29 INTRODUCTION TO QUANTUM PHYSICS

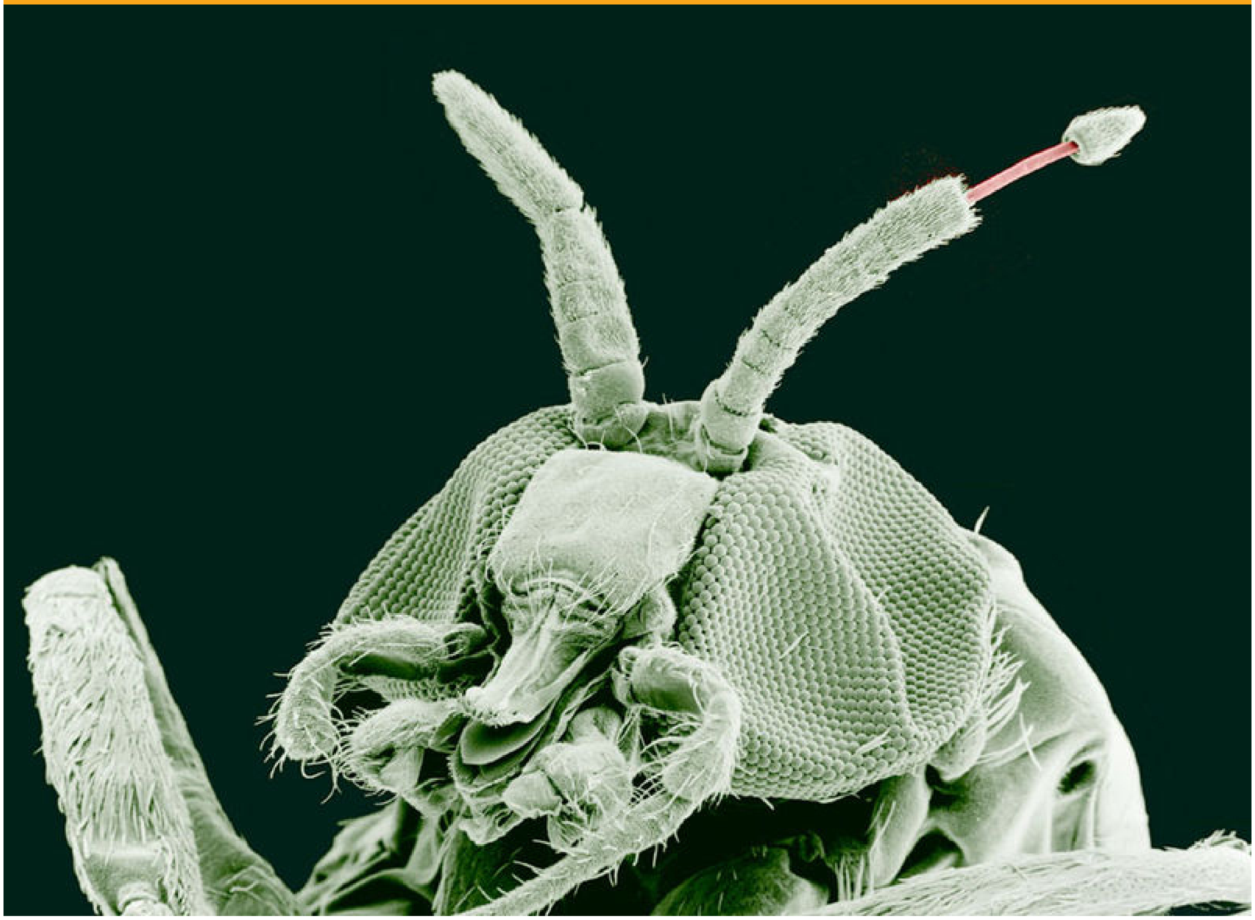

Figure 29.1 A black fly imaged by an electron microscope is as monstrous as any science-fiction creature. (credit: U.S. Department of Agriculture via Wikimedia Commons)

Chapter Outline

29.8. The Particle-Wave Duality Reviewed
- Explain the concept of particle-wave duality, and its scope.

Introduction to Quantum Physics

Quantum mechanics is the branch of physics needed to deal with submicroscopic objects. Because these objects are smaller than we can observe directly with our senses and generally must be observed with the aid of instruments, parts of quantum mechanics seem as foreign and bizarre as parts of relativity. But, like relativity, quantum mechanics has been shown to be valid—truth is often stranger than fiction.

Certain aspects of quantum mechanics are familiar to us. We accept as fact that matter is composed of atoms, the smallest unit of an element, and that these atoms combine to form molecules, the smallest unit of a compound. (See **Figure 29.2**.) While we cannot see the individual water molecules in a stream, for example, we are aware that this is because molecules are so small and so numerous in that stream. When introducing atoms, we commonly say that electrons orbit atoms in discrete shells around a tiny nucleus, itself composed of smaller particles called protons and neutrons. We are also aware that electric charge comes in tiny units carried almost entirely by electrons and protons. As with water molecules in a stream, we do not notice individual charges in the current through a lightbulb, because the charges are so small and so numerous in the macroscopic situations we sense directly.

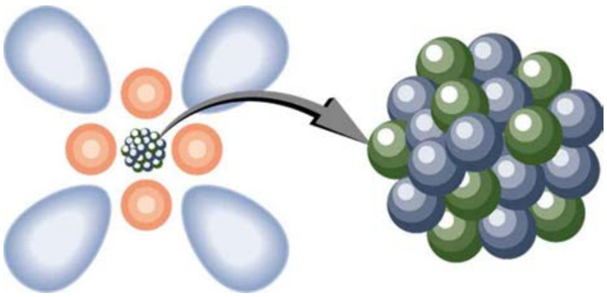

Figure 29.2 Atoms and their substructure are familiar examples of objects that require quantum mechanics to be fully explained. Certain of their characteristics, such as the discrete electron shells, are classical physics explanations. In quantum mechanics we conceptualize discrete "electron clouds" around the nucleus.

Making Connections: Realms of Physics

Classical physics is a good approximation of modern physics under conditions first discussed in the **The Nature of Science and Physics**. Quantum mechanics is valid in general, and it must be used rather than classical physics to describe small objects, such as atoms.

Atoms, molecules, and fundamental electron and proton charges are all examples of physical entities that are **quantized**—that is, they appear only in certain discrete values and do not have every conceivable value. Quantized is the opposite of continuous. We cannot have a fraction of an atom, or part of an electron's charge, or 14-1/3 cents, for example. Rather, everything is built of integral multiples of these substructures. Quantum physics is the branch of physics that deals with small objects and the quantization of various entities, including energy and angular momentum. Just as with classical physics, quantum physics has several subfields, such as mechanics and the study of electromagnetic forces. The **correspondence principle** states that in the classical limit (large, slow-moving objects), **quantum mechanics** becomes the same as classical physics. In this chapter, we begin the development of quantum mechanics and its description of the strange submicroscopic world. In later chapters, we will examine many areas, such as atomic and nuclear physics, in which quantum mechanics is crucial.

29.1 Quantization of Energy

Planck's Contribution

Energy is quantized in some systems, meaning that the system can have only certain energies and not a continuum of energies, unlike the classical case. This would be like having only certain speeds at which a car can travel because its kinetic energy can have only certain values. We also find that some forms of energy transfer take place with discrete lumps of energy. While most of us are familiar with the quantization of matter into lumps called atoms, molecules, and the like, we are less aware that energy, too, can be quantized. Some of the earliest clues about the necessity of quantum mechanics over classical physics came from the quantization of energy.

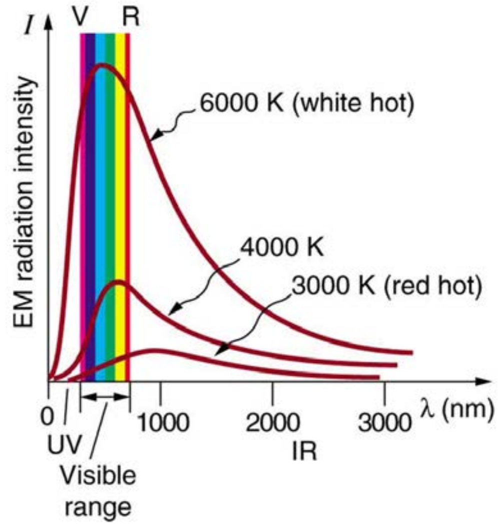

Figure 29.3 Graphs of blackbody radiation (from an ideal radiator) at three different radiator temperatures. The intensity or rate of radiation emission increases dramatically with temperature, and the peak of the spectrum shifts toward the visible and ultraviolet parts of the spectrum. The shape of the spectrum cannot be described with classical physics.

Where is the quantization of energy observed? Let us begin by considering the emission and absorption of electromagnetic (EM) radiation. The EM spectrum radiated by a hot solid is linked directly to the solid's temperature. (See **Figure 29.3**.) An ideal radiator is one that has an emissivity of 1 at all wavelengths and, thus, is jet black. Ideal radiators are therefore called **blackbodies**, and their EM radiation is called **blackbody radiation**. It was discussed that the total intensity of the radiation varies as T^4, the fourth power of the absolute temperature of the body, and that the peak of the spectrum shifts to shorter wavelengths at higher temperatures. All of this seems quite continuous, but it was the curve of the spectrum of intensity versus wavelength that gave a clue that the energies of the atoms in the solid are quantized. In fact, providing a theoretical explanation for the experimentally measured shape of the spectrum was a mystery at the turn of the century. When this "ultraviolet catastrophe" was eventually solved, the answers led to new technologies such as computers and the sophisticated imaging techniques described in earlier chapters. Once again, physics as an enabling science changed the way we live.

The German physicist Max Planck (1858–1947) used the idea that atoms and molecules in a body act like oscillators to absorb and emit radiation. The energies of the oscillating atoms and molecules had to be quantized to correctly describe the shape of the blackbody spectrum. Planck deduced that the energy of an oscillator having a frequency f is given by

$$E = \left(n + \frac{1}{2}\right)hf. \tag{29.1}$$

Here n is any nonnegative integer (0, 1, 2, 3, …). The symbol h stands for **Planck's constant**, given by

$$h = 6.626\times10^{-34}\ \text{J}\cdot\text{s}. \tag{29.2}$$

The equation $E = \left(n + \frac{1}{2}\right)hf$ means that an oscillator having a frequency f (emitting and absorbing EM radiation of frequency f) can have its energy increase or decrease only in *discrete* steps of size

$$\Delta E = hf. \tag{29.3}$$

It might be helpful to mention some macroscopic analogies of this quantization of energy phenomena. This is like a pendulum that has a characteristic oscillation frequency but can swing with only certain amplitudes. Quantization of energy also resembles a standing wave on a string that allows only particular harmonics described by integers. It is also similar to going up and down a hill using discrete stair steps rather than being able to move up and down a continuous slope. Your potential energy takes on discrete values as you move from step to step.

Using the quantization of oscillators, Planck was able to correctly describe the experimentally known shape of the blackbody spectrum. This was the first indication that energy is sometimes quantized on a small scale and earned him the Nobel Prize in Physics in 1918. Although Planck's theory comes from observations of a macroscopic object, its analysis is based on atoms and molecules. It was such a revolutionary departure from classical physics that Planck himself was reluctant to accept his own idea that energy states are not continuous. The general acceptance of Planck's energy quantization was greatly enhanced by Einstein's explanation of the photoelectric effect (discussed in the next section), which took energy quantization a step further. Planck was fully involved in the development of both early quantum mechanics and relativity. He quickly embraced Einstein's special relativity, published in 1905, and in 1906 Planck was the first to suggest the correct formula for relativistic momentum, $p = \gamma mu$.

