Engineering Optics, Homework 2

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1 Problem 1

Chapter 2, Problem 1

1.1 Positive Lens

1. $-\infty$, Image plane is at f.

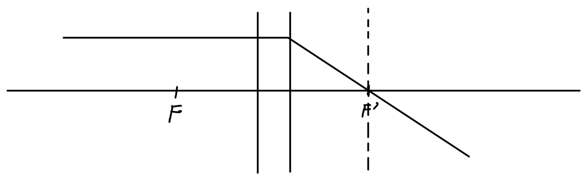

Figure 1: Object at $-\infty$

2. $-2f$, Image plane is at $2f$.

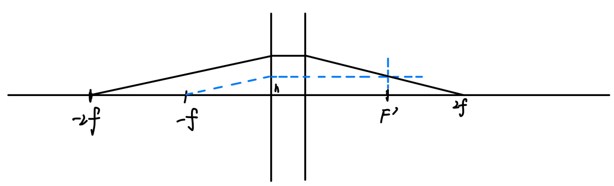

Figure 2: Object at $-2f$

3. $-f$, Image plane is at ∞.

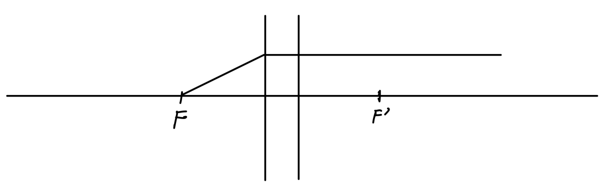

Figure 3: Object at $-f$

4. $-\frac{f}{2}$, Image plane is at $-f$.

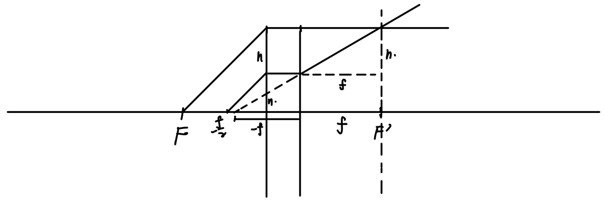

Figure 4: Object at $-0.5f$

5. 0, Image plane is at 0.

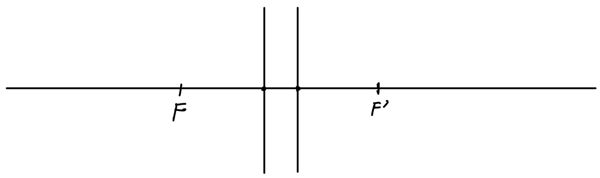

Figure 5: Object at 0

6. $\frac{f}{2}$, Image plane is at f.

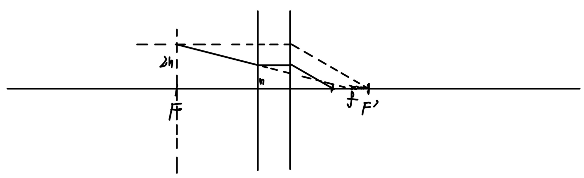

Figure 6: Object at $0.5f$

7. f, Image plane is at $-\infty$.

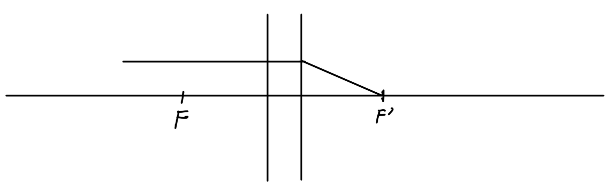

Figure 7: Object at f

8. $2f$, Image plane is at $-2f$.

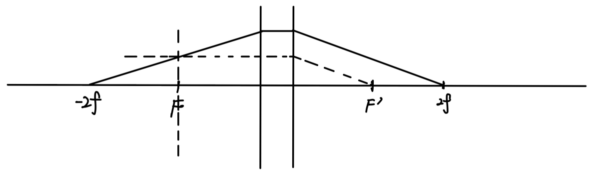

Figure 8: Object at $2f$

9. $+\infty$, Image plane is at $-f$.

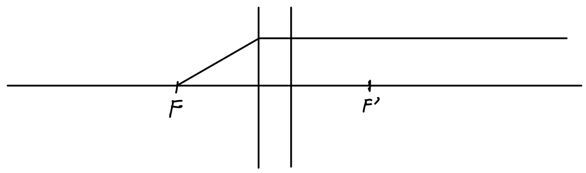

Figure 9: Object at $+\infty$

2 Problem 2

Chapter 2, Problem 2

Donated the F as the zero point of the axis, we can use the Newtonian form of the lens equation to solve the problem. The lens equation is given by

$$xx' = ff' \tag{1}$$

Therefore, the Object at $-\infty, -10m, -8m, -6m, -4m, -2m$ follows the equation

$$x_1 \cdot \infty = f \cdot f' \tag{2}$$
$$x_2 \cdot 10 = f \cdot f' \tag{3}$$
$$x_3 \cdot 8 = f \cdot f' \tag{4}$$
$$x_4 \cdot 6 = f \cdot f' \tag{5}$$
$$x_5 \cdot 4 = f \cdot f' \tag{6}$$
$$x_6 \cdot 2 = f \cdot f' \tag{7}$$
$$\tag{8}$$

