Steven Choi HW8

1. alpha : $p \rightarrow (q \rightarrow r)$

$\neg p \vee (q \rightarrow r)$: implication

$\neg p \vee (\neg q \vee r)$: implication

$\neg (p \wedge q) \vee r$: de Morgan's Law

$\neg p \vee \neg q \vee r$: distribute

beta : $(p \wedge q) \rightarrow (q \rightarrow r)$

$\neg(p \wedge q) \vee (q \rightarrow r)$: implication

$\neg(p \wedge q) \vee (\neg q \vee r)$: implication

$(\neg p \vee \neg q) \vee (\neg q \vee r)$: de Morgan's Law

$\neg p \vee \neg q \vee r$: distribute

Knowledge Base:

$\neg p, \neg q, r, \neg p \vee \neg q \vee r$

remove complements of each other, then knowledge base = null;

proof by contradiction

proved that alpha = beta

2a.

Child(x) = x is a child

Loves(x,y) = x loves y

Reindeer(x) = x is a reindeer

Rednose(x) = x has a red nose

Weird(x) = x is weird

Clown(x) = x is a clown

a. $\forall x\ (\text{Child}(x) \rightarrow \text{Loves}(x, \text{santa}))$

b. $\forall x\ (\text{Loves}(x, \text{Santa}) \rightarrow \forall y\ (\text{Reindeer}(y) \rightarrow \text{Loves}(x, y)))$

c. $\text{Reindeer}(\text{Rudolph}) \wedge \text{Rednose}(\text{Rudolph})$

d. $\forall x\ (\text{Rednose}(x) \rightarrow \text{Weird}(x) \vee \text{Clown}(x))$

e. $\neg \exists x\ (\text{Reindeer}(x) \wedge \text{Clown}(x))$

f. $\forall x\ (\text{Weird}(x) \rightarrow \neg \text{Loves}(\text{Scrooge}, x))$

g. $\neg \text{Child}(\text{Scrooge})$

2b.

Austinite(x) = x is a Austinite

Conservative(x) = x is conservative

Armadillo(y) = y is an armadillo

Loves(x,y) = x loves y

Wear_shirt(x) = x wears a maroon-and-white shirt

Dog(x) = x is a dog

a. $\forall x\ (\text{Austinite}(x) \wedge \neg \text{Conservative}(x) \rightarrow \exists y\ (\text{Armadillo}(y) \wedge \text{Loves}(x, y)))$

b. $\forall x\ (\text{Wear\_Shirt}(x) \rightarrow \text{Aggie}(x))$

c. $\forall x\ (\text{Aggie}(x) \rightarrow \forall y\ (\text{Dog}(y) \rightarrow \text{Loves}(x, y)))$

d. $\neg \exists x\ ((\forall y\ (\text{Dog}(y) \rightarrow \text{Loves}(x, y))) \wedge \exists z\ (\text{Armadillo}(z) \wedge \text{Loves}(x, z)))$

e. $\text{Austinite}(\text{Clem}) \wedge \text{Wear\_Shirt}(\text{Clem})$

f. $\exists x\ (\text{Austinite}(x) \wedge \text{Conservative}(x))$

3.

a. Alice : $\neg$ Murderer(alice) $\rightarrow$ (Friends(barney, victor) $\wedge$ $\neg$ Friends(caddy, victor))

Barney : $\neg$ Murderer(barney) $\rightarrow$ $\neg$ Friends(barney, victor)

Caddy: $\neg$ Murderer(caddy) $\rightarrow$ Friends(barney, victor)

b. $\forall$x $\forall$y Friends(x, y) $\rightarrow$ ($\neg$ Kill(x, y) $\wedge\neg$ Kill(y, x))

$\forall$x $\forall$y $\neg$ Murderer(x) $\wedge\neg$ Murderer(y)

$\forall$x (Person(x) $\wedge$ $\neg$ Murderer(x)) $\rightarrow$ $\neg$ lie(x)

c. (Murderer(alice) $\vee$ Friends(barney, victor)) $\wedge$ (Murderer(alice) $\vee$ $\neg$ Friends(caddy, victor))

Murderer(barney) $\vee$ $\neg$ Friends(barney, victor)

Murderer(caddy) $\vee$ Friends(barney, victor)

$\neg$friends(x, y) $\vee$ $\neg$kill(x, y) $\wedge$ ($\neg$friends(x, y) $\vee$ $\neg$kill(y, x))

$\neg$murderer(x) $\vee$ $\neg$murderer(y)

$\neg$ person(x) $\vee$ murderer(x) $\vee$ $\neg$lie(x)