Figure 29.4 The German physicist Max Planck had a major influence on the early development of quantum mechanics, being the first to recognize that energy is sometimes quantized. Planck also made important contributions to special relativity and classical physics. (credit: Library of Congress, Prints and Photographs Division via Wikimedia Commons)

Note that Planck's constant h is a very small number. So for an infrared frequency of $10^{14}\ \text{Hz}$ being emitted by a blackbody, for example, the difference between energy levels is only $\Delta E = hf = (6.63\times10^{-34}\ \text{J·s})(10^{14}\ \text{Hz}) = 6.63\times10^{-20}\ \text{J},$ or about 0.4 eV. This 0.4 eV of energy is significant compared with typical atomic energies, which are on the order of an electron volt, or thermal energies, which are typically fractions of an electron volt. But on a macroscopic or classical scale, energies are typically on the order of joules. Even if macroscopic energies are quantized, the quantum steps are too small to be noticed. This is an example of the correspondence principle. For a large object, quantum mechanics produces results indistinguishable from those of classical physics.

Atomic Spectra

Now let us turn our attention to the *emission and absorption of EM radiation by gases*. The Sun is the most common example of a body containing gases emitting an EM spectrum that includes visible light. We also see examples in neon signs and candle flames. Studies of emissions of hot gases began more than two centuries ago, and it was soon recognized that these emission spectra contained huge amounts of information. The type of gas and its temperature, for example, could be determined. We now know that these EM emissions come from electrons transitioning between energy levels in individual atoms and molecules; thus, they are called **atomic spectra**. Atomic spectra remain an important analytical tool today. **Figure 29.5** shows an example of an emission spectrum obtained by passing an electric discharge through a material. One of the most important characteristics of these spectra is that they are discrete. By this we mean that only certain wavelengths, and hence frequencies, are emitted. This is called a line spectrum. If frequency and energy are associated as $\Delta E = hf,$ the energies of the electrons in the emitting atoms and molecules are quantized. This is discussed in more detail later in this chapter.

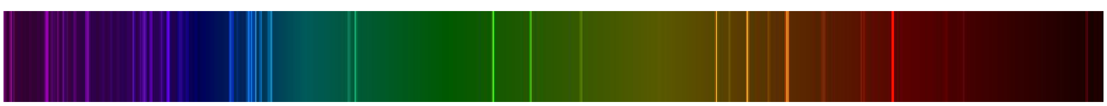

Figure 29.5 Emission spectrum of oxygen. When an electrical discharge is passed through a substance, its atoms and molecules absorb energy, which is reemitted as EM radiation. The discrete nature of these emissions implies that the energy states of the atoms and molecules are quantized. Such atomic spectra were used as analytical tools for many decades before it was understood why they are quantized. (credit: Teravolt, Wikimedia Commons)

It was a major puzzle that atomic spectra are quantized. Some of the best minds of 19th-century science failed to explain why this might be. Not until the second decade of the 20th century did an answer based on quantum mechanics begin to emerge. Again a macroscopic or classical body of gas was involved in the studies, but the effect, as we shall see, is due to individual atoms and molecules.

PhET Explorations: Models of the Hydrogen Atom

How did scientists figure out the structure of atoms without looking at them? Try out different models by shooting light at the atom. Check how the prediction of the model matches the experimental results.

PhET Interactive Simulation

Figure 29.6 Models of the Hydrogen Atom (http://cnx.org/content/m42554/1.4/hydrogen-atom_en.jar)

29.2 The Photoelectric Effect

When light strikes materials, it can eject electrons from them. This is called the **photoelectric effect**, meaning that light (*photo*) produces electricity. One common use of the photoelectric effect is in light meters, such as those that adjust the automatic iris on various types of cameras. In a similar way, another use is in solar cells, as you probably have in your calculator or have seen on a roof top or a roadside sign. These make use of the photoelectric effect to convert light into electricity for running different devices.

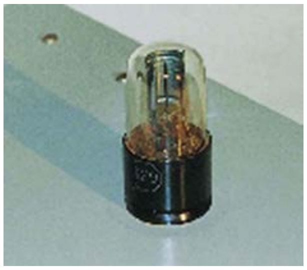

Figure 29.7 The photoelectric effect can be observed by allowing light to fall on the metal plate in this evacuated tube. Electrons ejected by the light are collected on the collector wire and measured as a current. A retarding voltage between the collector wire and plate can then be adjusted so as to determine the energy of the ejected electrons. For example, if it is sufficiently negative, no electrons will reach the wire. (credit: P.P. Urone)

This effect has been known for more than a century and can be studied using a device such as that shown in Figure 29.7. This figure shows an evacuated tube with a metal plate and a collector wire that are connected by a variable voltage source, with the collector more negative than the plate. When light (or other EM radiation) strikes the plate in the evacuated tube, it may eject electrons. If the electrons have energy in electron volts (eV) greater than the potential difference between the plate and the wire in volts, some electrons will be collected on the wire. Since the electron energy in eV is qV, where q is the electron charge and V is the potential difference, the electron energy can be measured by adjusting the retarding voltage between the wire and the plate. The voltage that stops the electrons from reaching the wire equals the energy in eV. For example, if -3.00 V barely stops the electrons, their energy is 3.00 eV. The number of electrons ejected can be determined by measuring the current between the wire and plate. The more light, the more electrons; a little circuitry allows this device to be used as a light meter.

What is really important about the photoelectric effect is what Albert Einstein deduced from it. Einstein realized that there were several characteristics of the photoelectric effect that could be explained only if *EM radiation is itself quantized*: the apparently continuous stream of energy in an EM wave is actually composed of energy quanta called photons. In his explanation of the photoelectric effect, Einstein defined a quantized unit or quantum of EM energy, which we now call a **photon**, with an energy proportional to the frequency of EM radiation. In equation form, the **photon energy** is

$$E = hf, \tag{29.4}$$

where E is the energy of a photon of frequency f and h is Planck's constant. This revolutionary idea looks similar to Planck's quantization of energy states in blackbody oscillators, but it is quite different. It is the quantization of EM radiation itself. EM waves are composed of photons and are not continuous smooth waves as described in previous chapters on optics. Their energy is absorbed and emitted in lumps, not continuously. This is exactly consistent with Planck's quantization of energy levels in blackbody oscillators, since these oscillators increase and decrease their energy in steps of hf by absorbing and emitting photons having $E = hf$. We do not observe this with our eyes, because there are so many photons in common light sources that individual photons go unnoticed. (See Figure 29.8.) The next section of the text (Photon Energies and the Electromagnetic Spectrum) is devoted to a discussion of photons and some of their characteristics and implications. For now, we will use the photon concept to explain the photoelectric effect, much as Einstein did.

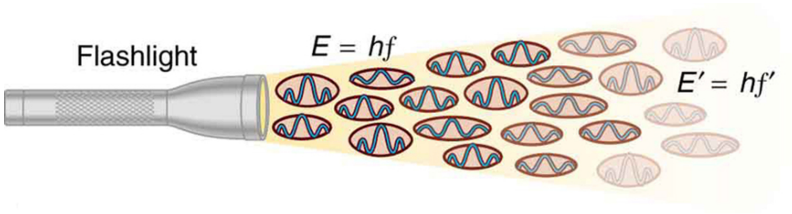

Figure 29.8 An EM wave of frequency f is composed of photons, or individual quanta of EM radiation. The energy of each photon is $E = hf$, where h is Planck's constant and f is the frequency of the EM radiation. Higher intensity means more photons per unit area. The flashlight emits large numbers of photons of many different frequencies, hence others have energy $E' = hf'$, and so on.

The photoelectric effect has the properties discussed below. All these properties are consistent with the idea that individual photons of EM radiation are absorbed by individual electrons in a material, with the electron gaining the photon's energy. Some of these properties are inconsistent with the idea that EM radiation is a simple wave. For simplicity, let us consider what happens with monochromatic EM radiation in which all photons have the same energy hf.

1. If we vary the frequency of the EM radiation falling on a material, we find the following: For a given material, there is a threshold frequency f_0 for the EM radiation below which no electrons are ejected, regardless of intensity. Individual photons interact with individual electrons. Thus if the photon energy is too small to break an electron away, no electrons will be ejected. If EM radiation was a simple wave, sufficient energy could be obtained by increasing the intensity.
2. *Once EM radiation falls on a material, electrons are ejected without delay*. As soon as an individual photon of a sufficiently high frequency is absorbed by an individual electron, the electron is ejected. If the EM radiation were a simple wave, several minutes would be required for sufficient energy to be deposited to the metal surface to eject an electron.
3. The number of electrons ejected per unit time is proportional to the intensity of the EM radiation and to no other characteristic. High-intensity EM radiation consists of large numbers of photons per unit area, with all photons having the same characteristic energy hf .
4. If we vary the intensity of the EM radiation and measure the energy of ejected electrons, we find the following: *The maximum kinetic energy of ejected electrons is independent of the intensity of the EM radiation*. Since there are so many electrons in a material, it is extremely unlikely that two photons will interact with the same electron at the same time, thereby increasing the energy given it. Instead (as noted in 3 above), increased intensity results in more electrons of the same energy being ejected. If EM radiation were a simple wave, a higher intensity could give more energy, and higher-energy electrons would be ejected.
5. The kinetic energy of an ejected electron equals the photon energy minus the binding energy of the electron in the specific material. An individual photon can give all of its energy to an electron. The photon's energy is partly used to break the electron away from the material. The remainder goes into the ejected electron's kinetic energy. In equation form, this is given by

$$\mathrm{KE}_e = hf - \mathrm{BE}, \tag{29.5}$$

where KE_e is the maximum kinetic energy of the ejected electron, hf is the photon's energy, and BE is the **binding energy** of the electron to the particular material. (BE is sometimes called the *work function* of the material.) This equation, due to Einstein in 1905, explains the properties of the photoelectric effect quantitatively. An individual photon of EM radiation (it does not come any other way) interacts with an individual electron, supplying enough energy, BE, to break it away, with the remainder going to kinetic energy. The binding energy is $\mathrm{BE} = hf_0$, where f_0 is the threshold frequency for the particular material. **Figure 29.9** shows a graph of maximum KE_e versus the frequency of incident EM radiation falling on a particular material.

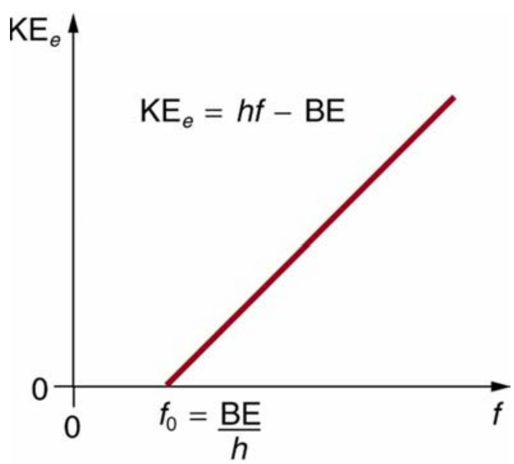

Figure 29.9 Photoelectric effect. A graph of the kinetic energy of an ejected electron, KE_e , versus the frequency of EM radiation impinging on a certain material. There is a threshold frequency below which no electrons are ejected, because the individual photon interacting with an individual electron has insufficient energy to break it away. Above the threshold energy, KE_e increases linearly with f , consistent with $\mathrm{KE}_e = hf - \mathrm{BE}$. The slope of this line is h —the data can be used to determine Planck's constant experimentally. Einstein gave the first successful explanation of such data by proposing the idea of photons—quanta of EM radiation.

Einstein's idea that EM radiation is quantized was crucial to the beginnings of quantum mechanics. It is a far more general concept than its explanation of the photoelectric effect might imply. All EM radiation can also be modeled in the form of photons, and the characteristics of EM radiation are entirely consistent with this fact. (As we will see in the next section, many aspects of EM radiation, such as the hazards of ultraviolet (UV) radiation, can be explained *only* by photon properties.) More famous for modern relativity, Einstein planted an important seed for quantum mechanics in 1905, the same year he published his first paper on special relativity. His explanation of the photoelectric effect was the basis for the Nobel Prize awarded to him in 1921. Although his other contributions to theoretical physics were also noted in that award, special and general relativity were not fully recognized in spite of having been partially verified by experiment by 1921. Although hero-worshipped, this great man never received Nobel recognition for his most famous work—relativity.

