Population Modeling

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Introduction

- exponential, logistic, Gompertz
- Populations: US and Munich, Germany
- Analyzing Models
  - The models ability to predict
  - How close the data fits the model

PPGR and Differential Equations

Per Unit Population Growth Rate (PPGR)$= \frac{1}{p}\frac{dp}{dt}$

Model	Differential Equation	PPGR	$\lim_{p\to 0}$ PPGR	$\lim_{p\to L}$ PPGR
exponential	$\frac{dp}{dt} = rp$	r	r	r
logistic	$\frac{dp}{dt} = rp\left(1-\frac{p}{L}\right)$	$r\left(1-\frac{p}{L}\right)$	r	0
Gompertz	$\frac{dp}{dt} = rp\log\left(\frac{L}{p}\right)$	$r\log\left(\frac{L}{p}\right)$	∞	0

The Project

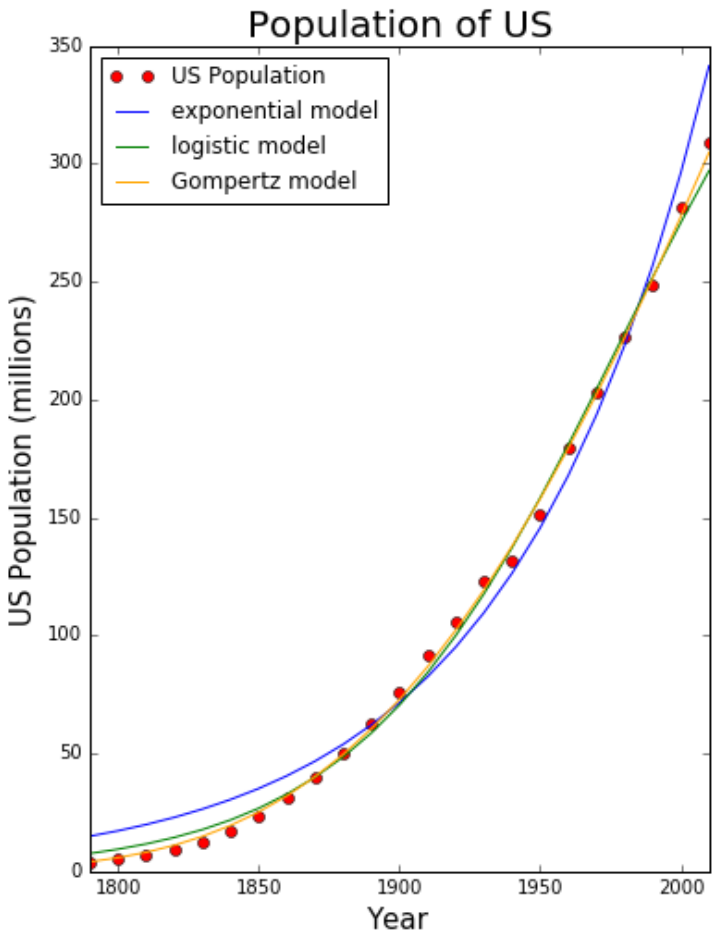
Solutions to differential equations	
Model	Solutions
exponential	$P(t) = Ae^{rt}$
logistic	$P(t) = \frac{L}{1+Ae^{-rt}}$
Gompertz	$P(t)Le^{-be^{-rt}}$

The project steps

- Data for US and Munich, Germany
- Used Python to estimate the parameters in the solutions
- Found R^2: This is our measure of fit
- Found % error for the actual last data point and the estimated point using the Leave One Out Method

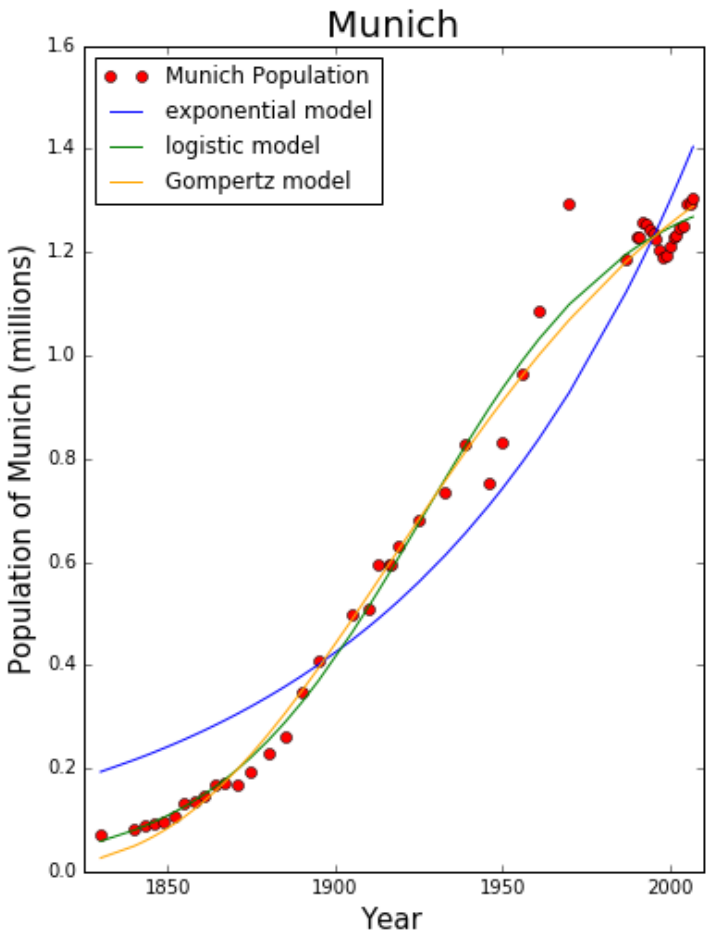
Testing the Model

Leave One Out Method		
	R^2 error	% error
exponential	0.9837	10.58%
logistic	0.9971	-3.81%
Gompertz	0.9989	-1.28%



Testing the Model

Leave One Out Method		
	R^2 error	% error
exponential	0.9383	7.56%
logistic	0.9909	-2.83%
Gompertz	0.9898	-1.26%



Conclusion

- Due to the Gompertz model, for both populations, having the lowest percent error and the largest R^2 error we decided it was the better model.
- Realistically none are very good:
  - Immigration and emigration
  - Always increasing