Principles of Database Systems
Assignment #3 - Relation Algebra

He Tianyang, 3022001441

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1 Exercise 2.4.1

This exercise builds upon the products schema of Exercise 2.3.1. Recall that the database schema consists of four relations, whose schemas are:

- `Product(maker, model, type)`
- `PC(model, speed, ram, hd, price)`
- `Laptop(model, speed, ram, hd, screen, price)`
- `Printer(model, color, type, price)`

Some sample data for the relation `Product` is shown in Fig. 1. Sample data for the other three relations is shown in Fig. 2. Manufacturers and model numbers have been "sanitized," but the data is typical of products on sale at the beginning of 2007.

Write expressions of relational algebra to answer the following queries. You may use the linear notation of Section 2.4.13 if you wish. For the data of Figs. 1 and 2, show the result of your query. However, your answer should work for arbitrary data, not just the data of these figures.

(a) What PC models have a speed of at least 3.00?

(b) Which manufacturers make laptops with a hard disk of at least 100GB?

(c) Find the model number and price of all products (of any type) made by manufacturer B.

(d) Find the model numbers of all color laser printers.

(e) Find those manufacturers that sell Laptops, but not PC's.

(f) Find those hard-disk sizes that occur in two or more PC's.

maker	*model*	*type*
A	1001	pc
A	1002	pc
A	1003	pc
A	2004	laptop
A	2005	laptop
A	2006	laptop
B	1004	pc
B	1005	pc
B	1006	pc
B	2007	laptop
C	1007	pc
D	1008	pc
D	1009	pc
D	1010	pc
D	3004	printer
D	3005	printer
E	1011	pc
E	1012	pc
E	1013	pc
E	2001	laptop
E	2002	laptop
E	2003	laptop
E	3001	printer
E	3002	printer
E	3003	printer
F	2008	laptop
F	2009	laptop
G	2010	laptop
H	3006	printer
H	3007	printer

Figure 1: Sample data for the relation `Product`.

model	*speed*	*ram*	*hd*	*price*
1001	2.66	1024	250	2114
1002	2.10	512	250	995
1003	1.42	512	80	478
1004	2.80	1024	250	649
1005	3.20	512	250	630
1006	3.20	1024	320	1049
1007	2.20	1024	200	510
1008	2.20	2048	250	770
1009	2.00	1024	250	650
1010	2.80	2048	300	770
1011	1.86	2048	160	959
1012	2.80	1024	160	649
1013	3.06	512	80	529

(a) Sample data for relation PC

model	*speed*	*ram*	*hd*	*screen*	*price*
2001	2.00	2048	240	20.1	3673
2002	1.73	1024	80	17.0	949
2003	1.80	512	60	15.4	549
2004	2.00	512	60	13.3	1150
2005	2.16	1024	120	17.0	2500
2006	2.00	2048	80	15.4	1700
2007	1.83	1024	120	13.3	1429
2008	1.60	1024	100	15.4	900
2009	1.60	512	80	14.1	680
2010	2.00	2048	160	15.4	2300

(b) Sample data for relation Laptop

model	*color*	*type*	*price*
3001	true	ink-jet	99
3002	false	laser	239
3003	true	laser	899
3004	true	ink-jet	120
3005	false	laser	120
3006	true	ink-jet	100
3007	true	laser	200

(c) Sample data for relation Printer

Figure 2: Sample data for the relations PC, Laptop, and Printer.

Solutions

(a) What PC models have a speed of at least 3.00?

$$\pi_{\text{model}}(\sigma_{\text{speed}\geq 3.00}(\text{PC}))$$

(b) Which manufacturers make laptops with a hard disk of at least 100GB?

$$\pi_{\text{maker}}(\sigma_{\text{hd}\geq 100}(\text{Laptop}) \bowtie \text{Product})$$

(c) Find the model number and price of all products (of any type) made by manufacturer B.

$$\pi_{\text{model, price}}\Big((\sigma_{\text{maker}='B'}(\text{Product}) \bowtie \text{PC}) \cup (\sigma_{\text{maker}='B'}(\text{Product}) \bowtie \text{Laptop}) \cup (\sigma_{\text{maker}='B'}(\text{Product}) \bowtie \text{Printer}) \Big)$$

(d) Find the model numbers of all color laser printers.

$$\pi_{\text{model}}(\sigma_{\text{color AND type='laser'}}(\text{Printer}))$$

(e) Find those manufacturers that sell Laptops, but not PC's.

$$\pi_{\text{maker}}(\sigma_{\text{type}='laptop'}(\text{Product})) \ - \ \pi_{\text{maker}}(\sigma_{\text{type}='pc'}(\text{Product}))$$

(f) Find those hard-disk sizes that occur in two or more PC's.

$$\pi_{\text{hd}}(\sigma_{\text{COUNT(hd)}\geq 2}(\text{PC}))$$

2 Exercise 2.4.3

This exercise builds upon Exercise 2.3.2 concerning World War II capital ships. Recall it involves the following relations:

- `Classes(class, type, country, numGuns, bore, displacement)`
- `Ships(name, class, launched)`
- `Battles(name, date)`
- `Outcomes(ship, battle, result)`

Figures 3 and 4 give some sample data for these four relations. Note that, unlike the data for Exercise 2.4.1, there are some "dangling tuples" in this data, e.g., ships mentioned in `Outcomes` that are not mentioned in `Ships`.