Example 29.1 Calculating Photon Energy and the Photoelectric Effect: A Violet Light

(a) What is the energy in joules and electron volts of a photon of 420-nm violet light? (b) What is the maximum kinetic energy of electrons ejected from calcium by 420-nm violet light, given that the binding energy (or work function) of electrons for calcium metal is 2.71 eV?

Strategy

To solve part (a), note that the energy of a photon is given by $E = hf$. For part (b), once the energy of the photon is calculated, it is a straightforward application of $\text{KE}_e = hf – \text{BE}$ to find the ejected electron's maximum kinetic energy, since BE is given.

Solution for (a)

Photon energy is given by

$$E = hf \tag{29.6}$$

Since we are given the wavelength rather than the frequency, we solve the familiar relationship $c = f\lambda$ for the frequency, yielding

$$f = \frac{c}{\lambda}. \tag{29.7}$$

Combining these two equations gives the useful relationship

$$E = \frac{hc}{\lambda}. \tag{29.8}$$

Now substituting known values yields

$$E = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{420\times10^{-9}\ \text{m}} = 4.74\times10^{-19}\ \text{J}. \tag{29.9}$$

Converting to eV, the energy of the photon is

$$E = \left(4.74\times10^{-19}\ \text{J}\right)\frac{1\ \text{eV}}{1.6\times10^{-19}\ \text{J}} = 2.96\ \text{eV}. \tag{29.10}$$

Solution for (b)

Finding the kinetic energy of the ejected electron is now a simple application of the equation $\text{KE}_e = hf – \text{BE}$. Substituting the photon energy and binding energy yields

$$\text{KE}_e = hf – \text{BE} = 2.96\ \text{eV} – 2.71\ \text{eV} = 0.246\ \text{eV}. \tag{29.11}$$

Discussion

The energy of this 420-nm photon of violet light is a tiny fraction of a joule, and so it is no wonder that a single photon would be difficult for us to sense directly—humans are more attuned to energies on the order of joules. But looking at the energy in electron volts, we can see that this photon has enough energy to affect atoms and molecules. A DNA molecule can be broken with about 1 eV of energy, for example, and typical atomic and molecular energies are on the order of eV, so that the UV photon in this example could have biological effects. The ejected electron (called a *photoelectron*) has a rather low energy, and it would not travel far, except in a vacuum. The electron would be stopped by a retarding potential of but 0.26 eV. In fact, if the photon wavelength were longer and its energy less than 2.71 eV, then the formula would give a negative kinetic energy, an impossibility. This simply means that the 420-nm photons with their 2.96-eV energy are not much above the frequency threshold. You can show for yourself that the threshold wavelength is 459 nm (blue light). This means that if calcium metal is used in a light meter, the meter will be insensitive to wavelengths longer than those of blue light. Such a light meter would be completely insensitive to red light, for example.

PhET Explorations: Photoelectric Effect

See how light knocks electrons off a metal target, and recreate the experiment that spawned the field of quantum mechanics.

PhET Interactive Simulation

Figure 29.10 Photoelectric Effect (http://cnx.org/content/m42558/1.5/photoelectric_en.jar)

29.3 Photon Energies and the Electromagnetic Spectrum

Ionizing Radiation

A photon is a quantum of EM radiation. Its energy is given by $E = hf$ and is related to the frequency f and wavelength λ of the radiation by

$$E = hf = \frac{hc}{\lambda}\text{(energy of a photon)}, \tag{29.12}$$

where E is the energy of a single photon and c is the speed of light. When working with small systems, energy in eV is often useful. Note that Planck's constant in these units is

$$h = 4.14\times10^{-15}\text{ eV}\cdot\text{s}. \tag{29.13}$$

Since many wavelengths are stated in nanometers (nm), it is also useful to know that

$$hc = 1240\text{ eV}\cdot\text{nm}. \tag{29.14}$$

These will make many calculations a little easier.

All EM radiation is composed of photons. **Figure 29.11** shows various divisions of the EM spectrum plotted against wavelength, frequency, and photon energy. Previously in this book, photon characteristics were alluded to in the discussion of some of the characteristics of UV, x rays, and γ rays, the first of which start with frequencies just above violet in the visible spectrum. It was noted that these types of EM radiation have characteristics much different than visible light. We can now see that such properties arise because photon energy is larger at high frequencies.

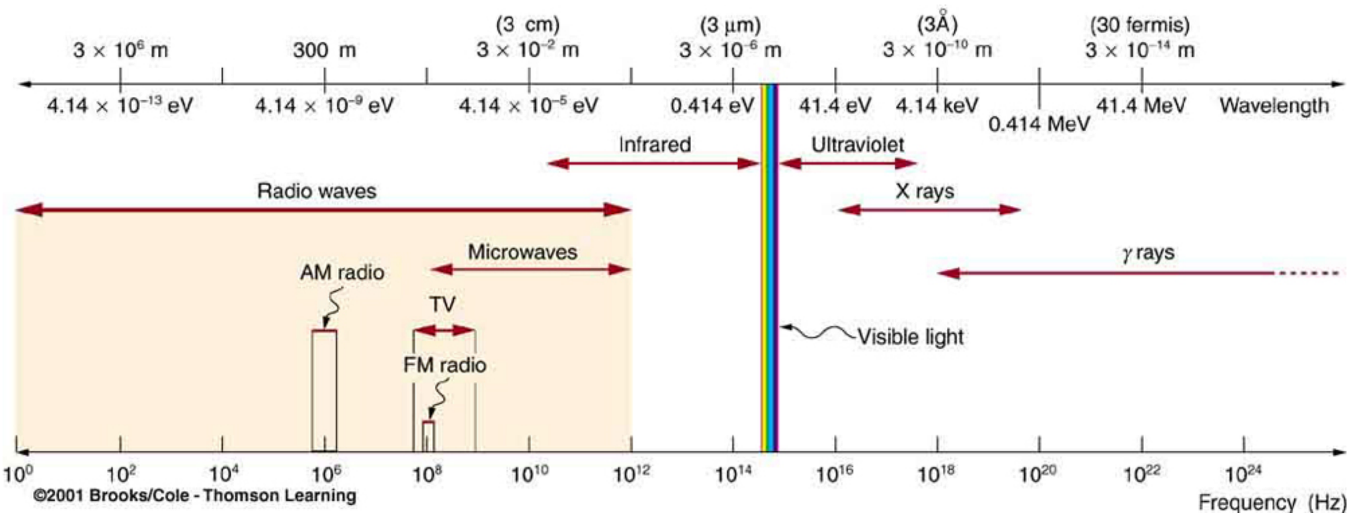

Figure 29.11 The EM spectrum, showing major categories as a function of photon energy in eV, as well as wavelength and frequency. Certain characteristics of EM radiation are directly attributable to photon energy alone.

Table 29.1 Representative Energies for Submicroscopic Effects (Order of Magnitude Only)

Rotational energies of molecules	10^{-5} eV
Vibrational energies of molecules	0.1 eV
Energy between outer electron shells in atoms	1 eV
Binding energy of a weakly bound molecule	1 eV
Energy of red light	2 eV
Binding energy of a tightly bound molecule	10 eV
Energy to ionize atom or molecule	10 to 1000 eV

Photons act as individual quanta and interact with individual electrons, atoms, molecules, and so on. The energy a photon carries is, thus, crucial to the effects it has. **Table 29.1** lists representative submicroscopic energies in eV. When we compare photon energies from the EM spectrum in **Figure 29.11** with energies in the table, we can see how effects vary with the type of EM radiation.

Gamma rays, a form of nuclear and cosmic EM radiation, can have the highest frequencies and, hence, the highest photon energies in the EM spectrum. For example, a γ -ray photon with $f = 10^{21}$ Hz has an energy $E = hf = 6.63 \times 10^{-13}\text{ J} = 4.14\text{ MeV}.$ This is sufficient energy to ionize thousands of atoms and molecules, since only 10 to 1000 eV are needed per ionization. In fact, γ rays are one type of **ionizing radiation**, as are x rays and UV, because they produce ionization in materials that absorb them. Because so much ionization can be produced, a single γ -ray photon can cause significant damage to biological tissue, killing cells or damaging their ability to properly reproduce. When cell reproduction is disrupted, the result can be cancer, one of the known effects of exposure to ionizing radiation. Since cancer cells are rapidly reproducing, they are exceptionally sensitive to the disruption produced by ionizing radiation. This means that ionizing radiation has positive uses in cancer treatment as well as risks in producing cancer.

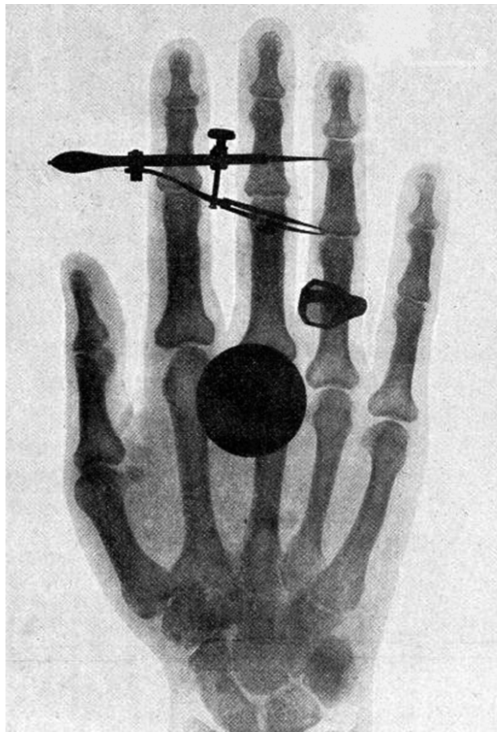

Figure 29.12 One of the first x-ray images, taken by Röentgen himself. The hand belongs to Bertha Röentgen, his wife. (credit: Wilhelm Conrad Röntgen, via Wikimedia Commons)

High photon energy also enables γ rays to penetrate materials, since a collision with a single atom or molecule is unlikely to absorb all the γ ray's energy. This can make γ rays useful as a probe, and they are sometimes used in medical imaging. **x rays**, as you can see in **Figure 29.11**, overlap with the low-frequency end of the γ ray range. Since x rays have energies of keV and up, individual x-ray photons also can produce large amounts of ionization. At lower photon energies, x rays are not as penetrating as γ rays and are slightly less hazardous. X rays are ideal for medical imaging, their most common use, and a fact that was recognized immediately upon their discovery in 1895 by the German physicist W. C. Roentgen (1845–1923). (See **Figure 29.12**.) Within one year of their discovery, x rays (for a time called Roentgen rays) were used for medical diagnostics. Roentgen received the 1901 Nobel Prize for the discovery of x rays.

Connections: Conservation of Energy

Once again, we find that conservation of energy allows us to consider the initial and final forms that energy takes, without having to make detailed calculations of the intermediate steps. **Example 29.2** is solved by considering only the initial and final forms of energy.

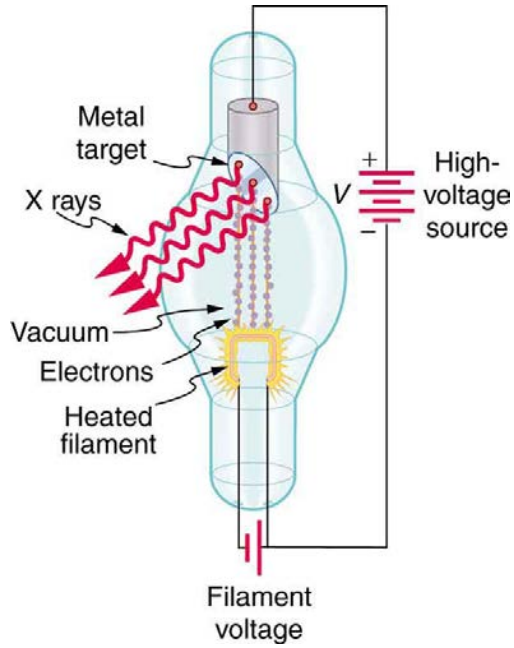

Figure 29.13 X rays are produced when energetic electrons strike the copper anode of this cathode ray tube (CRT). Electrons (shown here as separate particles) interact individually with the material they strike, sometimes producing photons of EM radiation.

