Pole length: $l = 0.5$

Pole mass: $m = 0.5$

Cart mass: $M = 0.5$

Friction constant: $b = 0.1$

Force due to gravity: $g = 9.82$

Let $x = \begin{bmatrix} x \\ \dot{x} \\ \dot{\theta} \\ \theta \end{bmatrix}$

$$\dot{x} = \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \ddot{\theta} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dfrac{2ml\dot{\theta}^2\sin\theta + 3mg\sin\theta\cos\theta + 4(u - b\dot{x})}{4(M+m) - 3m\cos^2\theta} \\ \dfrac{-3(ml\dot{\theta}^2\sin\theta\cos\theta + 2((M+m)g\sin\theta + (u-b\dot{x})\cos\theta))}{l(4(M+m) - 3m\cos^2\theta)} \\ \dot{\theta} \end{bmatrix}$$

$$\dot{x} = \nabla_{\vec{x}} f(\vec{x},u)[(\vec{x},u)]\vec{x} + \nabla_u f(\vec{x},u)[(\vec{x},u)]u$$

Let $x_1 = x$, $x_2 = \dot{x}$, $x_3 = \dot{\theta}$, $x_4 = \theta$

To get the Jacobian matrix, A, we do $J_x(x_0, u_0)$ where columns are $\left.\dfrac{\partial f_i}{\partial x_1}\right|_{x_0,u_0}, \dots, \left.\dfrac{\partial f_i}{\partial x_4}\right|$ $\quad i = [1, 4]$

Let $f_1 = x_2$

$$f_2 = \frac{2mlx_3^2\sin(x_4) + 3mg\sin(x_4)\cos(x_4) + 4(u - bx_2)}{4(M+m) - 3m\cos^2(x_4)}$$

$$f_3 = \frac{-3(mlx_3^2\sin(x_4)\cos(x_4) + 2((M+m)g\sin(x_4) + (u - bx_2)\cos(x_4)))}{l(4(M+m) - 3m\cos^2(x_4))}$$

$$= \frac{-3mlx_3^2\sin(x_4)\cos(x_4) - 6((M+m)g\sin(x_4)) - 6(u-bx_2)\cos(x_4)}{4l(M+m) - 3lm\cos^2(x_4)}$$

$f_4 = x_3 = \dot{\theta}$

$$\frac{\partial f_1}{\partial x_1} = 0 \qquad \frac{\partial f_1}{\partial x_2} = 1 \qquad \frac{\partial f_1}{\partial x_3} = 0 \qquad \frac{\partial f_1}{\partial x_4} = 0$$

$$\frac{\partial f_2}{\partial x_1} = 0 \qquad \frac{\partial f_2}{\partial x_2} = 0 + 0 + \frac{4b}{8m - 3m\cos^2(\theta)}$$

$$\frac{\partial f_3}{\partial x_1} = 0 \qquad \frac{\partial f_3}{\partial x_2} = 0 + 0 + \frac{6b\cos(\theta)}{8lm - 3lm\cos^2(\theta)}$$

$$\frac{\partial f_4}{\partial x_1} = 0 \qquad \frac{\partial f_4}{\partial x_2} = 0$$

$$\frac{\partial f_2}{\partial x_3} = \frac{4 \cdot l \cdot \dot{\theta}\sin(\theta)}{8 - 3\cos^2(\theta)} + 0 + 0$$

$$\frac{\partial f_3}{\partial x_3} = \frac{3\dot{\theta}\sin(2\theta)}{8 - 3\cos^2(\theta)}$$

$$\frac{\partial f_4}{\partial x_3} = 1$$

$$\frac{\partial f_2}{\partial x_4} = \frac{\left(2 \cdot l \cdot m \cdot \dot{\theta}^2\cos(\theta) + 29.4m\cos(2\theta)\right)\left(8m - 3m\cos^2(\theta)\right) - 3m\sin(2\theta)\left(2lm\dot{\theta}^2\sin(\theta) + 14.7m\sin(2\theta) + 4(u - b \cdot \dot{x})\right)}{\left(8m - 3m\cos^2(\theta)\right)^2}$$

$$\frac{\partial f_3}{\partial x_4} = \frac{\left[-3 \cdot l \cdot m \cdot \dot{\theta}^2\cos(2\theta) - 117.6m\cos(\theta) + 6\sin(\theta)(u - b \cdot \dot{x})\right] \cdot \left[8lm - 3lm\cos^2(\theta)\right] - 3lm\sin(2\theta)\left[-1.5lmz^2\sin(2\theta) - 6\cos(\theta)(u - b \cdot \dot{x})\right]}{\left(8 \cdot l \cdot m - 3 \cdot lm\cos^2(\theta)\right)^2}$$

$$\frac{\partial f_4}{\partial x_4} = 0$$

$A =$ linearized at $x = [0, 0, 0, \pi]^T$

$u = 0$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{4b}{5m} & 0 & 5.88 \\ 0 & -\frac{6b}{5m} & 0 & \text{[redacted]}\ \frac{23.52}{l} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \frac{\partial f_1}{\partial u} = 0$$

$$\frac{\partial f_2}{\partial u} = \frac{4}{8m - 3m\cos^2(\theta)}$$

$$\frac{\partial f_3}{\partial u} = \frac{-6\cos(\theta)}{8lm - 3lm\cos^2(\theta)}$$

$$\frac{\partial f_4}{\partial u} = 0$$

$\Rightarrow$ linearized at $x = [0\ 0\ 0\ \pi]^T$

$u = 0$

$$\begin{bmatrix} 0 & \frac{4}{5m} & \frac{6}{5lm} & 0 \end{bmatrix}$$