Engineering Optics, Homework 7

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1 Problem 1

Chapter 11, Problem 1

The standerdized electric wave equation is given by

$$\mathbf{E} = \mathbf{A}cos\left[2\pi\nu(\frac{z}{c} - t) + \phi\right] \tag{1}$$

form the given equation, we can see that

$$\begin{aligned}
&\omega = 2\pi\nu = 2\pi \cdot 10^{14}\mathbf{rad/s} \\
&\nu = 10^{14}\mathbf{Hz} \\
&\lambda = cT = \frac{c}{\nu} = \frac{3\times 10^8}{10^{14}} = 3\times 10^{-6}\mathbf{m} \\
&A = 2\mathbf{V/m}
\end{aligned}$$

when $z = 0$, $t = 0$, we can solve that $\boldsymbol{\phi_0 = \frac{\pi}{2}}$. Also, we can know from the expression that electric vector is along the **y-axis direction**.

2 Problem 2

Chapter 11, Problem 2

The standerdized electric wave equation is given by

$$\mathbf{E} = \mathbf{A}cos\left[2\pi\nu(\frac{z}{c/n} - t)\right] = \mathbf{A}cos\left[2\pi\nu(\frac{z}{\lambda} - \nu t)\right] = 10^2\cos\left[2\pi \cdot \frac{10^15}{2} \cdot \left(\frac{z}{0.65c} - t\right)\right] \tag{2}$$

Therefore, comparing the given equation with the standerdized electric wave equation, we can solve that

$$\begin{aligned}
&\nu = \frac{10^15}{2} = 5\times 10^{14}\mathbf{Hz} \\
&\lambda = \frac{2\times 0.65c}{10^15} = 3.9\cdot 10^{-7}\mathbf{m} = 390\mathbf{nm} \\
&n = \frac{1}{0.65} = 1.538
\end{aligned}$$

3 Problem 3

Chapter 11, Problem 3

The standerdized complex form of the electric wave equation is given by

$$\mathbf{E} = \mathbf{A} \cdot \exp\left[i\mathbf{k}(x\cos\alpha + y\cos\beta + z\cos\gamma)\right] \tag{3}$$

By given, we have

$$\cos\alpha = \frac{\sqrt{3}}{2} \tag{4}$$
$$\cos\beta = \frac{1}{2} \tag{5}$$
$$\cos\gamma = 0 \tag{6}$$

Therefore, we can solve that

$$A_0 = x_0 cos120^\circ + y_0 cos30^\circ$$
$$k_0 = x_0 \cos 30^\circ + y_0 \cos 60^\circ$$

4 Problem 4

Chapter 11, Problem 4

the optical path change Δ is given by

$$\Delta = (n-1)h = 0.005\mathbf{mm}$$

and the phase difference δ is given by

$$\delta = \frac{\Delta}{\lambda} \cdot 2\pi = \frac{0.005 \times 10^6}{0.5} \cdot 2\pi = 20\pi\mathbf{rad}$$

5 Problem 5

Chapter 11, Problem 12

The incident angle i_b is given by the Brewster's law

$$\tan i_b = \arctan\frac{n_2}{n_1}$$

and based on the geometry relationship, we have

$$\tan i_z = \frac{\pi}{2} - i_b$$

Therefore,

$$\tan i_z = \cot i_b = \frac{n1}{n2} = \frac{n3}{n2} = i_{zb}$$

That means the full polarization will also occur at the bottom surface of the glas plate.

6 Problem 6

Chapter 11, Problem 23

By given, we have

$$\begin{aligned}
E = E_1 + E_2 &= a_1 \cos \omega t - \alpha_1 + a_2 \cos(\omega t - \alpha_2) \\
&= Acos(\alpha - \omega t) \\
A^2 &= a_1^2 + a_2^2 + 2a_1 a_2 \cos(\alpha_1 - \alpha_2) \\
&= 10\mathbf{V/m} \\
\tan \alpha &= \frac{a_1 \sin \alpha_1 + a_2 \sin \alpha_2}{a_1 \cos \alpha_1 + a_2 \cos \alpha_2} \\
&= \frac{4}{3}
\end{aligned}$$

Therefore, we can solve that

$$E = 10 \cos(53.13^\circ - 2\pi \times 10^{15} t)$$

7 Problem 7

Chapter 11, Problem 24

$$\begin{aligned}
E_1 &= \alpha \cos(kx + \omega t) = A \exp[i(-kx - \omega t)] \\
E_2 &= -\alpha \cos(kx - \omega t) = -A \exp[i(kx - \omega t)]
\end{aligned}$$

Therefore

$$\begin{aligned}
E = E_1 + E_2 &= Aexp[i(-kx - \omega t)] - Aexp[i(kx - \omega t)] \\
&= 2iA \sin kx \exp[i(-\omega t)] \\
\Rightarrow E &= -2a \sin kx \cos \omega t
\end{aligned}$$

8 Problem 8

Chapter 11, Problem 30

When $z = 0$, we have

$$\begin{aligned}
\omega t = 0 &\Rightarrow E_x = A, E_y = -\frac{A}{\sqrt{2}} \\
\omega t = \frac{\pi}{4} &\Rightarrow E_x = \frac{A}{\sqrt{2}}, E_y = -A \\
\omega t = \frac{\pi}{2} &\Rightarrow E_x = 0, E_y = -\frac{A}{\sqrt{2}} \\
\omega t = \pi &\Rightarrow E_x = -A, E_y = \frac{A}{\sqrt{2}} \\
\omega t = \frac{3\pi}{2} &\Rightarrow E_x = 0, E_y = \frac{A}{\sqrt{2}}
\end{aligned}$$

Therefore, this is **right-polarized light**.