1.1) a) $P(X, Y \mid Z) =$

$$\frac{P(X,Y,Z)}{P(Z)} = \frac{P(Z \mid X,Y)\, P(X,Y)}{P(Z)}$$

$$= \boxed{\frac{P(Z \mid X,Y)\, P(Y)\, P(X \mid Y)}{P(Z)}}$$

b) Not possible

No expression can be given with the tables

$$P(X,Y,Z) = P(X,Y \mid Z)\, P(Z)$$

$$= P(X/Z)\, P(Y/Z)\, P(Z)$$

since independence

no way to yet rid of P(Z) as P(Z) is not given

$\sum_{Y}$

$= \sum_{Y}$

d) $P(X)$, $P(Y \mid X)$, $P(Z \mid Y)$, $P(X \mid Y, Z)$ $X \perp\!\!\!\perp Y$ $P(Z)$

$P(Y/X) = \frac{P(Y,X)}{P(X)} = P(Y)$ $P(X \mid Y,Z) = \frac{P(X,Y,Z)}{P(Y,Z)} = P(X,Z)$

$P(Z \mid Y) = \frac{P(Z,Y)}{P(Y)}$ No such expression for P(Z)

d) $P(X)$, $P(Y|X)$, $P(X|Y,Z)$, $P(Z|X,Y)$ = $P(Y|X,Z)$

$P(X|Y,Z) = \frac{P(X,Y,Z)}{P(Y,Z)}$ $\quad$ $P(Z|X,Y) = \frac{P(Z,X,Y)}{P(X,Y)}$

$P(Y|X,Z) = \frac{P(Y,X,Z)}{P(X,Z)}$ $\quad$ No such expression can be given with the given tables

$\sum_{y \in Y} \frac{P(\ldots)P(Z)}{P(Z)} = \sum_{y \in Y} P(X \ldots)$

$X \ldots Z$

$P(Y,Z) \ldots$

d) $P(Y|X,Z) = \frac{P(X,Y,Z)}{P(X,Z)}$

$P(X) \cdot P(Y|X) = P(Y,X)$ multiply $\rightarrow$ $P(X,Y,Z)$

$P(Z|Y,X) = \frac{P(X,Y,Z)}{P(Y,X)}$

$\sum_{y \in Y} P(X) \cdot P(y|X) P(Z|y,X) = P(X,Z)$

$$\boxed{\frac{P(X) \cdot P(Y|X) \cdot P(Z|Y,X)}{\sum_{y \in Y} P(X) P(y|X) P(Z|y,X)}}$$

1.2) a) $P(X,Y) = P(X|Z)\,P(Y)$

(conditional independence)

Then, $P(X,Y) = P(X|Y)\,P(Y)$ chain rule

from above, $P(X|Y) = P(X|Z)$ iff $\boxed{X \perp\!\!\!\perp Y \text{ and } X \perp\!\!\!\perp Z}$

b) $\boxed{\text{No need independence since}}$

$$P(X|Y) = \frac{P(X,Y)}{P(Y)} = \frac{P(Y|X)\,P(X)}{P(Y)} \quad \text{(Baye's rule)}$$

c) $$P(Z|X,Y) = \frac{P(X,Y,Z)}{P(X,Y)} = \frac{P(X|Y,Z)\cdot P(Y|Z)\cdot P(Z)}{P(X|Y)\,P(Y)}$$

$$P(X|Y,Z)\cdot P(Y|Z)\cdot P(Z) = \frac{P(X,Y,Z)}{\cancel{P(Y,Z)}} \cdot \frac{\cancel{P(Y,Z)}}{\cancel{P(Z)}} \times \cancel{P(Z)}$$

always holds true

if $$P(Z|X,Y) = \frac{P(X|Z)\cdot P(Y|Z)\,P(Z)}{P(X|Y)\,P(Y)}$$

then $P(X|Y,Z)$ must equal $P(X|Z)$ therefore

$$\boxed{Y \perp\!\!\!\perp X|Z \quad \text{or} \quad Y \perp\!\!\!\perp Z|X}$$

d) $P(X,Y) = P(X|Y)\,P(Y)$ (chain rule)

$P(Y) = \sum_{Y \in Z} P(Y|Z)\,P(Z=z)$ (theorem of total probability)

then $P(X,Y) = P(X|Y) \sum_{Y \in Z} P(Y|Z)\,P(Z=z)$ $\boxed{\text{No assumption needed}}$

sum is under z

$= \sum_{Y \in Z} P(X|Y)\,P(Y|z)\,P(Z=z)$

sum of row = $P(x,y,z)$

1, 3) a) i) $\boxed{\sum_{w \in W} \frac{P(X,Y,Z,w)}{P(Y,Z)}}$ $\boxed{\text{true}}$

$\rightarrow P(y,z)$

since $P(X|Y,Z) = \frac{P(X,Y,Z)}{P(Y,Z)}$

in Baye's rule applied to multiple variables.

