Optics Homework 13-E

1 Q1

To find the angular width of the central maximum, we use the single slit diffraction formula. The angular width of the central maximum is given by:

$$\theta = \frac{\lambda}{a}$$

Given:

$$\lambda = 500 \times 10^{-9} \text{ m} \quad \text{(wavelength)}$$
$$a = 1.50 \times 10^{-6} \text{ m} \quad \text{(slit width)}$$

Substituting the values:

$$\theta = \frac{500 \times 10^{-9}}{1.50 \times 10^{-6}} = \frac{500}{1500} \times 10^{-3} = 0.333 \text{ radians}$$

Since the angular width of the central maximum is from $-\theta$ to θ, the total angular width is 2θ:

$$\text{Total angular width} = 2\theta = 2 \times 0.333 = 0.666 \text{ radians}$$

Converting this to degrees:

$$\theta_{\text{deg}} = \theta \times \frac{180}{\pi} = 0.333 \times \frac{180}{\pi} \approx 19.1^\circ$$

The angular width of the central maximum is approximately 0.666 radians or 38.2°.

2 Q2

To find the position of the first minima, we use the single slit diffraction formula. The position of the first minima on the observing screen is given by:

$$y = \frac{m\lambda L}{a}$$

where:

- $m = 1$ for the first minima
- $\lambda = 580 \times 10^{-9}$ m (wavelength)
- $a = 0.30 \times 10^{-3}$ m (slit width)
- $L = 2.0$ m (distance to the screen)

Substituting the values:

$$y = \frac{1 \times 580 \times 10^{-9} \times 2.0}{0.30 \times 10^{-3}} = \frac{580 \times 2.0}{0.30} \times 10^{-6} = \frac{1160}{0.30} \times 10^{-6} = 3866.67 \times 10^{-6} \text{ m}$$

Converting this to millimeters:

$$y = 3866.67 \times 10^{-6} \times 1000 = 3.87 \text{ mm}$$

Therefore, the position of the first minima is approximately 3.87 mm from the center of the central maximum.

3 Q3

To determine which higher-order maxima are missing, we use the condition for missing maxima in a diffraction grating. The missing orders occur when the grating spacing d is an integer multiple of the slit width a. This can be expressed as:

$$m\lambda = a \sin\theta$$

where:

- m is the order of the maximum.
- $a = 0.15$ mm (slit width).
- $d = 0.6$ mm (slit spacing).

The missing orders occur when:

$$\frac{d}{a} = \text{integer}$$

Substituting the values:

$$\frac{0.6 \text{ mm}}{0.15 \text{ mm}} = 4$$

Therefore, the missing higher-order maxima are for the order:

$$m = 4$$

The higher-order maxima that are missing are the 4th order maxima.

4 Q4

a) Separation between a set of slits

To find the separation between the slits, we use the formula for the position of dark fringes in a diffraction pattern:

$$y = \frac{L\lambda}{d}$$

where:

- $\lambda = 450 \times 10^{-9}$ m (wavelength)
- $L = 1.80$ m (distance to the screen)
- $y = 4.20 \times 10^{-3}$ m$/2 = 2.10 \times 10^{-3}$ m (distance from central maximum to one dark fringe)

Rearranging to solve for d:

$$d = \frac{L\lambda}{y}$$

Substituting the values:

$$d = \frac{1.80 \times 450 \times 10^{-9}}{2.10 \times 10^{-3}} = 3.857 \times 10^{-4} \text{ m} = 0.000386 \text{ m}$$

b) Number of lines per meter

The number of lines per meter N is the reciprocal of the slit separation d:

$$N = \frac{1}{d}$$

Substituting the value of d:

$$N = \frac{1}{0.000386} \approx 2592.59 \text{ lines per meter}$$

5 Q5

To find the size of the details that can be resolved by the telescope, we use the Rayleigh criterion for the minimum resolvable detail:

$$\theta = 1.22\frac{\lambda}{D}$$

where:

- $\lambda = 500 \times 10^{-9}$ m (wavelength)
- $D = 7.6 \times 10^{-2}$ m (aperture diameter)

- $L = 12.5 \times 10^3$ m (distance to the target)

Substituting the values:

$$\theta = 1.22 \frac{500 \times 10^{-9}}{7.6 \times 10^{-2}} = 8.026 \times 10^{-6} \text{ radians}$$

The linear size of the resolvable detail is given by:

$$s = \theta \times L$$

Substituting the values:

$$s = 8.026 \times 10^{-6} \times 12.5 \times 10^3 = 0.1003 \text{ m}$$

Converting this to millimeters:

$$s = 0.1003 \times 1000 = 100.3 \text{ mm}$$