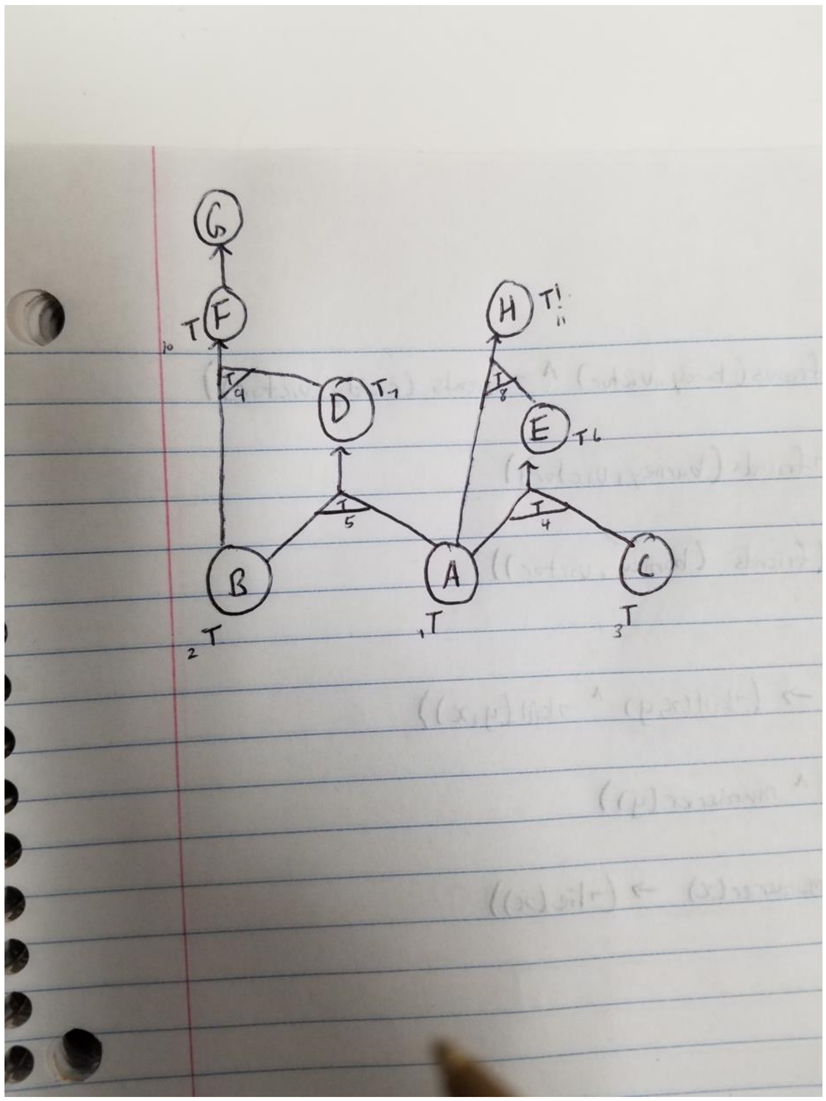
d. $\exists$x kill(x, victor)

e. $\neg$ Murderer(caddy) $\rightarrow$ (Friends(barney, victor) $\wedge$ Friends(caddy, victor))

f. No, it is not satisfiable.

knowledge base after removing complements : murderer(alice) , murderer(barney), murderer(caddy) as we see, the knowledge base is not null, therefore proved that the knowledge base is unsatisfied.

4.



a.

A

B

C

$A \land B \Rightarrow D$

$B \land D \Rightarrow F$

$F \Rightarrow G$

$A \land E \Rightarrow H$

$A \land C \Rightarrow E$

$A \land C$

$A \land B$

E

D

$A \land E$

$B \land E$

H

F

H is true.

b.