And

$$-f = f' = 75mm$$

Solving the equation, we can get the position of the image plane for each object plane. The result is shown in the table below.

$$x_1 = \frac{ff'}{\infty} = 0mm \tag{9}$$

$$x_2 = \frac{ff'}{10} = \frac{5.625 \times 10^{-3}}{10} = 5.625 \times 10^{-4}m = 0.5625mm \tag{10}$$

$$x_3 = \frac{ff'}{8} = \frac{5.625 \times 10^{-3}}{8} = 7.03125 \times 10^{-4}m = 0.703125mm \tag{11}$$

$$x_4 = \frac{ff'}{6} = \frac{5.625 \times 10^{-3}}{6} = 9.375 \times 10^{-4}m = 0.9375mm \tag{12}$$

$$x_5 = \frac{ff'}{4} = \frac{5.625 \times 10^{-3}}{4} = 1.40625 \times 10^{-3}m = 1.40625mm \tag{13}$$

$$x_6 = \frac{ff'}{2} = \frac{5.625 \times 10^{-3}}{2} = 2.8125 \times 10^{-3}m = 2.8125mm \tag{14}$$

(15)

3 Problem 3

Chapter 2, Problem 3

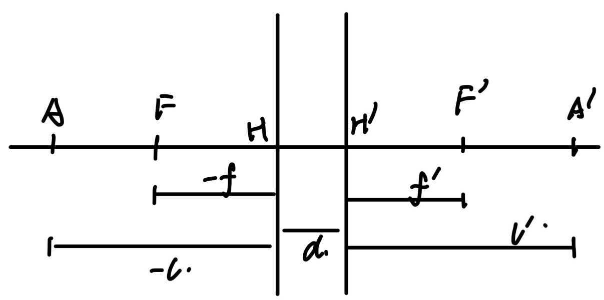

Figure 10: Optical System

The Optical system is placed in the air, so we have

$$n' = n, \quad -f = f' \tag{16}$$

And assume the optical system is formed like Figure.10, we can use the Newtonian form of the lens equation to solve the problem. The lens equation is given by

$$\begin{cases} \beta = \frac{l'}{l} \\ f' + (-f) + d = 1140 \\ l' + (-l) + d = 7200 \\ \frac{1}{l'} - \frac{1}{l} = \frac{1}{f'} \end{cases} \tag{17}$$

Solving the equation, we have

$$\begin{cases} f' = 600mm \\ d = -60mm \end{cases} \tag{18}$$

Therefore, the real base point and base plane are shown in Figure.11.

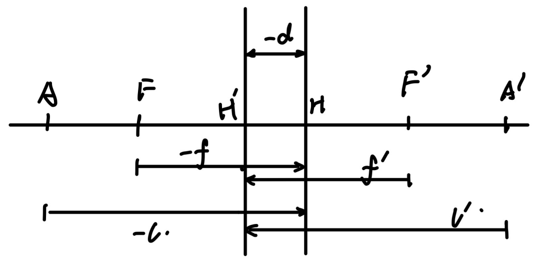

Figure 11: Real Base Point and Base Plane

4 Problem 4

Chapter 2, Problem 5

Assume the focus length of two lens are f_1 and f_2. We have.

$$\begin{cases} \beta_1 = \frac{l_1'}{l_1} = -1 \\ l_1' = l' + 20 \\ \beta = \frac{l'}{l} = -\frac{3}{4} \\ l1 = l \end{cases} \tag{19}$$

Where l_1 and l_1' are the object-side and image-side distance, l',l are the object-side and image-side distance of the composed system. By solving the equation, we have

$$\begin{cases} l_1 = -80 \\ l_1' = 80 \\ l = -80 \\ l' = 60 \end{cases} \tag{20}$$

According to the composed lens equation, we have

$$\frac{1}{l_i'} - \frac{1}{l_i} = \frac{1}{f_i'} = \mathit{\Phi}_i \tag{21}$$

Therefore,

$$\begin{cases} \mathit{\Phi_1} = \frac{1}{f_1'} = \frac{1}{40} \\ \mathit{\Phi_2} = \frac{1}{f_2'} = \frac{7}{240} \end{cases} \tag{22}$$