Write expressions of relational algebra to answer the following queries. You may use the linear notation of Section 2.4.13 if you wish. For the data of Figs. 3 and 4, show the result of your query. However, your answer should work for arbitrary data, not just the data of these figures.

(a) Give the class names and countries of the classes that carried guns of at least 16-inch bore.

(b) Find the ships launched prior to 1921.

(c) Find the ships sunk in the battle of the Denmark Strait.

(d) The Treaty of Washington in 1921 prohibited capital ships heavier than 35,000 tons. List the ships that violated the Treaty of Washington.

(e) List the name, displacement, and number of guns of the ships engaged in the battle of Guadalcanal.

(f) List all the capital ships mentioned in the database. (Remember that all these ships may not appear in the `Ships` relation.)

Solutions

(a) Give the class names and countries of the classes that carried guns of at least 16-inch bore.

$$\pi_{\text{class, country}}(\sigma_{\text{bore}\geq 16}(\text{Classes}))$$

class	*type*	*country*	*numGuns*	*bore*	*displacement*
Bismarck	bb	Germany	8	15	42000
Iowa	bb	USA	9	16	46000
Kongo	bc	Japan	8	14	32000
North Carolina	bb	USA	9	16	37000
Renown	bc	Gt. Britain	6	15	32000
Revenge	bb	Gt. Britain	8	15	29000
Tennessee	bb	USA	12	14	32000
Yamato	bb	Japan	9	18	65000

(a) Sample data for relation `Classes`

name	*date*
Denmark Strait	5/24-27/41
Guadalcanal	11/15/42
North Cape	12/26/43
Surigao Strait	10/25/44

(b) Sample data for relation `Battles`

ship	*battle*	*result*
Arizona	Pearl Harbor	sunk
Bismarck	Denmark Strait	sunk
California	Surigao Strait	ok
Duke of York	North Cape	ok
Fuso	Surigao Strait	sunk
Hood	Denmark Strait	sunk
King George V	Denmark Strait	ok
Kirishima	Guadalcanal	sunk
Prince of Wales	Denmark Strait	damaged
Rodney	Denmark Strait	ok
Scharnhorst	North Cape	sunk
South Dakota	Guadalcanal	damaged
Tennessee	Surigao Strait	ok
Washington	Guadalcanal	ok
West Virginia	Surigao Strait	ok
Yamashiro	Surigao Strait	sunk

(c) Sample data for relation `Outcomes`

Figure 3: Sample data for the relations `Classes`, `Ships`, `Battles`, and `Outcomes`.

name	*class*	*launched*
California	Tennessee	1921
Haruna	Kongo	1915
Hiei	Kongo	1914
Iowa	Iowa	1943
Kirishima	Kongo	1915
Kongo	Kongo	1913
Missouri	Iowa	1944
Musashi	Yamato	1942
New Jersey	Iowa	1943
North Carolina	North Carolina	1941
Ramillies	Revenge	1917
Renown	Renown	1916
Repulse	Renown	1916
Resolution	Revenge	1916
Revenge	Revenge	1916
Royal Oak	Revenge	1916
Royal Sovereign	Revenge	1916
Tennessee	Tennessee	1920
Washington	North Carolina	1941
Wisconsin	Iowa	1944
Yamato	Yamato	1941

Figure 4: Sample data for the relations `Ships` .

(b) Find the ships launched prior to 1921.

$$\pi_{\text{name}}(\sigma_{\text{launched}<1921}(\text{Ships}))$$

(c) Find the ships sunk in the battle of the Denmark Strait.

$$\pi_{\text{ship}}(\sigma_{\text{battle}='\text{Denmark Strait}'}(\text{Outcomes}))$$

(d) The Treaty of Washington in 1921 prohibited capital ships heavier than 35,000 tons. List the ships that violated the Treaty of Washington.

$$\pi_{\text{name}}(\sigma_{\text{displacement}>35000}(\text{Ships} \bowtie \text{Classes}))$$

(e) List the name, displacement, and number of guns of the ships engaged in the battle of Guadalcanal.

$$\pi_{\text{name, displacement, numGuns}}(\sigma_{\text{battle}='\text{Guadalcanal}'}(\text{Outcomes} \bowtie \text{Ships} \bowtie \text{Classes}))$$

(f) List all the capital ships mentioned in the database. (Remember that all these ships may not appear in the `Ships` relation.)