$\boxed{\frac{\sum_w P(X,Y,Z,w)}{\sum_w P(Y,Z,w)}}$

ii) $\boxed{\cancel{\sum_{w \in W} P(X,w|Y,Z)}}$

$\boxed{\cancel{\sum_{w \in W} P(X|Y,Z,w)}} = \frac{\sum P(X,Y,Z,w)}{\sum P(Y,Z,w)} = \frac{P(X,Y,Z)}{P(Y,Z)} = P(X|Y,Z)$

b)

$P(X,Y|Z) = P(X|Z)\,P(Y|Z)$ (conditional independence)

$P(X|Z) = \boxed{\frac{P(X,Y|Z)}{P(Y|Z)}}$

$\sum_{y \in Y} \frac{P(X,y|Z)\,P(Z)}{P(Z)} = \sum_{y \in Y} P(X,y|Z) = P(X|Z) \sum_{y \in Y} \underbrace{P(y|Z)}_{1}$

$= P(X|Z)$

$\boxed{\sum_{y \in Y} \frac{P(X,y|Z)\,P(Z)}{P(Z)}}$

Question 2.

2.1) A = -2

B = -2

C = 11

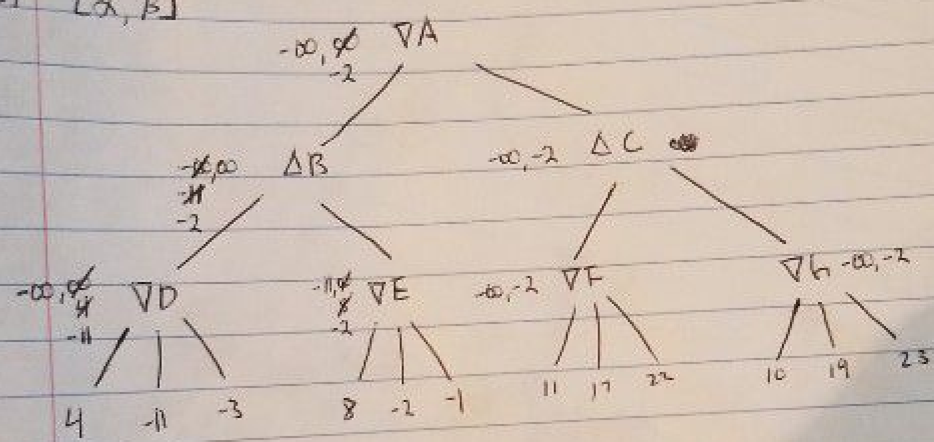
D = -11

E = -2

F = 11

G = 10

2.2) [α, β]