And for the thin lens, we have

$$\mathit{\Phi} = \mathit{\Phi_1} + \mathit{\Phi_2} \tag{23}$$

As a result, we have

$$\mathit{\Phi_2} = \mathit{\Phi} - \mathit{\Phi_1} = \frac{1}{240} \tag{24}$$

Therefore,

$$\boxed{\begin{aligned} f_2' &= \frac{1}{\mathit{\Phi_2}} = 240mm \\ f_1' &= \frac{1}{\mathit{\Phi_1}} = 40mm \end{aligned}} \tag{25}$$

5 Problem 5

Chapter 2, Problem 6

Based on the Newtonian form of the lens equation, we have

$$xx' = ff' \tag{26}$$

$$\beta = \frac{f}{x} \tag{27}$$

At the beginning, $\beta_1 = -\frac{1}{2}^{\times}$, and at the second time, $\beta_2 = -1^{\times}$. Therefore, we have

$$\begin{cases} \beta_1 = \frac{f}{x_1} = -\frac{1}{2} \\ \beta_2 = \frac{f}{x_2} = -1 \\ x_1 - x_2 = 100mm \end{cases} \tag{28}$$

Solving the equation, we have

$$\begin{cases} x_1 = 200mm \\ x_2 = 100mm \end{cases} \tag{29}$$

Therefore, the focus length of the lens is

$$\boxed{f = 100mm} \tag{30}$$

6 Problem 6

Chapter 2, Problem 7

According to the condition, $f' > L$, therefore this is a telephoto optical group. The optical path is shown in Figure.12.

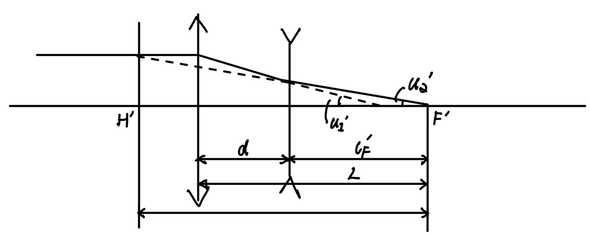

Figure 12: Telephoto Optical Group

we have,

$$d = L - l_F' = 300mm \tag{31}$$

$$f' = 1200mm \tag{32}$$

$$l_F' = 400mm \tag{33}$$

$$l_F' = f'"(1 - \frac{d}{f_1'}) \tag{34}$$

Therefore,

$$\boxed{f_1' = \frac{d}{1-\frac{l_F'}{f'}} = \frac{300mm}{1-\frac{400}{1200}} = 450mm} \tag{35}$$

Also we have the formula of the composed lens

$$f' = -\frac{f_1' f_2'}{\Delta} = -\frac{f_1' f_2'}{d - f_1' - f_2'} \tag{36}$$

So the focus length of the second lens is

$$\boxed{f_2' = \frac{d - f_1'}{1-\frac{f_1'}{f'}} = \frac{300mm - 450mm}{1-\frac{450}{1200}} = -240mm} \tag{37}$$

7 Problem 7

Chapter 2, Problem 9

According to the formula of lens, we have

$$\begin{aligned} f' =& \frac{nr_1r_2}{(n-1)\left[n(r_2-r_1)+(n-1)d\right]} && (38)\\ =& \frac{1.5 \cdot -200mm \cdot -300mm}{0.5 \cdot [1.5 \cdot (-300mm - (-200mm)) + (1.5-1) \cdot 50mm]} && (39)\\ =& \boxed{600mm} && (40)\\ \Phi =& \frac{1}{f'} = \frac{1}{600mm} = 0.001 && (41) \end{aligned}$$

8 Problem 8

Chapter 2, Problem 14

the optical path is shown in Figure.13.

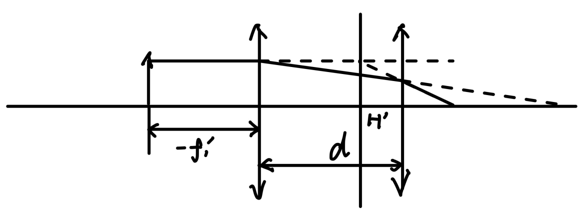

Figure 13: Optical System

According to the condition, we have

$$\begin{aligned} x_F = \frac{f_1 f_1'}{\Delta} = -\frac{f_1'^2}{\Delta} && (42)\\ \Delta = d - f_1' - f_2' && (43)\\ && (44) \end{aligned}$$

Therefore, the focus length is

$$f = \frac{f_1 f_2}{\Delta} = \frac{f_1' f_2'}{d - f_1' - f_2'} \tag{45}$$

The object is at the focus point of the first lens, so we have

$$l = f - x_F l = x + f \tag{46}$$

That is,

$$x = -x_F = \frac{f_1'^2}{\Delta} \tag{47}$$

And the vertical magnification is

$$\beta = -\frac{f}{x} = -\frac{f_2'}{f_1'} \tag{48}$$