

Summary



- 1 Function
- 2 First-class Function (FcF)
- 3 Higher-order Function (HoF)
- 4 Currying and Uncurrying
- 5 Function Application
- 6 Evaluation and Termination



Function

Function is an organized block that contains expression(s) to perform one or more action.

or **in Math**: to map from *input* to *output*.



Function's Definition

- Function declaration with a name. In Scala, called **method**.
- Function expression without a name (anonymous / lambda). In Scala, called **function**.



Functions terms in this Praktikum

But, in this Praktikum use these terms to distinguish function:

- Function declaration with a name but not a member of an object or class, called **function**.
- Function declaration with a name and a member of an object or class, called **method**.
- Function expression without a name (anonymous), called **lambda**.



Method / Function

```

1 def funName [typParam] (param1: typ1, ..., paramN: typN) :
  retType =
2   exp1
3   ...
4   expN
5   lastExp

```

Listing: Method declaration format

def method annotation (keyword)

funName method's name

typParam type parameter

param1 ... paramN parameter

typ1 ... typN type of parameter

retType type of return value

exp1 ... expN lastExp list of expressions in the body of the method

lastExp last expression is the return value for method. Type of lastExp must be the same with retType.

Method / Function - example

```
1 def multiply (fstNum: Int, sndNum: Int) : Int =  
2   val returnValue = fstNum * sndNum  
3   returnValue  
4  
5 val nine = multiply(3,3)
```

Listing: Method/function - example1



Method / Function - example

```
1 def idFun[A](a: A) : A = a
2
3 val nineIsNime = idFun(9)
4
5 val mulFun = idFun(multiply)
6 val anotherNine = mulFun(3,3)
```

Listing: Method/function - example2

The **identity function** is a function that always returns the value that was used as its argument, unchanged. That is, when f is the identity function, the equality $f(X) = X$ is true for all values of X to which f can be applied.



Method / Function - Nested

```
1 def multThenSquare(a: Int, b: Int) : Int =  
2   def squareIt(x: Int) : Int =  
3     x * x  
4     squareIt(a * b)  
5  
6 val _36 = multThenSquare(2,3)
```

Listing: Method/function - nested



Function (a.k.a Lambda)

```
1 (param1: typ1, ... , paramN: typN) =>  
2   exp1  
3   ...  
4   expN  
5   lastExp
```

Listing: Lambda declaration format

`param1 ... paramN` parameter

`=>` separator between param and body

`typ1 ... typN` type of parameter

`exp1 ... expN lastExp` list of expressions in the body of the method

`lastExp` last expression is the return value for lambda.



Lambda - example

```
1 val add = (a: Int, b: Int) => a + b
2
3 val seven = add(3,4)
```

Listing: Lambda - example1



First-class Function (FcF)

FcF is a function that **only accepts values** as input (parameter) and returns **only values** as output.



FcF - example

```
1 def multiply (fstNum: Int, sndNum: Int) : Int =  
2   val returnValue = fstNum * sndNum  
3   returnValue  
4  
5 val nine = multiply(3,3)
```

Listing: FcF - example1



Is this an FcF?

```
1 def pow(n: Int, pw: Int): Int = pw match
2   case 0 => 1
3   case x if x < 0 => sys.error("It can only be raised to the
4     power of positive numbers.")
5   case 1 => n
6   case x if x > 1 => n * pow(n, (pw-1))
```

Listing: Is this an FcF?



Higher-order Function (HoF)

HoF is a function that can takes value(s) or function(s) as parameter(s) and return a value or function.



