the statement "x has a ferret." Express each of these statements in terms of $C(x), D(x), F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.

a) A student in your class has a cat, a dog and a ferret.

$\exists x(C(x) \land D(x) \land F(x))$

b) All students in your class have a cat, a dog, or a ferret.

$\forall x(C(x) \lor D(x) \lor F(x))$

c) Some student in your class has a cat and a ferret, but not a dog.

$\exists x(C(x) \land \neg D(x) \land F(x))$

d) No student in your class has a cat, a dog, and a ferret.

$\forall x \neg(C(x) \land D(x) \land F(x))$

e) For each of the three animals, cats, dogs and ferrets, there is a student in your class who has this animal as a pet.

$\exists x(C(x) \lor D(x) \lor F(x))$

11) Let $P(x)$ be the statement "$x = x^2$." If the domain consists of the integers, what are these truth values?

a) P(0)

True. $0^2 = 0$

b) P(1)

True. $1^2 = 1 = 1$

c) P(2)

False. $2^2 = 4 \neq 2$

d) P(-1)

False. $-1^2 = 1 \neq -1$

e) $\exists x P(x)$

True. For some integer x there exists an integer when squared, the value of which is equal to the integer x.

f) $\forall x P(x)$

False. For all values of x, some integers are not equal after squaring to the initial value x.

15) Determine the truth value of each of these statements if the domain for all variables consists of all integers.

a) $\forall n(n^2 \geq 0)$

True. Any integer squared will always be greater than or equal to 0.

b) $\exists n(n^2 = 2)$

False. There doesn't exist an integer whose value when squared is equal to 2.

c) $\forall n(n^2 \geq n)$

True. For all integers, when the integer n is squared, it is always greater than the integer n.

d) $\exists n(n^2 < 0)$

False. There is no integer n when n is squared is less than 0.

16) Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

a) $\exists x(x^2 = 2)$

True. There exists a real number x that when squared = 2. $\sqrt{2}$

b) $\exists x(x^2 = -1)$

False. There exists no real number that when squared equals -1.

c) $\forall x(x^2 + 2 \geq 1)$

True. For all real numbers, when real number x is squared and 2 added to it, the resulting number will always be greater than 1.

d) $\forall x(x^2 \neq x)$

False. For all real numbers, there exists one number x that when squared is equal to the number x.

32) Express each of these statements using quantifiers. Then form the negation of the statement so that no negation to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase "It is not the case that.")

a) All dogs have fleas.

$\forall x D(x)$

Negation: $\exists x \neg D(x)$

There exists a dog that does not have fleas.

b) There is a horse that can add.

$\exists x H(x)$

Negation: $\exists x \neg H(x)$

There exists a horse that can not add.

c) Every koala can climb.

$\forall x C(x)$

Negation: $\exists x \neg C(x)$

There exists a koala that can't climb.

d) No monkey can speak French.

$\neg \forall x M(x)$

Negation: $\exists x M(x)$

There exists a monkey who can speak French.

e) There exists a pig that can swim and catch fish.
$\exists x S(x) \wedge F(x)$
Negation: $\exists x \neg S(x) \wedge \neg F(x)$
There exists a pig that can not swim and catch fish.

51) Show that $\exists x P(x) \wedge \exists x Q(x)$ and $\exists x(P(x) \wedge (Q(x))$ are not logically equivalent.
The same letter is being used to represent variables bound by different quantifiers with scopes that do not overlap. Imagine $P(x) =$ "beach is sunny today" and $Q(x) =$"beach is overcast today." The first example reads as "There exists a beach that is sunny today AND there exists a beach that is overcast today." While the second statement reads as "There exists a beach that is sunny and overcast." Since the first example uses two different scopes and the second example uses only one, they are not logically equivalent.

52) As mentioned in the text, the notation $\exists! x P(x)$ denotes
"There exists a unique x such that P(x) is true."
If the domain consists of all integers, what are the truth values of these statements?
a) $\exists! x(x > 1)$
False. There are many integers greater than 1.
b) $\exists! x(x^2 = 1)$
False. The integers 1 and negative 1 are both 1 when squared.
c) $\exists! x(x + 3 = 2x)$
True. Only positive 1 will satisfy x+3 = 2x.
d) $\exists! x(x = x + 1)$
False. There exists no integer x that when 1 is added to it becomes x.

60) Let P(x), Q(x), and R(x) be the statements "x is a clear explanation," "x is satisfactory," and "x is an excuse," respectively. Suppose that the domain for x consists of all English text. Express each of these statements using quantifiers, logical connectives, and P(x), Q(x), and R(x).
a) All clear explanations are satisfactory.
$\forall x P(x) \rightarrow Q(x)$
b) Some excuses are unsatisfactory.
$\exists x R(x) \rightarrow \neg Q(x)$
c) Some excuses are not clear explanations.
$\exists x R(x) \rightarrow \neg P(x)$
d) Does (c) follow from (a) and (b)?
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