A: $-\infty, -2$

B: $-2, \infty$

C: $-\infty, -2$

D: $-\infty, -11$

E: $-11, -2$

F: $-\infty, -2$

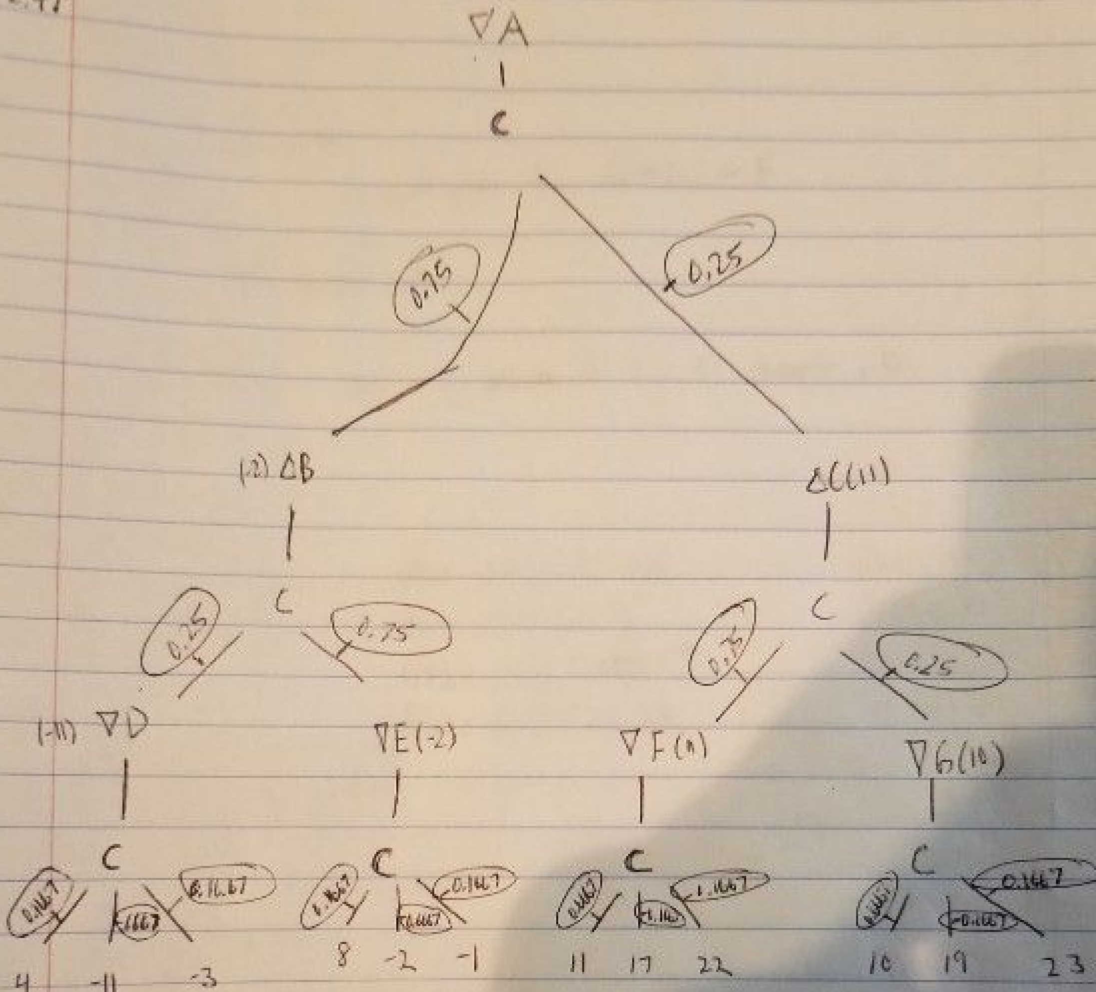
G: $-\infty, -2$

no nodes can be pruned,

For all nodes, the α value $<$ β value

2.3) Since there is no pruning in this tree, the alpha-beta pruning does not do anything, so it would not have an advantage over the naive minimax. However in larger trees, alpha-beta pruning will be more efficient since it does not search unnecessary nodes if there are any.

24)



25)

$A = 0.75(-2) + 11(0.25) = \boxed{1.25}$

$B = 0.25(-11) + 0.75(-2) = \boxed{-4.25}$

$C = 0.75(11) + 0.25(10) = \boxed{10.75}$

$D = 0.1667(4) + 0.6667(-11) + 0.1667(-3)$
$= \boxed{-7.167}$

$E = 0.1667(8) + 0.6667(-2) + 0.1667(-1)$
$= \boxed{-0.1665}$

$F = 0.6667(11) + 0.1667(17) + 0.1667(22)$
$= \boxed{13.835}$

$G = 0.6667(10) + 0.1667(19) + 0.1667(23)$
$= \boxed{13.6684}$

Question 3.

3.1) h_1 is not admissible.

$h_1(E) = 6$ cheapest path from $E \to G = 5$

$h(E) >$ cheapest path

h_2 is admissible since all values of h_2 <= cheapest path

3.2) A^*: using h_1

$f(b) = 2+14 = \underline{16}$

$f(d) = 5+5 = \underline{10}$

$f(b) = 5+9+14 = \underline{28}$

$f(c) = 5+1+10 = \underline{16}$

$f(f) = 5+2+2 = 9$

$f(e) = 5+9+6 = 20$

$f(e) = 5+2+4+6 = \underline{17}$

$f(g) = 5+2+8+0 = \underline{15}$

$\boxed{A \to D \to F \to G}$

A^*: using h_2

$f(b) = 2+12 = \underline{14}$

$f(d) = 5+10 = \underline{15}$

$f(d) = 2+9+10 = \underline{21}$

$f(c) = 2+1+11 = \underline{14}$

$f(d) = 2+1+1+10 = \underline{14}$

$f(f) = 4+2+5.5 = \underline{11.5}$

$f(e) = 4+9+5 = \underline{18}$

$f(e) = 6+4+5 = 15$

$f(g) = 6+8+0 = 14$

$\boxed{A \to B \to C \to D \to F \to G}$

they return different values