While γ rays originate in nuclear decay, x rays are produced by the process shown in **Figure 29.13**. Electrons ejected by thermal agitation from a hot filament in a vacuum tube are accelerated through a high voltage, gaining kinetic energy from the electrical potential energy. When they strike the anode, the electrons convert their kinetic energy to a variety of forms, including thermal energy. But since an accelerated charge radiates EM waves, and since the electrons act individually, photons are also produced. Some of these x-ray photons obtain the kinetic energy of the electron. The accelerated electrons originate at the cathode, so such a tube is called a cathode ray tube (CRT), and various versions of them are found in older TV and computer screens as well as in x-ray machines.

Example 29.2 X-ray Photon Energy and X-ray Tube Voltage

Find the maximum energy in eV of an x-ray photon produced by electrons accelerated through a potential difference of 50.0 kV in a CRT like the one in **Figure 29.13**.

Strategy

Electrons can give all of their kinetic energy to a single photon when they strike the anode of a CRT. (This is something like the photoelectric effect in reverse.) The kinetic energy of the electron comes from electrical potential energy. Thus we can simply equate the maximum photon energy to the electrical potential energy—that is, $hf = qV.$ (We do not have to calculate each step from beginning to end if we know that all of the starting energy qV is converted to the final form $hf.$)

Solution

The maximum photon energy is $hf = qV$, where q is the charge of the electron and V is the accelerating voltage. Thus,

$$hf = (1.60\times10^{-19}\text{ C})(50.0\times10^{3}\text{ V}). \tag{29.15}$$

From the definition of the electron volt, we know $1\text{ eV} = 1.60\times10^{-19}\text{ J}$, where $1\text{ J} = 1\text{ C}\cdot\text{V}.$ Gathering factors and converting energy to eV yields

$$hf = (50.0\times10^{3})(1.60\times10^{-19}\text{ C}\cdot\text{V})\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ C}\cdot\text{V}}\right) = (50.0\times10^{3})(1\text{ eV}) = 50.0\text{ keV}. \tag{29.16}$$

Discussion

This example produces a result that can be applied to many similar situations. If you accelerate a single elementary charge, like that of an electron, through a potential given in volts, then its energy in eV has the same numerical value. Thus a 50.0-kV potential generates 50.0 keV electrons, which in turn can produce photons with a maximum energy of 50 keV. Similarly, a 100-kV potential in an x-ray tube can generate up to 100-keV x-ray photons. Many x-ray tubes have adjustable voltages so that various energy x rays with differing energies, and therefore differing abilities to penetrate, can be generated.

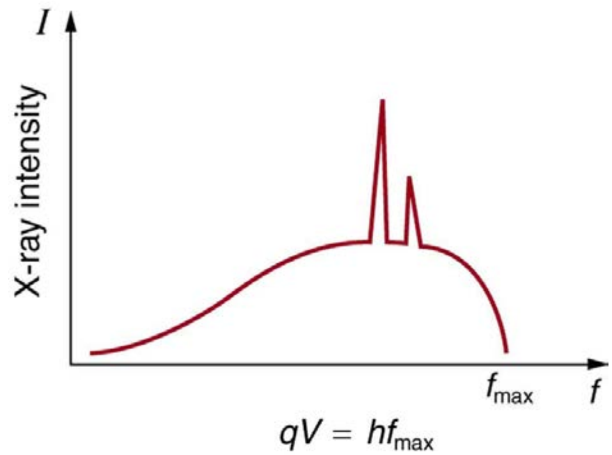

Figure 29.14 X-ray spectrum obtained when energetic electrons strike a material. The smooth part of the spectrum is bremsstrahlung, while the peaks are characteristic of the anode material. Both are atomic processes that produce energetic photons known as x-ray photons.

Figure 29.14 shows the spectrum of x rays obtained from an x-ray tube. There are two distinct features to the spectrum. First, the smooth distribution results from electrons being decelerated in the anode material. A curve like this is obtained by detecting many photons, and it is apparent that the maximum energy is unlikely. This decelerating process produces radiation that is called **bremsstrahlung** (German for *braking radiation*). The second feature is the existence of sharp peaks in the spectrum; these are called **characteristic x rays**, since they are characteristic of the anode material. Characteristic x rays come from atomic excitations unique to a given type of anode material. They are akin to lines in atomic spectra, implying the energy levels of atoms are quantized. Phenomena such as discrete atomic spectra and characteristic x rays are explored further in **Atomic Physics**.

Ultraviolet radiation (approximately 4 eV to 300 eV) overlaps with the low end of the energy range of x rays, but UV is typically lower in energy. UV comes from the de-excitation of atoms that may be part of a hot solid or gas. These atoms can be given energy that they later release as UV by numerous processes, including electric discharge, nuclear explosion, thermal agitation, and exposure to x rays. A UV photon has sufficient energy to ionize atoms and molecules, which makes its effects different from those of visible light. UV thus has some of the same biological effects as γ rays and x rays. For example, it can cause skin cancer and is used as a sterilizer. The major difference is that several UV photons are required to disrupt cell reproduction or kill a bacterium, whereas single γ -ray and X-ray photons can do the same damage. But since UV does have the energy to alter molecules, it can do what visible light cannot. One of the beneficial aspects of UV is that it triggers the production of vitamin D in the skin, whereas visible light has insufficient energy per photon to alter the molecules that trigger this production. Infantile jaundice is treated by exposing the baby to UV (with eye protection), called phototherapy, the beneficial effects of which are thought to be related to its ability to help prevent the buildup of potentially toxic bilirubin in the blood.

Example 29.3 Photon Energy and Effects for UV

Short-wavelength UV is sometimes called vacuum UV, because it is strongly absorbed by air and must be studied in a vacuum. Calculate the photon energy in eV for 100-nm vacuum UV, and estimate the number of molecules it could ionize or break apart.

Strategy

Using the equation $E = hf$ and appropriate constants, we can find the photon energy and compare it with energy information in **Table 29.1**.

Solution

The energy of a photon is given by

$$E = hf = \frac{hc}{\lambda}. \tag{29.17}$$

Using $hc = 1240 \text{ eV} \cdot \text{nm},$ we find that

$$E = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{100 \text{ nm}} = 12.4 \text{ eV}. \tag{29.18}$$

Discussion

According to **Table 29.1**, this photon energy might be able to ionize an atom or molecule, and it is about what is needed to break up a tightly bound molecule, since they are bound by approximately 10 eV. This photon energy could destroy about a dozen weakly bound molecules. Because of its high photon energy, UV disrupts atoms and molecules it interacts with. One good consequence is that all but the longest-wavelength UV is strongly absorbed and is easily blocked by sunglasses. In fact, most of the Sun’s UV is absorbed by a thin layer of ozone in the upper atmosphere, protecting sensitive organisms on Earth. Damage to our ozone layer by the addition of such chemicals as CFC’s has reduced this protection for us.

Visible Light

The range of photon energies for **visible light** from red to violet is 1.63 to 3.26 eV, respectively (left for this chapter’s Problems and Exercises to verify). These energies are on the order of those between outer electron shells in atoms and molecules. This means that these photons can be absorbed by atoms and molecules. A *single* photon can actually stimulate the retina, for example, by altering a receptor molecule that then triggers a nerve impulse. Photons can be absorbed or emitted only by atoms and molecules that have precisely the correct quantized energy step to do so. For example, if a red photon of frequency f encounters a molecule that has an energy step, $\Delta E,$ equal to $hf,$ then the photon can be absorbed. Violet flowers absorb red and reflect violet; this implies there is no energy step between levels in the receptor molecule equal to the violet photon’s energy, but there is an energy step for the red.

There are some noticeable differences in the characteristics of light between the two ends of the visible spectrum that are due to photon energies. Red light has insufficient photon energy to expose most black-and-white film, and it is thus used to illuminate darkrooms where such film is developed. Since violet light has a higher photon energy, dyes that absorb violet tend to fade more quickly than those that do not. (See **Figure 29.15**.) Take a look at some faded color posters in a storefront some time, and you will notice that the blues and violets are the last to fade. This is because other dyes, such as red and green dyes, absorb blue and violet photons, the higher energies of which break up their weakly bound molecules. (Complex molecules such as those in dyes and DNA tend to be weakly bound.) Blue and violet dyes reflect those colors and, therefore, do not absorb these more energetic photons, thus suffering less molecular damage.

Figure 29.15 Why do the reds, yellows, and greens fade before the blues and violets when exposed to the Sun, as with this poster? The answer is related to photon energy. (credit: Deb Collins, Flickr)

Transparent materials, such as some glasses, do not absorb any visible light, because there is no energy step in the atoms or molecules that could absorb the light. Since individual photons interact with individual atoms, it is nearly impossible to have two photons absorbed simultaneously to reach a large energy step. Because of its lower photon energy, visible light can sometimes pass through many kilometers of a substance, while higher frequencies like UV, x ray, and γ rays are absorbed, because they have sufficient photon energy to ionize the material.

Example 29.4 How Many Photons per Second Does a Typical Light Bulb Produce?

Assuming that 10.0% of a 100-W light bulb’s energy output is in the visible range (typical for incandescent bulbs) with an average wavelength of 580 nm, calculate the number of visible photons emitted per second.

Strategy

Power is energy per unit time, and so if we can find the energy per photon, we can determine the number of photons per second. This will best be done in joules, since power is given in watts, which are joules per second.

Solution

The power in visible light production is 10.0% of 100 W, or 10.0 J/s. The energy of the average visible photon is found by substituting the given average wavelength into the formula

$$E = \frac{hc}{\lambda}. \tag{29.19}$$

This produces

$$E = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{580\times10^{-9}\ \text{m}} = 3.43\times10^{-19}\ \text{J}. \tag{29.20}$$

The number of visible photons per second is thus

$$\text{photon/s} = \frac{10.0\ \text{J/s}}{3.43\times10^{-19}\ \text{J/photon}} = 2.92\times10^{19}\ \text{photon/s}. \tag{29.21}$$

Discussion

This incredible number of photons per second is verification that individual photons are insignificant in ordinary human experience. It is also a verification of the correspondence principle—on the macroscopic scale, quantization becomes essentially continuous or classical. Finally, there are so many photons emitted by a 100-W lightbulb that it can be seen by the unaided eye many kilometers away.

Lower-Energy Photons

Infrared radiation (IR) has even lower photon energies than visible light and cannot significantly alter atoms and molecules. IR can be absorbed and emitted by atoms and molecules, particularly between closely spaced states. IR is extremely strongly absorbed by water, for example, because water molecules have many states separated by energies on the order of $10^{-5}\ \text{eV}$ to $10^{-2}\ \text{eV},$ well within the IR and microwave energy ranges. This is why in the IR range, skin is almost jet black, with an emissivity near 1—there are many states in water molecules in the skin that can absorb a large range of IR photon energies. Not all molecules have this property. Air, for example, is nearly transparent to many IR frequencies.

Microwaves are the highest frequencies that can be produced by electronic circuits, although they are also produced naturally. Thus microwaves are similar to IR but do not extend to as high frequencies. There are states in water and other molecules that have the same frequency and energy as microwaves, typically about $10^{-5}\ \text{eV}.$ This is one reason why food absorbs microwaves more strongly than many other materials, making microwave ovens an efficient way of putting energy directly into food.

Photon energies for both IR and microwaves are so low that huge numbers of photons are involved in any significant energy transfer by IR or microwaves (such as warming yourself with a heat lamp or cooking pizza in the microwave). Visible light, IR, microwaves, and all lower frequencies cannot produce ionization with single photons and do not ordinarily have the hazards of higher frequencies. When visible, IR, or microwave radiation *is* hazardous, such as the inducement of cataracts by microwaves, the hazard is due to huge numbers of photons acting together (not to an accumulation of photons, such as sterilization by weak UV). The negative effects of visible, IR, or microwave radiation can be thermal effects, which could be produced by any heat source. But one difference is that at very high intensity, strong electric and magnetic fields can be produced by photons acting together. Such electromagnetic fields (EMF) can actually ionize materials.