$$\pi_{\text{name}}(\text{Ships}) \cup \rho_{S(\text{name})}(\pi_{\text{ship}}(\text{Outcomes}))$$

3 Exercise 2.5.1

Express the following constraints about the relations of Exercise 2.3.1, reproduced here:

- `Product(maker, model, type)`
- `PC(model, speed, ram, hd, price)`
- `Laptop(model, speed, ram, hd, screen, price)`
- `Printer(model, color, type, price)`

You may write your constraints either as containments or by equating an expression to the empty set. For the data of Exercise 2.4.1, indicate any violations to your constraints:

(a) A PC with a processor speed less than 2.00 must not sell for more than \$500

(b) A laptop with a screen size less than 15.4 inches must have at least a 100 gigabyte hard disk or sell for less than \$1000

(c) No manufacturer of PCs may also make laptops

(d) A manufacturer of a PC must also make a laptop with at least as great a processor speed

(e) If a laptop has a larger main memory than a PC, then the laptop must also have a higher price than the PC

Solutions

(a) A PC with a processor speed less than 2.00 must not sell for more than \$500

$$\sigma_{speed<2.00 \textbf{ AND } price>500} = \emptyset$$

(b) A laptop with a screen size less than 15.4 inches must have at least a 100 gigabyte hard disk or sell for less than \$1000

$$\sigma_{screen<15.4 \textbf{ AND } hd<100 \textbf{ AND} price\geq 1000}(Laptop) = \emptyset$$

(c) No manufacturer of PCs may also make laptops

$$\pi_{maker}(\sigma_{type=pc}(Product)) \cap \pi_{maker}(\sigma_{type=laptop}(Product)) = \emptyset$$

(d) A manufacturer of a PC must also make a laptop with at least as great a processor speed

$$\begin{aligned} R1 &:= \pi_{maker,model,speed}(Product \bowtie PC) \\ R2 &:= \pi_{maker,speed}(Product \bowtie Laptop) \\ R3 &:= \pi_{model}(R1 \bowtie_{R1.maker=R2.maker \textbf{ AND } R1.speed>R2.speed} R2) \\ R4 &:= \pi_{model}(R3) = \emptyset \end{aligned}$$

(e) If a laptop has a larger main memory than a PC, then the laptop must also have a higher price than the PC

$$R1 := \pi_{ram,price}(PC)$$
$$R2 := \pi_{ram,price}(Laptop)$$

And we can get constraints such that

$$R1 \bowtie_{R2.ram > R1.ram \textbf{ AND } R2.price \leq R1.price} R2 = \emptyset$$

4 Exercise 2.5.2

Express the following constraints in relational algebra. The constraints are based on the relations of Exercise 2.3.2, reproduced here:

- `Classes(class, type, country, numGuns, bore, displacement)`
- `Ships(name, class, launched)`
- `Battles(name, date)`
- `Outcomes(ship, battle, result)`

You may write your constraints either as containments or by equating an expression to the empty set. For the data of Exercise 2.4.3, indicate any violations to your constraints:

(a) No class of ships may have guns with larger than 16-inch bore.

(b) If a class of ships has more than 9 guns, then their bore must be no larger than 14 inches.

(c) No class may have more than 2 ships.

(d) No country may have both battleships and battlecruisers.

(e) No ship with more than 9 guns may be in a battle with a ship having fewer than 9 guns that was sunk.

Solutions

(a) No class of ships may have guns with larger than 16-inch bore.

$$\pi_{Class}(\sigma_{bore>16}(Classes)) = \emptyset$$

(b) If a class of ships has more than 9 guns, then their bore must be no larger than 14 inches.

$$\pi_{Class}(\sigma_{bore>16 \textbf{ AND } numGuns>9}(Classes)) = \emptyset$$

(c) No class may have more than 2 ships.

$$\begin{aligned} R_1 &= \pi_{class,\mathrm{COUNT}(name)}(ship) \\ R_2 &= \sigma_{\mathrm{COUNT}(name)>2}(R_1) \\ R_2 &= \emptyset \end{aligned}$$

(d) No country may have both battleships and battlecruisers.

$$\pi_{country}(\sigma_{type=battlecruiser}(Classes)) \cap \pi_{country}(\sigma_{type=battleship}(Classes)) = \emptyset$$

(e) No ship with more than 9 guns may be in a battle with a ship having fewer than 9 guns that was sunk.

$$\begin{aligned} R1 &:= \sigma_{numGuns>9}(Classes) \\ R2 &:= \sigma_{numGuns<9}(Classes) \\ R3 &:= (R1 \bowtie Ships) \bowtie Battles \\ R4 &:= (R2 \bowtie Ships) \bowtie \sigma_{result=sunk}(Battles) \\ R5 &:= \pi_{battlename}(R3) \bowtie \pi_{battlename}(R4) = \emptyset \end{aligned}$$