P ⇒ Q

E ⇒ B

R ⇒ Q

M ∧ N ⇒ Q

A ∧ B ⇒ P

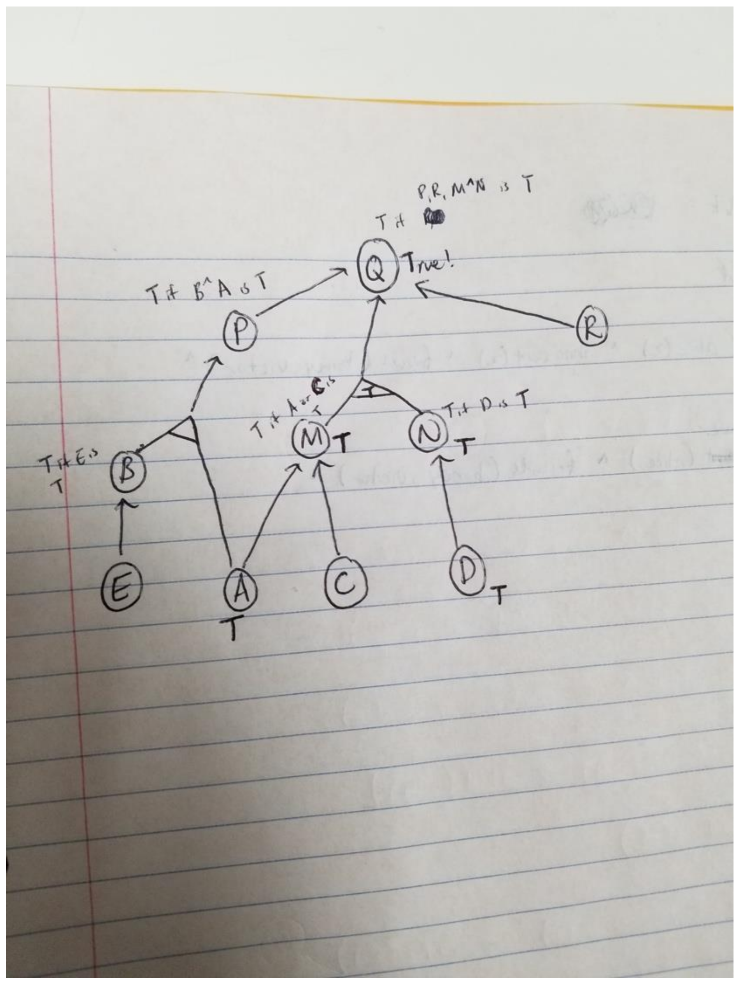
A ⇒ M

C ⇒ M

D ⇒ N

D

A



M

N

M ∧ N

Q : Q is true

5a.

a.

BOS and NY : 156.43

(BOS, NY) & DC : 249.79

DEN & SLC : 371.59

SF & LA : 542.14

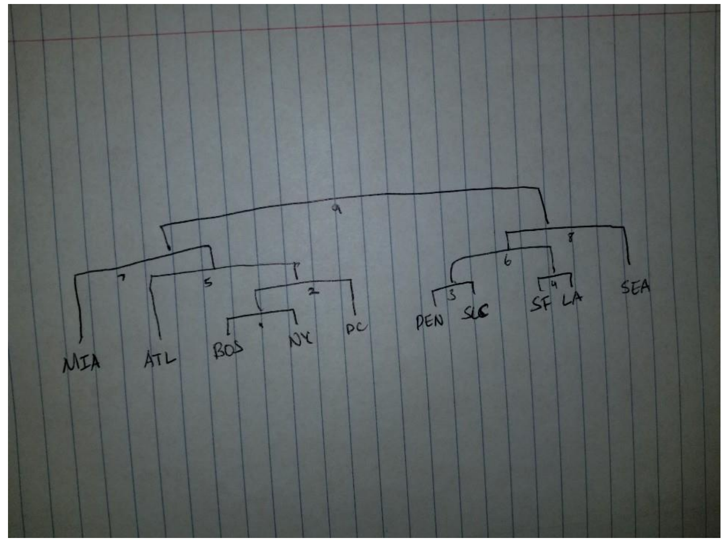
(BOS, NY, DC) & ATL : 577.29

(SF, LA) & (SLC, DEN) : 577.29

(BOS, NY, DC, ATL) & MIA : 598.5

(SF, LA, SLC, DEN) & SEA : 677.1

Final Cluster : 1215.87



b.

(MIA, ATL, BOS, NY, DC)
(DEN, SLC, SF, LA)
(SEA)

5b.

a.

City	Distance x	Distance y	Cluster
BOS	20.371	31.448	1
NY	18.025	28.511	1
DC	17.094	24.662	1
MIA	26.109	20.241	2
SLC	23.754	16.070	2
SEA	32.389	28.409	2

SF	34.621	23.719	2
LA	32.374	18.656	2
DEN	18.196	10.913	2

b.

City	New coordinate x	New coordinate y
BOS	46.2	80.55
NY	45.85	82
DC	44.45	83.5
MIA	27.9	90.1
SLC	35.4	105.95
SEA	38.8	111.15
SF	33.9	111.2
LA	32.05	109.1
DEN	34.85	102.5
ATL	31.85	92.15

c.

City	Distance x	Distance y
BOS	10.18541	25.31289
NY	9.012353	23.9838
DC	8.547076	21.93291
MIA	22.10023	10.12028
SLC	21.62319	8.035079

SEA	23.93246	14.20431
SF	26.62048	11.8596
LA	26.21092	9.328049
DEN	19.64109	5.456418
ATL	18.2769	8.065048

Cluster 1 : BOS, NY, DC

Cluster 2 : MIA, SLC, SEA, SF, LA, DEN, ATL