Misconception Alert: High-Voltage Power Lines

Although some people think that living near high-voltage power lines is hazardous to one's health, ongoing studies of the transient field effects produced by these lines show their strengths to be insufficient to cause damage. Demographic studies also fail to show significant correlation of ill effects with high-voltage power lines. The American Physical Society issued a report over 10 years ago on power-line fields, which concluded that the scientific literature and reviews of panels show no consistent, significant link between cancer and power-line fields. They also felt that the "diversion of resources to eliminate a threat which has no persuasive scientific basis is disturbing."

It is virtually impossible to detect individual photons having frequencies below microwave frequencies, because of their low photon energy. But the photons are there. A continuous EM wave can be modeled as photons. At low frequencies, EM waves are generally treated as time- and position-varying electric and magnetic fields with no discernible quantization. This is another example of the correspondence principle in situations involving huge numbers of photons.

PhET Explorations: Color Vision

Make a whole rainbow by mixing red, green, and blue light. Change the wavelength of a monochromatic beam or filter white light. View the light as a solid beam, or see the individual photons.

PhET Interactive Simulation

Figure 29.16 Color Vision (http://cnx.org/content/m42563/1.7/color-vision_en.jar)

29.4 Photon Momentum

Measuring Photon Momentum

The quantum of EM radiation we call a **photon** has properties analogous to those of particles we can see, such as grains of sand. A photon interacts as a unit in collisions or when absorbed, rather than as an extensive wave. Massive quanta, like electrons, also act like macroscopic particles—something we expect, because they are the smallest units of matter. Particles carry momentum as well as energy. Despite photons having no mass, there has long been evidence that EM radiation carries momentum. (Maxwell and others who studied EM waves predicted that they would carry momentum.) It is now a well-established fact that photons *do* have momentum. In fact, photon momentum is suggested by the photoelectric effect, where photons knock electrons out of a substance. **Figure 29.17** shows macroscopic evidence of photon momentum.

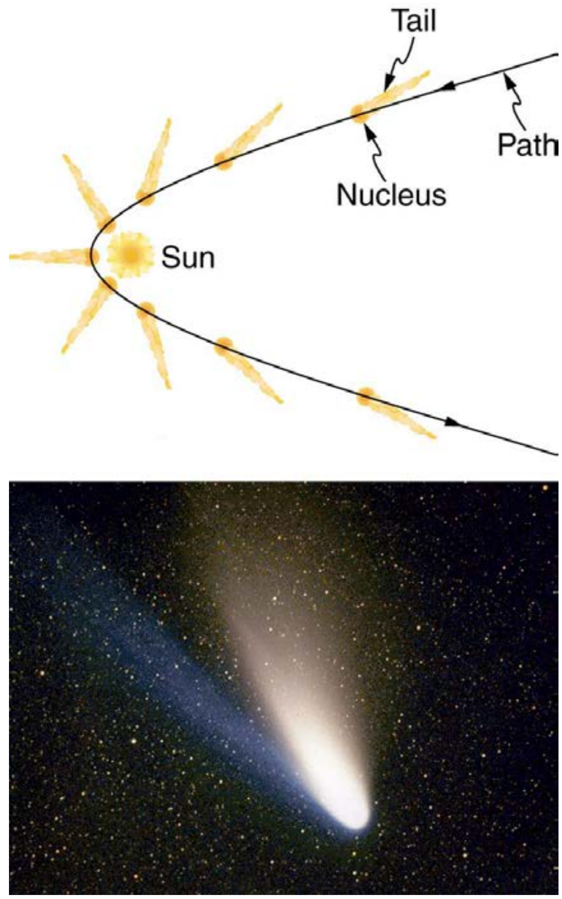

Figure 29.17 The tails of the Hale-Bopp comet point away from the Sun, evidence that light has momentum. Dust emanating from the body of the comet forms this tail. Particles of dust are pushed away from the Sun by light reflecting from them. The blue ionized gas tail is also produced by photons interacting with atoms in the comet material. (credit: Geoff Chester, U.S. Navy, via Wikimedia Commons)

Figure 29.17 shows a comet with two prominent tails. What most people do not know about the tails is that they always point *away* from the Sun rather than trailing behind the comet (like the tail of Bo Peep's sheep). Comet tails are composed of gases and dust evaporated from the body of the comet and ionized gas. The dust particles recoil away from the Sun when photons scatter from them. Evidently, photons carry momentum in the direction of their motion (away from the Sun), and some of this momentum is transferred to dust particles in collisions. Gas atoms and molecules in the blue tail are most affected by other particles of radiation, such as protons and electrons emanating from the Sun, rather than by the momentum of photons.

Connections: Conservation of Momentum

Not only is momentum conserved in all realms of physics, but all types of particles are found to have momentum. We expect particles with mass to have momentum, but now we see that massless particles including photons also carry momentum.

Momentum is conserved in quantum mechanics just as it is in relativity and classical physics. Some of the earliest direct experimental evidence of this came from scattering of x-ray photons by electrons in substances, named Compton scattering after the American physicist Arthur H. Compton (1892–1962). Around 1923, Compton observed that x rays scattered from materials had a decreased energy and correctly analyzed this as being due to the scattering of photons from electrons. This phenomenon could be handled as a collision between two particles—a photon and an electron at rest in the material. Energy and momentum are conserved in the collision. (See **Figure 29.18**) He won a Nobel Prize in 1929 for the discovery of this scattering, now called the **Compton effect**, because it helped prove that **photon momentum** is given by

$$p = \frac{h}{\lambda}, \tag{29.22}$$

where h is Planck's constant and λ is the photon wavelength. (Note that relativistic momentum given as $p = \gamma mu$ is valid only for particles having mass.)

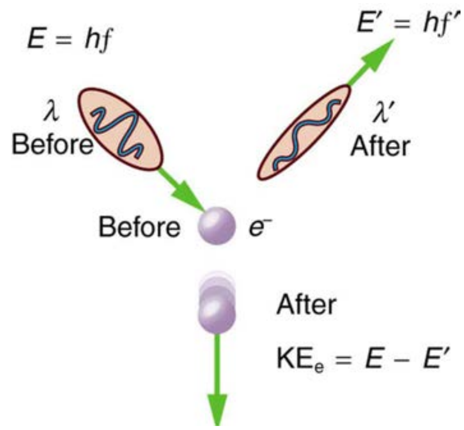

Figure 29.18 The Compton effect is the name given to the scattering of a photon by an electron. Energy and momentum are conserved, resulting in a reduction of both for the scattered photon. Studying this effect, Compton verified that photons have momentum.

We can see that photon momentum is small, since $p = h / \lambda$ and h is very small. It is for this reason that we do not ordinarily observe photon momentum. Our mirrors do not recoil when light reflects from them (except perhaps in cartoons). Compton saw the effects of photon momentum because he was observing x rays, which have a small wavelength and a relatively large momentum, interacting with the lightest of particles, the electron.

Example 29.5 Electron and Photon Momentum Compared

(a) Calculate the momentum of a visible photon that has a wavelength of 500 nm. (b) Find the velocity of an electron having the same momentum. (c) What is the energy of the electron, and how does it compare with the energy of the photon?

Strategy

Finding the photon momentum is a straightforward application of its definition: $p = \frac{h}{\lambda}$. If we find the photon momentum is small, then we can assume that an electron with the same momentum will be nonrelativistic, making it easy to find its velocity and kinetic energy from the classical formulas.

Solution for (a)

Photon momentum is given by the equation:

$$p = \frac{h}{\lambda}. \tag{29.23}$$

Entering the given photon wavelength yields

$$p = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{500\times10^{-9}\ \text{m}} = 1.33\times10^{-27}\ \text{kg}\cdot\text{m/s}. \tag{29.24}$$

Solution for (b)

Since this momentum is indeed small, we will use the classical expression $p = mv$ to find the velocity of an electron with this momentum. Solving for v and using the known value for the mass of an electron gives

$$v = \frac{p}{m} = \frac{1.33\times10^{-27}\ \text{kg}\cdot\text{m/s}}{9.11\times10^{-31}\ \text{kg}} = 1460\ \text{m/s} \approx 1460\ \text{m/s}. \tag{29.25}$$

Solution for (c)

The electron has kinetic energy, which is classically given by

$$\text{KE}_e = \frac{1}{2}mv^2. \tag{29.26}$$

Thus,

$$\text{KE}_e = \frac{1}{2}(9.11\times10^{-3}\ \text{kg})(1455\ \text{m/s})^2 = 9.64\times10^{-25}\ \text{J}. \tag{29.27}$$

Converting this to eV by multiplying by $(1\ \text{eV})/(1.602\times10^{-19}\ \text{J})$ yields

$$\text{KE}_e = 6.02\times10^{-6}\ \text{eV}. \tag{29.28}$$

The photon energy E is

$$E = \frac{hc}{\lambda} = \frac{1240\ \text{eV}\cdot\text{nm}}{500\ \text{nm}} = 2.48\ \text{eV}, \tag{29.29}$$

which is about five orders of magnitude greater.

Discussion

Photon momentum is indeed small. Even if we have huge numbers of them, the total momentum they carry is small. An electron with the same momentum has a 1460 m/s velocity, which is clearly nonrelativistic. A more massive particle with the same momentum would have an even smaller velocity. This is borne out by the fact that it takes far less energy to give an electron the same momentum as a photon. But on a quantum-mechanical scale, especially for high-energy photons interacting with small masses, photon momentum is significant. Even on a large scale, photon momentum can have an effect if there are enough of them and if there is nothing to prevent the slow recoil of matter. Comet tails are one example, but there are also proposals to build space sails that use huge low-mass mirrors (made of aluminized Mylar) to reflect sunlight. In the vacuum of space, the mirrors would gradually recoil and could actually take spacecraft from place to place in the solar system. (See **Figure 29.19**.)

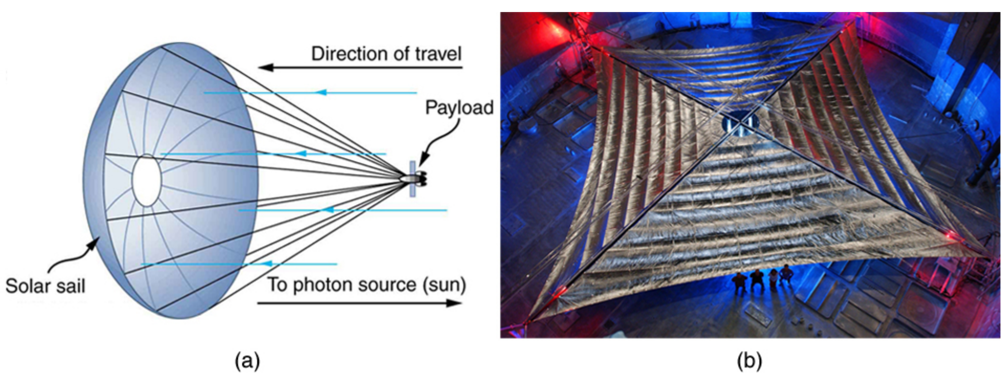

Figure 29.19 (a) Space sails have been proposed that use the momentum of sunlight reflecting from gigantic low-mass sails to propel spacecraft about the solar system. A Russian test model of this (the Cosmos 1) was launched in 2005, but did not make it into orbit due to a rocket failure. (b) A U.S. version of this, labeled LightSail-1, is scheduled for trial launches in the first part of this decade. It will have a 40-m^2 sail. (credit: Kim Newton/NASA)

Relativistic Photon Momentum

There is a relationship between photon momentum p and photon energy E that is consistent with the relation given previously for the relativistic total energy of a particle as $E^2 = (pc)^2 + (mc)^2$. We know m is zero for a photon, but p is not, so that $E^2 = (pc)^2 + (mc)^2$ becomes

$$E = pc, \tag{29.30}$$

or

$$p = \frac{E}{c}(\text{photons}). \tag{29.31}$$

To check the validity of this relation, note that $E = hc/\lambda$ for a photon. Substituting this into $p = E/c$ yields

$$p = (hc/\lambda)/c = \frac{h}{\lambda}, \tag{29.32}$$

as determined experimentally and discussed above. Thus, $p = E/c$ is equivalent to Compton's result $p = h/\lambda$. For a further verification of the relationship between photon energy and momentum, see **Example 29.6**.

