HW #1, Chapter 1

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Problem 1. Chapter 1 P8

Suppose users share a 10 Mbps link. Also suppose each user requires 200 kbps when transmitting, but each user transmits only 10 percent of the time. (See the discussion of packet switching versus circuit switching in Section 1.3)

a. When circuit switching is used, how many users can be supported?

b. For the remainder of this problem, suppose packet switching is used. Find the probability that a given user is transmitting.

c. Suppose there are 120 users. Find the probability that at any given time, exactly n users are transmitting simultaneously. (Hint: Use the binomial distribution.)

d. Find the probability that there are 51 or more users transmitting simultaneously.

Solutions:

a. When circuit switching, the paralleled user can be calculated as:

$$n = \frac{Totalbandwidth}{Bandwidthperuser} = \frac{10\textbf{Mbps}}{200\textbf{kbps}} = 500users$$

Therefore, the curcuit switching can support up to 500 users.

b. Given that each user transmits only 10% of the time:

$$\boxed{P(transmits) = 0.10}$$

c. The number of users transmitting at any given time follows binomial distribution with parameters $n = 120$ and $p = 0.1$.

Therefore, the probability of exactly n users are transmitting is:

$$\boxed{P(X = n) = \binom{120}{n} \cdot (0.1)^n \cdot (0.9)^{120-n}}$$

The expression $\binom{120}{n}$, which represents C_{120}^n, stands for "n choose k," and X denotes the random variable representing the number of simultaneous users.

d. We can use the central limit theorem to approximate the answer. Let X_i as a independent random variable stands for the user i is transmitting at the moments, we have.

$$E[X_i] = 0.1$$

$$E(\sum_{i=1}^{120} X_i) = 120 \times 0.1 = 12$$
$$Var(X_i) = p(1-p) = 0.1 \times 0.9 = 0.09$$
$$Var(\sum_{i=1}^{120} X_i) = 120 \times 0.09 = 10.8$$

Therefore, the distribution of the sum can be approximated as:

$$\sum_{i=1}^{120} X_i \sim N(12, 10.8)$$

Let.

$$Z = \frac{\sum_{i=1}^{120} X_i - \mu}{\sigma} = \frac{51-12}{\sqrt{10.8}} \approx 11.56$$

By querying the table of normal distribution, we can calculated the answer

$$P(\sum_{i=1}^{120} X_i \geq 51) = 1 - P(\sum_{i=1}^{120} X_i \leq 51) = 1 - P(Z \leq 11.56) \approx= 1 - 1 = 0$$

Therefore, the probability is approximately zero; the probability of 51 or more users transmitting is negligible.