HoF - Method / Function

```

1  def funName [typParam] (param1: typ1, ..., paramN: typN) :
    retType =
2  exp1
3  ...
4  expN
5  lastExp
    
```

Listing: Method declaration format

def method annotation (keyword)

funName method's name

typParam type parameter

param1 ... paramN parameter, **value(s) or function(s)**

typ1 ... typN type of parameter

retType type of return value

exp1 ... expN lastExp list of expressions in the body of the method

lastExp last expression is the return value for method. Type of lastExp must be the same with retType, can be **value(s) or function(s)**

HoF - Lambda

```
1 (param1: typ1, ... , paramN: typN) =>  
2   exp1  
3   ...  
4   expN  
5   lastExp
```

Listing: Lambda declaration format

`param1 ... paramN` parameter, **value(s) or function(s)**

`=>` separator between param and body

`typ1 ... typN` type of parameter

`exp1 ... expN lastExp` list of expressions in the body of the method

`lastExp` last expression is the return value for lambda.



Higher-order Function (HoF)

```
1 def hofArithOperator(a: Int, b: Int, f: (Int, Int) => Int) :  
  Int =  
2   f(a,b)  
3  
4 def addOptr(a: Int, b: Int) : Int = a + b  
5 def multOptr(a: Int, b: Int) : Int = a * b  
6 def subtrOptr(a: Int, b: Int) : Int = a - b  
7 def divOptr(a: Int, b: Int) : Int = b match  
8   case 0 => sys.error("arithmetic error, devide by zero")  
9   case _ => a / b  
10  
11 val twoPlusThree = hofArithOperator(2,3,addOptr)
```

Listing: HoF - Ex-1



Higher-order Function (HoF)

```
1 def urlBuilder(ssl: Boolean, domainName: String) : (String,  
   String) => String =  
2   val schema = if ssl then "https://" else "http://"  
3   (endPoint: String, query: String) => s"$schema$domainName/  
   $endPoint?$query"  
4  
5   val domName = "www.example.com"  
6   def getUrl = urlBuilder(true, domName)  
7   val endPoint = "users"  
8   val query = "id=1"  
9   val url = getUrl(endPoint, query)
```

Listing: HoF - Ex-2



Higher-order Function (HoF)

```
1 def urlBuilder(ssl: Boolean, domainName: String) : (String,  
  String) => String =  
2   val schema = if ssl then "https://" else "http://"  
3   (endPoint: String, query: String) => s"$schema$domainName/  
    $endPoint?$query"  
4  
5   val domName = "www.example.com"  
6   def getUrl = urlBuilder(true, domName)  
7   val endPoint = "users"  
8   val query = "id=1"  
9   val url = getUrl(endPoint, query)
```

Listing: HoF - Ex-2



Currying

Currying is a function that takes an input (one parameter) at a time.

```
1 def fHoFArithOptCurrying (a: Int) (b: Int) (f: (Int, Int) =>
    Int) : Int = f(a,b)
```

Listing: Currying

Take a parameter at a time

```
2 val needB = fHoFArithOptCurrying(4)
3 val needF = needB(3)
4 val ex2_seven = needF(add0ptr)
```

Listing: Currying - function application1

Take all parameters at a time

```
5 val ex1_seven = fHoFArithOptCurrying(4)(3)(add0ptr)
```

Listing: Currying - function application2



Uncurrying

Uncurrying is a function that takes all inputs (parameters) at a time.

```
1 def fHoFArithOptUncurrying (a: Int, b: Int, f: (Int,Int)=>  
   Int) : Int = f(a,b)
```

Listing: Uncurrying

Take all parameters at a time

```
2 // val anError = fHoFArithOptUncurrying(4)  
3 val ex3_seven = fHoFArithOptUncurrying(4, 3, addOpttr)
```

Listing: Uncurrying - function application



Uncurrying to Currying

In Scala, the wildcard '_' will make the **Uncurrying function** act like a **Currying function**.

```
def fHoFArithOptUncurrying (a: Int, b: Int, f: (Int,Int)=>
  Int) : Int = f(a,b)
```

Listing: Uncurrying

Take all parameters at a time

```
2 val needBUc = fHoFArithOptUncurrying(4, _ , _)
3 val needFUc = needBUc(3, _)
4 val exUc_seven = needFUc(add0ptr)
```