Photon Detectors

Almost all detection systems talked about thus far—eyes, photographic plates, photomultiplier tubes in microscopes, and CCD cameras—rely on particle-like properties of photons interacting with a sensitive area. A change is caused and either the change is cascaded or zillions of points are recorded to form an image we detect. These detectors are used in biomedical imaging systems, and there is ongoing research into improving the efficiency of receiving photons, particularly by cooling detection systems and reducing thermal effects.

Example 29.6 Photon Energy and Momentum

Show that $p = E/c$ for the photon considered in the **Example 29.5**.

Strategy

We will take the energy E found in **Example 29.5**, divide it by the speed of light, and see if the same momentum is obtained as before.

Solution

Given that the energy of the photon is 2.48 eV and converting this to joules, we get

$$p = \frac{E}{c} = \frac{(2.48\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{3.00\times10^{8}\text{ m/s}} = 1.33\times10^{-27}\text{ kg}\cdot\text{m/s}. \tag{29.33}$$

Discussion

This value for momentum is the same as found before (note that unrounded values are used in all calculations to avoid even small rounding errors), an expected verification of the relationship $p = E/c$. This also means the relationship between energy, momentum, and mass given by $E^2 = (pc)^2 + (mc)^2$ applies to both matter and photons. Once again, note that p is not zero, even when m is.

Problem-Solving Suggestion

Note that the forms of the constants $h = 4.14\times10^{-15}\text{ eV}\cdot\text{s}$ and $hc = 1240\text{ eV}\cdot\text{nm}$ may be particularly useful for this section's Problems and Exercises.

29.5 The Particle-Wave Duality

We have long known that EM radiation is a wave, capable of interference and diffraction. We now see that light can be modeled as photons, which are massless particles. This may seem contradictory, since we ordinarily deal with large objects that never act like both wave and particle. An ocean wave, for example, looks nothing like a rock. To understand small-scale phenomena, we make analogies with the large-scale phenomena we observe directly. When we say something behaves like a wave, we mean it shows interference effects analogous to those seen in overlapping water waves. (See **Figure 29.20**.) Two examples of waves are sound and EM radiation. When we say something behaves like a particle, we mean that it interacts as a discrete unit with no interference effects. Examples of particles include electrons, atoms, and photons of EM radiation. How do we talk about a phenomenon that acts like both a particle and a wave?

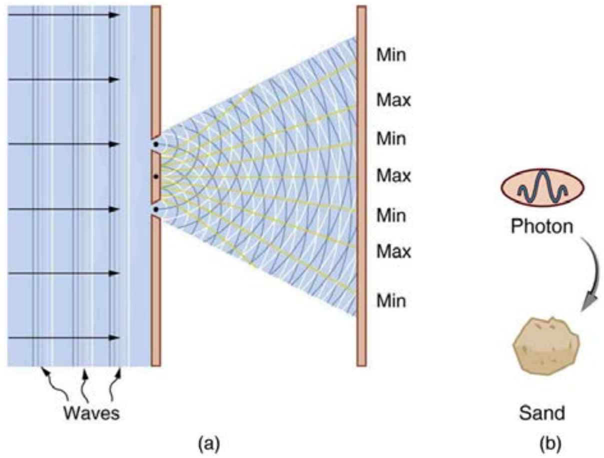

Figure 29.20 (a) The interference pattern for light through a double slit is a wave property understood by analogy to water waves. (b) The properties of photons having quantized energy and momentum and acting as a concentrated unit are understood by analogy to macroscopic particles.

There is no doubt that EM radiation interferes and has the properties of wavelength and frequency. There is also no doubt that it behaves as particles—photons with discrete energy. We call this twofold nature the **particle-wave duality**, meaning that EM radiation has both particle and wave properties. This so-called duality is simply a term for properties of the photon analogous to phenomena we can observe directly, on a macroscopic scale. If this term seems strange, it is because we do not ordinarily observe details on the quantum level directly, and our observations yield either particle *or* wavelike properties, but never both simultaneously.

Since we have a particle-wave duality for photons, and since we have seen connections between photons and matter in that both have momentum, it is reasonable to ask whether there is a particle-wave duality for matter as well. If the EM radiation we once thought to be a pure wave has particle properties, is it possible that matter has wave properties? The answer is yes. The consequences are tremendous, as we will begin to see in the next section.

PhET Explorations: Quantum Wave Interference

When do photons, electrons, and atoms behave like particles and when do they behave like waves? Watch waves spread out and interfere as they pass through a double slit, then get detected on a screen as tiny dots. Use quantum detectors to explore how measurements change the waves and the patterns they produce on the screen.

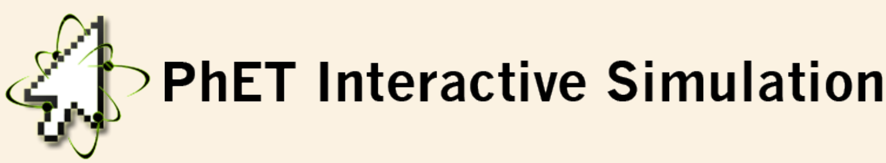

Figure 29.21 Quantum Wave Interference (http://cnx.org/content/m42573/1.3/quantum-wave-interference_en.jar)

29.6 The Wave Nature of Matter

De Broglie Wavelength

In 1923 a French physics graduate student named Prince Louis-Victor de Broglie (1892–1987) made a radical proposal based on the hope that nature is symmetric. If EM radiation has both particle and wave properties, then nature would be symmetric if matter also had both particle and wave properties. If what we once thought of as an unequivocal wave (EM radiation) is also a particle, then what we think of as an unequivocal particle (matter) may also be a wave. De Broglie's suggestion, made as part of his doctoral thesis, was so radical that it was greeted with some skepticism. A copy of his thesis was sent to Einstein, who said it was not only probably correct, but that it might be of fundamental importance. With the support of Einstein and a few other prominent physicists, de Broglie was awarded his doctorate.

De Broglie took both relativity and quantum mechanics into account to develop the proposal that *all particles have a wavelength*, given by

$$\lambda = \frac{h}{p}\text{(matter and photons)}, \tag{29.34}$$

where h is Planck's constant and p is momentum. This is defined to be the **de Broglie wavelength**. (Note that we already have this for photons, from the equation $p = h / \lambda$.) The hallmark of a wave is interference. If matter is a wave, then it must exhibit constructive and destructive interference. Why isn't this ordinarily observed? The answer is that in order to see significant interference effects, a wave must interact with an object about the same size as its wavelength. Since h is very small, λ is also small, especially for macroscopic objects. A 3-kg bowling ball moving at 10 m/s, for example, has

$$\lambda = h / p = (6.63\times10^{-34}\text{ J}\cdot\text{s}) / [(3\text{ kg})(10\text{ m/s})] = 2\times10^{-35}\text{ m}. \tag{29.35}$$

This means that to see its wave characteristics, the bowling ball would have to interact with something about 10^{-35} m in size—far smaller than anything known. When waves interact with objects much larger than their wavelength, they show negligible interference effects and move in straight lines (such as light rays in geometric optics). To get easily observed interference effects from particles of matter, the longest wavelength and hence smallest mass possible would be useful. Therefore, this effect was first observed with electrons.

American physicists Clinton J. Davisson and Lester H. Germer in 1925 and, independently, British physicist G. P. Thomson (son of J. J. Thomson, discoverer of the electron) in 1926 scattered electrons from crystals and found diffraction patterns. These patterns are exactly consistent with interference of electrons having the de Broglie wavelength and are somewhat analogous to light interacting with a diffraction grating. (See **Figure 29.22**.)

Connections: Waves

All microscopic particles, whether massless, like photons, or having mass, like electrons, have wave properties. The relationship between momentum and wavelength is fundamental for all particles.

De Broglie's proposal of a wave nature for all particles initiated a remarkably productive era in which the foundations for quantum mechanics were laid. In 1926, the Austrian physicist Erwin Schrödinger (1887–1961) published four papers in which the wave nature of particles was treated explicitly with wave equations. At the same time, many others began important work. Among them was German physicist Werner Heisenberg (1901–1976) who, among many other contributions to quantum mechanics, formulated a mathematical treatment of the wave nature of matter that used matrices rather than wave equations. We will deal with some specifics in later sections, but it is worth noting that de Broglie's work was a watershed for the development of quantum mechanics. De Broglie was awarded the Nobel Prize in 1929 for his vision, as were Davisson and G. P. Thomson in 1937 for their experimental verification of de Broglie's hypothesis.

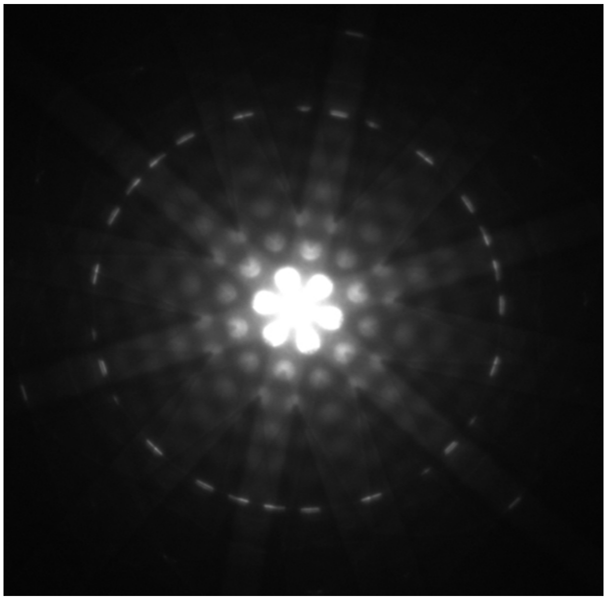

Figure 29.22 This diffraction pattern was obtained for electrons diffracted by crystalline silicon. Bright regions are those of constructive interference, while dark regions are those of destructive interference. (credit: Ndthe, Wikimedia Commons)

Example 29.7 Electron Wavelength versus Velocity and Energy

For an electron having a de Broglie wavelength of 0.167 nm (appropriate for interacting with crystal lattice structures that are about this size): (a) Calculate the electron's velocity, assuming it is nonrelativistic. (b) Calculate the electron's kinetic energy in eV.

Strategy

For part (a), since the de Broglie wavelength is given, the electron's velocity can be obtained from $\lambda = h / p$ by using the nonrelativistic formula for momentum, $p = mv.$ For part (b), once v is obtained (and it has been verified that v is nonrelativistic), the classical kinetic energy is simply $(1 / 2)mv^2$.

Solution for (a)

Substituting the nonrelativistic formula for momentum ($p = mv$) into the de Broglie wavelength gives

$$\lambda = \frac{h}{p} = \frac{h}{mv}. \tag{29.36}$$

Solving for v gives

$$v = \frac{h}{m\lambda}. \tag{29.37}$$

Substituting known values yields

$$v = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(0.167\times10^{-9}\ \text{m})} = 4.36\times10^{6}\ \text{m/s}. \tag{29.38}$$

Solution for (b)

While fast compared with a car, this electron's speed is not highly relativistic, and so we can comfortably use the classical formula to find the electron's kinetic energy and convert it to eV as requested.

$$\begin{aligned} \text{KE} &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(9.11\times10^{-31}\ \text{kg})(4.36\times10^{6}\ \text{m/s})^2 \\ &= (86.4\times10^{-18}\ \text{J})\left(\frac{1\ \text{eV}}{1.602\times10^{-19}\ \text{J}}\right) \\ &= 54.0\ \text{eV} \end{aligned} \tag{29.39}$$

Discussion

This low energy means that these 0.167-nm electrons could be obtained by accelerating them through a 54.0-V electrostatic potential, an easy task. The results also confirm the assumption that the electrons are nonrelativistic, since their velocity is just over 1% of the speed of light and the kinetic energy is about 0.01% of the rest energy of an electron (0.511 MeV). If the electrons had turned out to be relativistic, we would have had to use more involved calculations employing relativistic formulas.

