# HW #3, Chapter 3

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## Chapter 3, P32.

Consider the TCP procedure for estimating RTT. Suppose that $\alpha = 0.1$. Let SampleRTT1 be the most recent sample RTT, let SampleRTT2 be the next most recent sample RTT, and so on.

a. For a given TCP connection, suppose four acknowledgments have been returned with corresponding sample RTTs: SampleRTT4, SampleRTT3, SampleRTT2, and SampleRTT1. Express EstimatedRTT in terms of the four sample RTTs.

b. Generalize your formula for $n$ sample RTTs.

c. For the formula in part (b) let $n$ approach infinity. Comment on why this averaging procedure is called an exponential moving average.

### Solutions

**a.** We can express the Estimated RTT after receiving four acknowledgments with sample RTTs $\text{SampleRTT}_4, \text{SampleRTT}_3, \text{SampleRTT}_2, \text{SampleRTT}_1$ using the exponential weighted moving average formula. Starting with the most recent sample $\text{SampleRTT}_1$, we expand the recursive formula as follows:

$$\begin{aligned}
\text{EstimatedRTT} &= (1-\alpha) \times \text{EstimatedRTT}_3 + \alpha \times \text{SampleRTT}_4 \\
&= (1-\alpha)\left[(1-\alpha) \times \text{EstimatedRTT}_2 + \alpha \times \text{SampleRTT}_3\right] + \alpha \times \text{SampleRTT}_4 \\
&= (1-\alpha)^2 \times \text{EstimatedRTT}_2 + (1-\alpha)\alpha \times \text{SampleRTT}_3 + \alpha \times \text{SampleRTT}_4 \\
&= (1-\alpha)^2\left[(1-\alpha) \times \text{EstimatedRTT}_1 + \alpha \times \text{SampleRTT}_2\right] \\
&\quad + (1-\alpha)\alpha \times \text{SampleRTT}_3 + \alpha \times \text{SampleRTT}_4 \\
&= (1-\alpha)^3 \times \text{EstimatedRTT}_1 + (1-\alpha)^2\alpha \times \text{SampleRTT}_2 \\
&\quad + (1-\alpha)\alpha \times \text{SampleRTT}_3 + \alpha \times \text{SampleRTT}_4
\end{aligned}$$

Assuming an initial $\text{EstimatedRTT}_0$, we can include it in the expression:

$$\text{EstimatedRTT} = (1-\alpha)^4 \times \text{EstimatedRTT}_0 + \alpha \sum_{k=1}^{4} (1-\alpha)^{4-k} \times \text{SampleRTT}_k$$

With $\alpha = 0.1$, the expression becomes:

$$\begin{aligned} \text{EstimatedRTT} = {} & 0.9^4 \times \text{EstimatedRTT}_0 \\ & + 0.1 \left(0.9^3 \times \text{SampleRTT}_1\right) \\ & + 0.1 \left(0.9^2 \times \text{SampleRTT}_2\right) \\ & + 0.1 \left(0.9 \times \text{SampleRTT}_3\right) \\ & + 0.1 \times \text{SampleRTT}_4 \end{aligned}$$

**b.** Generalizing for $n$ sample RTTs, the formula becomes:

$$\text{EstimatedRTT} = (1-\alpha)^n \times \text{EstimatedRTT}_0 + \alpha \sum_{k=1}^{n} (1-\alpha)^{n-k} \times \text{SampleRTT}_k$$

This shows that the EstimatedRTT is a weighted sum of all past sample RTTs, where each sample is weighted by $\alpha(1-\alpha)^{n-k}$.

**c.** As $n$ approaches infinity, the term $(1-\alpha)^n \times \text{EstimatedRTT}_0$ approaches zero because $0 < (1-\alpha) < 1$. This means the influence of the initial EstimatedRTT diminishes over time. The weights assigned to older SampleRTTs decrease exponentially, emphasizing more recent samples. This behavior characterizes an **exponential moving average**, where:

- Recent samples have a higher impact on the EstimatedRTT.
- The sum of the weights approaches 1 as $n$ approaches infinity.
- The averaging gives exponentially less weight to older samples, capturing recent network conditions more accurately.

This method efficiently smooths out short-term variations while still being responsive to changes, which is essential for adaptive protocols like TCP.