Listing: Uncurrying - function application



Function application (Calling a function)

There are two type of function application:

- **Total application**, call a function with **complete** input (parameter).
- **Partial application**, call a function with **incomplete** input (parameter).

```
1 def fHoFArithOptUncurrying (a: Int, b: Int, f: (Int,Int)=>
   Int) : Int = f(a,b)
```

Listing: Uncurrying

Total application

```
2 val ex3_seven = fHoFArithOptUncurrying(4, 3, addOptpr)
```

Listing: Function application - Total

Partial application

```
3 val needBUc = fHoFArithOptUncurrying(4, _ , _)
4 val needFUc = needBUc(3, _)
```

Listing: Function application - Partial



Evaluation and Termination

This topic aims to explain how an expression is evaluated and how it has ended. There are two types of evaluation:

- **Call by Value** (CbV), *immediately evaluate an expression to a value*. This is the default expression evaluation method in Scala.
- **Call by name** (CbN), *substitute the variable with the expression and evaluate it when needed*. The annotation to distinct is ' \Rightarrow '.



Evaluation and Termination

```
1 def addOptr(a: Int, b: Int) : Int = a + b
```

Listing: a square function

```
2 def sumOfSquareCbV(a: Int, b: Int) : Int =  
3   multOptr(a) + multOptr(b)  
4  
5 val cbv_exm = sumOfSquareCbV(2, 3+3)
```

Listing: CbV

```
6 def sumOfSquareCbN(a: Int, b: => Int) =  
7   multOptr(a) + multOptr(b)  
8  
9 val cbv_exm = sumOfSquareCbN(2, 3+3)
```

Listing: CbN



Evaluation and Termination

Table: CbV vs CbN

Step	CbV	CbN
1	sumOfSquareCbV(2, 3+3)	sumOfSquareCbN(2, 3+3)



Evaluation and Termination

Table: CbV vs CbN

Step	CbV	CbN
1	sumOfSquareCbV(2, 3+3)	sumOfSquareCbN(2, 3+3)
2	sumOfSquareCbV(2, 6)	multOptr(2) + multOptr(3+3)



Evaluation and Termination

Table: CbV vs CbN

Step	CbV	CbN
1	sumOfSquareCbV(2, 3+3)	sumOfSquareCbN(2, 3+3)
2	sumOfSquareCbV(2, 6)	multOptr(2) + multOptr(3+3)
3	multOptr(2) + multOptr(6)	(2 * 2) + multOptr(3+3)



Evaluation and Termination

Table: CbV vs CbN

Step	CbV	CbN
1	sumOfSquareCbV(2, 3+3)	sumOfSquareCbN(2, 3+3)
2	sumOfSquareCbV(2, 6)	multOptr(2) + multOptr(3+3)
3	multOptr(2) + multOptr(6)	(2 * 2) + multOptr(3+3)
4	(2 * 2) + multOptr(6)	(4) + multOptr(3+3)



Evaluation and Termination

Table: CbV vs CbN

Step	CbV	CbN
1	sumOfSquareCbV(2, 3+3)	sumOfSquareCbN(2, 3+3)
2	sumOfSquareCbV(2, 6)	multOptr(2) + multOptr(3+3)
3	multOptr(2) + multOptr(6)	(2 * 2) + multOptr(3+3)
4	(2 * 2) + multOptr(6)	(4) + multOptr(3+3)
5	(4) + multOptr(6)	(4) + multOptr(6)



Evaluation and Termination

Table: CbV vs CbN

Step	CbV	CbN
1	sumOfSquareCbV(2, 3+3)	sumOfSquareCbN(2, 3+3)
2	sumOfSquareCbV(2, 6)	multOptr(2) + multOptr(3+3)
3	multOptr(2) + multOptr(6)	(2 * 2) + multOptr(3+3)
4	(2 * 2) + multOptr(6)	(4) + multOptr(3+3)
5	(4) + multOptr(6)	(4) + multOptr(6)
6	(4) + (6 * 6)	(4) + (6 * 6)