Electron Microscopes

One consequence or use of the wave nature of matter is found in the electron microscope. As we have discussed, there is a limit to the detail observed with any probe having a wavelength. Resolution, or observable detail, is limited to about one wavelength. Since a potential of only 54 V can produce electrons with sub-nanometer wavelengths, it is easy to get electrons with much smaller wavelengths than those of visible light (hundreds of nanometers). Electron microscopes can, thus, be constructed to detect much smaller details than optical microscopes. (See **Figure 29.23**.)

There are basically two types of electron microscopes. The transmission electron microscope (TEM) accelerates electrons that are emitted from a hot filament (the cathode). The beam is broadened and then passes through the sample. A magnetic lens focuses the beam image onto a fluorescent screen, a photographic plate, or (most probably) a CCD (light sensitive camera), from which it is transferred to a computer. The TEM is similar to the optical microscope, but it requires a thin sample examined in a vacuum. However it can resolve details as small as 0.1 nm (10^{-10} m), providing magnifications of 100 million times the size of the original object. The TEM has allowed us to see individual atoms and structure of cell nuclei.

The scanning electron microscope (SEM) provides images by using secondary electrons produced by the primary beam interacting with the surface of the sample (see **Figure 29.23**). The SEM also uses magnetic lenses to focus the beam onto the sample. However, it moves the beam around electrically to "scan" the sample in the *x* and *y* directions. A CCD detector is used to process the data for each electron position, producing images like the one at the beginning of this chapter. The SEM has the advantage of not requiring a thin sample and of providing a 3-D view. However, its resolution is about ten times less than a TEM.

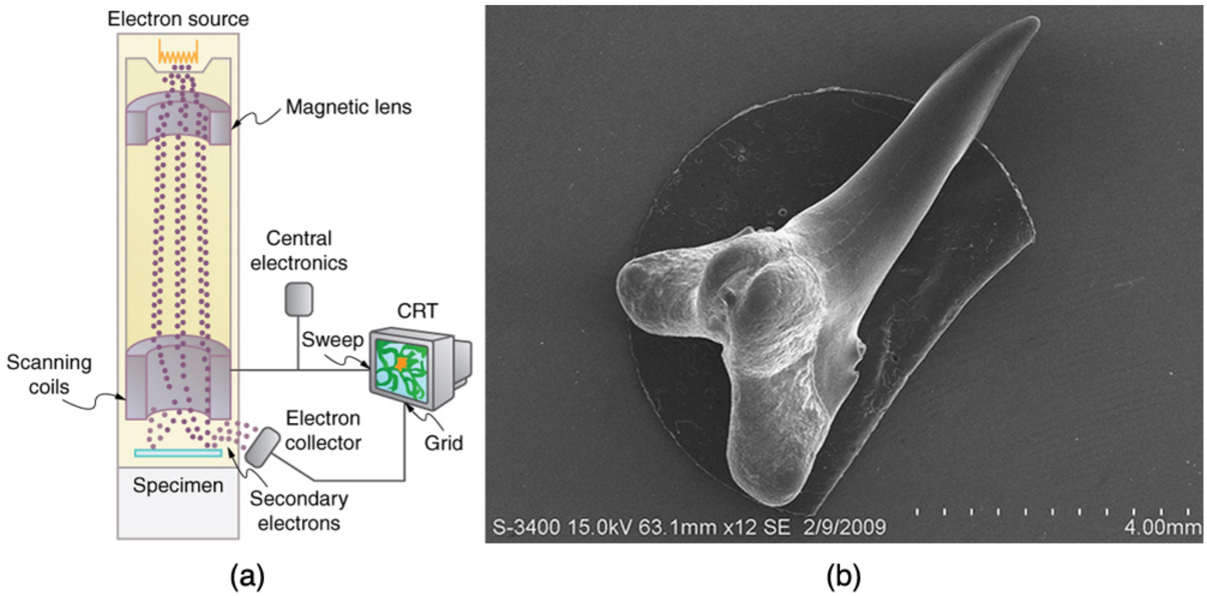

Figure 29.23 Schematic of a scanning electron microscope (SEM) (a) used to observe small details, such as those seen in this image of a tooth of a *Himipristis*, a type of shark (b). (credit: Dallas Krentzel, Flickr)

Electrons were the first particles with mass to be directly confirmed to have the wavelength proposed by de Broglie. Subsequently, protons, helium nuclei, neutrons, and many others have been observed to exhibit interference when they interact with objects having sizes similar to their de Broglie wavelength. The de Broglie wavelength for massless particles was well established in the 1920s for photons, and it has since been observed that all massless particles have a de Broglie wavelength $\lambda = h / p.$ The wave nature of all particles is a universal characteristic of nature. We shall see in following sections that implications of the de Broglie wavelength include the quantization of energy in atoms and molecules, and an alteration of our basic view of nature on the microscopic scale. The next section, for example, shows that there are limits to the precision with which we may make predictions, regardless of how hard we try. There are even limits to the precision with which we may measure an object's location or energy.

Making Connections: A Submicroscopic Diffraction Grating

The wave nature of matter allows it to exhibit all the characteristics of other, more familiar, waves. Diffraction gratings, for example, produce diffraction patterns for light that depend on grating spacing and the wavelength of the light. This effect, as with most wave phenomena, is most pronounced when the wave interacts with objects having a size similar to its wavelength. For gratings, this is the spacing between multiple slits.) When electrons interact with a system having a spacing similar to the electron wavelength, they show the same types of interference patterns as light does for diffraction gratings, as shown at top left in **Figure 29.24**.

Atoms are spaced at regular intervals in a crystal as parallel planes, as shown in the bottom part of **Figure 29.24**. The spacings between these planes act like the openings in a diffraction grating. At certain incident angles, the paths of electrons scattering from successive planes differ by one wavelength and, thus, interfere constructively. At other angles, the path length differences are not an integral wavelength, and there is partial to total destructive interference. This type of scattering from a large crystal with well-defined lattice planes can produce dramatic interference patterns. It is called *Bragg reflection*, for the father-and-son team who first explored and analyzed it in some detail. The expanded view also shows the path-length differences and indicates how these depend on incident angle θ in a manner similar to the diffraction patterns for x rays reflecting from a crystal.

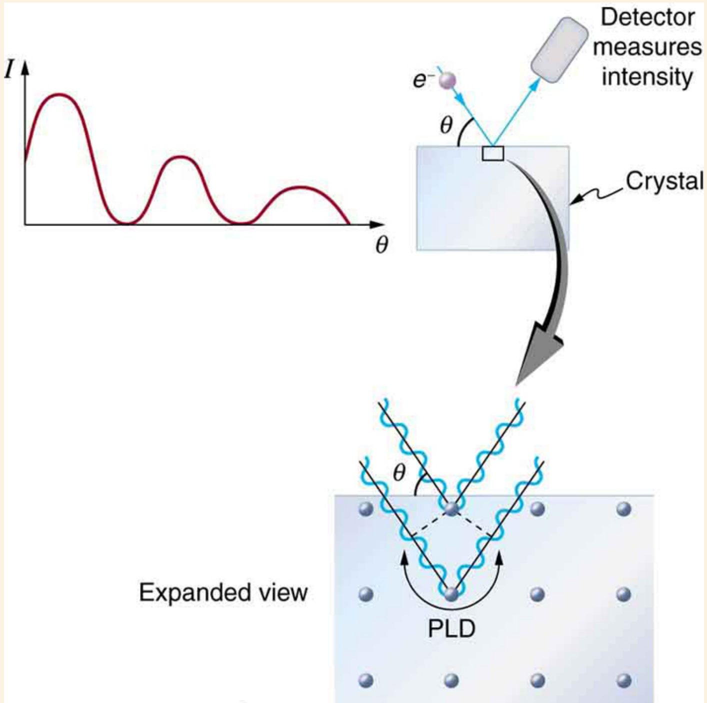

Figure 29.24 The diffraction pattern at top left is produced by scattering electrons from a crystal and is graphed as a function of incident angle relative to the regular array of atoms in a crystal, as shown at bottom. Electrons scattering from the second layer of atoms travel farther than those scattered from the top layer. If the path length difference (PLD) is an integral wavelength, there is constructive interference.

Let us take the spacing between parallel planes of atoms in the crystal to be d. As mentioned, if the path length difference (PLD) for the electrons is a whole number of wavelengths, there will be constructive interference—that is, $\text{PLD} = n\lambda (n = 1, 2, 3, \ldots)$. Because $\text{AB} = \text{BC} = d \sin \theta,$ we have constructive interference when $n\lambda = 2d \sin \theta.$ This relationship is called the *Bragg equation* and applies not only to electrons but also to x rays.

The wavelength of matter is a submicroscopic characteristic that explains a macroscopic phenomenon such as Bragg reflection. Similarly, the wavelength of light is a submicroscopic characteristic that explains the macroscopic phenomenon of diffraction patterns.

29.7 Probability: The Heisenberg Uncertainty Principle

Probability Distribution

Matter and photons are waves, implying they are spread out over some distance. What is the position of a particle, such as an electron? Is it at the center of the wave? The answer lies in how you measure the position of an electron. Experiments show that you will find the electron at some definite location, unlike a wave. But if you set up exactly the same situation and measure it again, you will find the electron in a different location, often far outside any experimental uncertainty in your measurement. Repeated measurements will display a statistical distribution of locations that appears wavelike. (See **Figure 29.25**.)

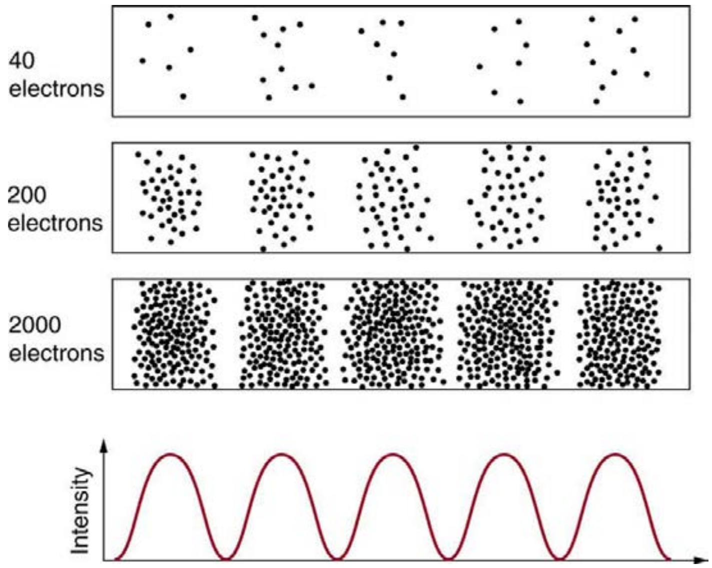

Figure 29.25 The building up of the diffraction pattern of electrons scattered from a crystal surface. Each electron arrives at a definite location, which cannot be precisely predicted. The overall distribution shown at the bottom can be predicted as the diffraction of waves having the de Broglie wavelength of the electrons.

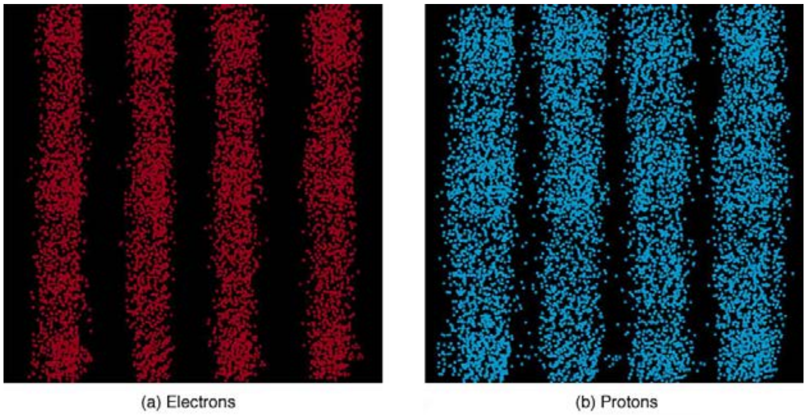

Figure 29.26 Double-slit interference for electrons (a) and protons (b) is identical for equal wavelengths and equal slit separations. Both patterns are probability distributions in the sense that they are built up by individual particles traversing the apparatus, the paths of which are not individually predictable.