Evaluation and Termination

Table: CbV vs CbN

Step	CbV	CbN
1	sumOfSquareCbV(2, 3+3)	sumOfSquareCbN(2, 3+3)
2	sumOfSquareCbV(2, 6)	multOptr(2) + multOptr(3+3)
3	multOptr(2) + multOptr(6)	(2 * 2) + multOptr(3+3)
4	(2 * 2) + multOptr(6)	(4) + multOptr(3+3)
5	(4) + multOptr(6)	(4) + multOptr(6)
6	(4) + (6 * 6)	(4) + (6 * 6)
7	(4) + (36)	(4) + (36)



Evaluation and Termination

Table: CbV vs CbN

Step	CbV	CbN
1	sumOfSquareCbV(2, 3+3)	sumOfSquareCbN(2, 3+3)
2	sumOfSquareCbV(2, 6)	multOptr(2) + multOptr(3+3)
3	multOptr(2) + multOptr(6)	(2 * 2) + multOptr(3+3)
4	(2 * 2) + multOptr(6)	(4) + multOptr(3+3)
5	(4) + multOptr(6)	(4) + multOptr(6)
6	(4) + (6 * 6)	(4) + (6 * 6)
7	(4) + (36)	(4) + (36)
8	40	40

Evaluation and Termination

Will all expressions terminate?

```
1 def loop : Int = loop
2
3 def exCbV (a: Int, b: Int) : Int = a
4 def exCbN (a: Int, b: => Int) : Int = a
5
6 val cbv = exCbV(4, loop)
7 val cbn = exCbN(4, loop)
```

Listing: cbv vs cbn



Evaluation and Termination

Table: CbV vs CbN

Step	CbV	CbN
1	exCbV(4, loop)	exCbN(4, loop)



Evaluation and Termination

Table: CbV vs CbN

Step	CbV	CbN
1	exCbV(4, loop)	exCbN(4, loop)
2	evaluate 4 then evaluate <i>loop</i>	evaluate 4 and binding <i>b</i> with <i>loop</i>



Evaluation and Termination

Table: CbV vs CbN

Step	CbV	CbN
1	exCbV(4, loop)	exCbN(4, loop)
2	evaluate 4 then evaluate <i>loop</i>	evaluate 4 and binding <i>b</i> with <i>loop</i>
3	evaluate <i>loop</i> because it is waiting for the value of loop	return 4 because it doesn't need b, so b has never evaluated



Evaluation and Termination

Table: CbV vs CbN

Step	CbV	CbN
1	exCbV(4, loop)	exCbN(4, loop)
2	evaluate 4 then evaluate <i>loop</i>	evaluate 4 and binding <i>b</i> with <i>loop</i>
3	evaluate <i>loop</i> because it is waiting for the value of loop	return 4 because it doesn't need b, so b has never evaluated
4	evaluate <i>loop</i>	



Evaluation and Termination

Table: CbV vs CbN

Step	CbV	CbN
1	exCbV(4, loop)	exCbN(4, loop)
2	evaluate 4 then evaluate <i>loop</i>	evaluate 4 and binding <i>b</i> with <i>loop</i>
3	evaluate <i>loop</i> because it is waiting for the value of loop	return 4 because it doesn't need b, so b has never evaluated
4	evaluate <i>loop</i>	
5	...	



Evaluation and Termination

Table: CbV vs CbN

Step	CbV	CbN
1	exCbV(4, loop)	exCbN(4, loop)
2	evaluate 4 then evaluate <i>loop</i>	evaluate 4 and binding <i>b</i> with <i>loop</i>
3	evaluate <i>loop</i> because it is waiting for the value of loop	return 4 because it doesn't need b, so b has never evaluated
4	evaluate <i>loop</i>	
5	...	
6	infinite evaluate <i>loop</i>	