After de Broglie proposed the wave nature of matter, many physicists, including Schrödinger and Heisenberg, explored the consequences. The idea quickly emerged that, *because of its wave character, a particle's trajectory and destination cannot be precisely predicted for each particle individually*. However, each particle goes to a definite place (as illustrated in **Figure 29.25**). After compiling enough data, you get a distribution related to the particle's wavelength and diffraction pattern. There is a certain *probability* of finding the particle at a given location, and the overall pattern is called a **probability distribution**. Those who developed quantum mechanics devised equations that predicted the probability distribution in various circumstances.

It is somewhat disquieting to think that you cannot predict exactly where an individual particle will go, or even follow it to its destination. Let us explore what happens if we try to follow a particle. Consider the double-slit patterns obtained for electrons and photons in **Figure 29.26**. First, we note that these patterns are identical, following $d \sin \theta = m\lambda$, the equation for double-slit constructive interference developed in **Photon Energies and the Electromagnetic Spectrum**, where d is the slit separation and λ is the electron or photon wavelength.

Both patterns build up statistically as individual particles fall on the detector. This can be observed for photons or electrons—for now, let us concentrate on electrons. You might imagine that the electrons are interfering with one another as any waves do. To test this, you can lower the intensity until there is never more than one electron between the slits and the screen. The same interference pattern builds up! This implies that a particle's probability distribution spans both slits, and the particles actually interfere with themselves. Does this also mean that the electron goes through both slits? An electron is a basic unit of matter that is not divisible. But it is a fair question, and so we should look to see if the electron traverses one slit or the other, or both. One possibility is to have coils around the slits that detect charges moving through them. What is observed is that an electron always goes through one slit or the other; it does not split to go through both. But there is a catch. If you determine that the electron went through one of the slits, you no longer get a double slit pattern—instead, you get single slit interference. There is no escape by using another method of determining which slit the electron went through. Knowing the particle went through one slit forces a single-slit pattern. If you do not observe which slit the electron goes through, you obtain a double-slit pattern.

Heisenberg Uncertainty

How does knowing which slit the electron passed through change the pattern? The answer is fundamentally important—*measurement affects the system being observed*. Information can be lost, and in some cases it is impossible to measure two physical quantities simultaneously to exact precision. For example, you can measure the position of a moving electron by scattering light or other electrons from it. Those probes have momentum themselves, and by scattering from the electron, they change its momentum *in a manner that loses information*. There is a limit to absolute knowledge, even in principle.

Figure 29.27 Werner Heisenberg was one of the best of those physicists who developed early quantum mechanics. Not only did his work enable a description of nature on the very small scale, it also changed our view of the availability of knowledge. Although he is universally recognized for his brilliance and the importance of his work (he received the Nobel Prize in 1932, for example), Heisenberg remained in Germany during World War II and headed the German effort to build a nuclear bomb, permanently alienating himself from most of the scientific community. (credit: Author Unknown, via Wikimedia Commons)

It was Werner Heisenberg who first stated this limit to knowledge in 1929 as a result of his work on quantum mechanics and the wave characteristics of all particles. (See **Figure 29.27**). Specifically, consider simultaneously measuring the position and momentum of an electron (it could be any particle). There is an **uncertainty in position** Δx that is approximately equal to the wavelength of the particle. That is,

$$\Delta x \approx \lambda. \tag{29.40}$$

As discussed above, a wave is not located at one point in space. If the electron's position is measured repeatedly, a spread in locations will be observed, implying an uncertainty in position Δx. To detect the position of the particle, we must interact with it, such as having it collide with a detector. In the collision, the particle will lose momentum. This change in momentum could be anywhere from close to zero to the total momentum of the particle, $p = h / \lambda$. It is not possible to tell how much momentum will be transferred to a detector, and so there is an **uncertainty in momentum** Δp, too. In fact, the uncertainty in momentum may be as large as the momentum itself, which in equation form means that

$$\Delta p \approx \frac{h}{\lambda}. \tag{29.41}$$

The uncertainty in position can be reduced by using a shorter-wavelength electron, since $\Delta x \approx \lambda$. But shortening the wavelength increases the uncertainty in momentum, since $\Delta p \approx h / \lambda$. Conversely, the uncertainty in momentum can be reduced by using a longer-wavelength electron, but this increases the uncertainty in position. Mathematically, you can express this trade-off by multiplying the uncertainties. The wavelength cancels, leaving

$$\Delta x \Delta p \approx h. \tag{29.42}$$

So if one uncertainty is reduced, the other must increase so that their product is $\approx h$.

With the use of advanced mathematics, Heisenberg showed that the best that can be done in a *simultaneous measurement of position and momentum* is

$$\Delta x \Delta p \geq \frac{h}{4\pi}. \tag{29.43}$$

This is known as the **Heisenberg uncertainty principle**. It is impossible to measure position x and momentum p simultaneously with uncertainties Δx and Δp that multiply to be less than $h/4\pi$. Neither uncertainty can be zero. Neither uncertainty can become small without the other becoming large. A small wavelength allows accurate position measurement, but it increases the momentum of the probe to the point that it further disturbs the momentum of a system being measured. For example, if an electron is scattered from an atom and has a wavelength small enough to detect the position of electrons in the atom, its momentum can knock the electrons from their orbits in a manner that loses information about their original motion. It is therefore impossible to follow an electron in its orbit around an atom. If you measure the electron's position, you will find it in a definite location, but the atom will be disrupted. Repeated measurements on identical atoms will produce interesting probability distributions for electrons around the atom, but they will not produce motion information. The probability distributions are referred to as electron clouds or orbitals. The shapes of these orbitals are often shown in general chemistry texts and are discussed in **The Wave Nature of Matter Causes Quantization**.

Example 29.8 Heisenberg Uncertainty Principle in Position and Momentum for an Atom

(a) If the position of an electron in an atom is measured to an accuracy of 0.0100 nm, what is the electron's uncertainty in velocity? (b) If the electron has this velocity, what is its kinetic energy in eV?

Strategy

The uncertainty in position is the accuracy of the measurement, or $\Delta x = 0.0100\text{ nm}$. Thus the smallest uncertainty in momentum Δp can be calculated using $\Delta x \Delta p \geq h/4\pi$. Once the uncertainty in momentum Δp is found, the uncertainty in velocity can be found from $\Delta p = m\Delta v$.

Solution for (a)

Using the equals sign in the uncertainty principle to express the minimum uncertainty, we have

$$\Delta x \Delta p = \frac{h}{4\pi}. \tag{29.44}$$

Solving for Δp and substituting known values gives

$$\Delta p = \frac{h}{4\pi\Delta x} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{4\pi(1.00\times10^{-11}\text{ m})} = 5.28\times10^{-24}\text{ kg}\cdot\text{m/s}. \tag{29.45}$$

Thus,

$$\Delta p = 5.28\times10^{-24}\text{ kg}\cdot\text{m/s} = m\Delta v. \tag{29.46}$$

Solving for Δv and substituting the mass of an electron gives

$$\Delta v = \frac{\Delta p}{m} = \frac{5.28\times10^{-24}\text{ kg}\cdot\text{m/s}}{9.11\times10^{-31}\text{ kg}} = 5.79\times10^{6}\text{ m/s}. \tag{29.47}$$

Solution for (b)

Although large, this velocity is not highly relativistic, and so the electron's kinetic energy is

$$\begin{aligned} \text{KE}_e &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(9.11\times10^{-31}\text{ kg})(5.79\times10^{6}\text{ m/s})^2 \\ &= (1.53\times10^{-17}\text{ J})\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = 95.5\text{ eV}. \end{aligned} \tag{29.48}$$

Discussion

Since atoms are roughly 0.1 nm in size, knowing the position of an electron to 0.0100 nm localizes it reasonably well inside the atom. This would be like being able to see details one-tenth the size of the atom. But the consequent uncertainty in velocity is large. You certainly could not follow it very well if its velocity is so uncertain. To get a further idea of how large the uncertainty in velocity is, we assumed the velocity of the electron was equal to its uncertainty and found this gave a kinetic energy of 95.5 eV. This is significantly greater than the typical energy difference between levels in atoms (see **Table 29.1**), so that it is impossible to get a meaningful energy for the electron if we know its position even moderately well.

Why don't we notice Heisenberg's uncertainty principle in everyday life? The answer is that Planck's constant is very small. Thus the lower limit in the uncertainty of measuring the position and momentum of large objects is negligible. We can detect sunlight reflected from Jupiter and follow the planet in its orbit around the Sun. The reflected sunlight alters the momentum of Jupiter and creates an uncertainty in its momentum, but this is totally negligible compared with Jupiter's huge momentum. The correspondence principle tells us that the predictions of quantum mechanics become indistinguishable from classical physics for large objects, which is the case here.

Heisenberg Uncertainty for Energy and Time

There is another form of **Heisenberg's uncertainty principle** for *simultaneous measurements of energy and time*. In equation form,

$$\Delta E \Delta t \geq \frac{h}{4\pi}, \tag{29.49}$$

where ΔE is the **uncertainty in energy** and Δt is the **uncertainty in time**. This means that within a time interval Δt, it is not possible to measure energy precisely—there will be an uncertainty ΔE in the measurement. In order to measure energy more precisely (to make ΔE smaller), we must increase Δt. This time interval may be the amount of time we take to make the measurement, or it could be the amount of time a particular state exists, as in the next **Example 29.9**.

Example 29.9 Heisenberg Uncertainty Principle for Energy and Time for an Atom

An atom in an excited state temporarily stores energy. If the lifetime of this excited state is measured to be $1.0\times10^{-10}\ \text{s}$, what is the minimum uncertainty in the energy of the state in eV?

Strategy

The minimum uncertainty in energy ΔE is found by using the equals sign in $\Delta E \Delta t \geq h/4\pi$ and corresponds to a reasonable choice for the uncertainty in time. The largest the uncertainty in time can be is the full lifetime of the excited state, or $\Delta t = 1.0\times10^{-10}\ \text{s}$.

Solution

Solving the uncertainty principle for ΔE and substituting known values gives

$$\Delta E = \frac{h}{4\pi\Delta \text{t}} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{4\pi(1.0\times10^{-10}\ \text{s})} = 5.3\times10^{-25}\ \text{J}. \tag{29.50}$$

Now converting to eV yields

$$\Delta E = (5.3\times10^{-25}\ \text{J})\left(\frac{1\ \text{eV}}{1.6\times10^{-19}\ \text{J}}\right) = 3.3\times10^{-6}\ \text{eV}. \tag{29.51}$$

Discussion

The lifetime of $10^{-10}\ \text{s}$ is typical of excited states in atoms—on human time scales, they quickly emit their stored energy. An uncertainty in energy of only a few millionths of an eV results. This uncertainty is small compared with typical excitation energies in atoms, which are on the order of 1 eV. So here the uncertainty principle limits the accuracy with which we can measure the lifetime and energy of such states, but not very significantly.

The uncertainty principle for energy and time can be of great significance if the lifetime of a system is very short. Then Δt is very small, and ΔE is consequently very large. Some nuclei and exotic particles have extremely short lifetimes (as small as $10^{-25}\ \text{s}$), causing uncertainties in energy as great as many GeV ($10^{9}\ \text{eV}$). Stored energy appears as increased rest mass, and so this means that there is significant uncertainty in the rest mass of short-lived particles. When measured repeatedly, a spread of masses or decay energies are obtained. The spread is ΔE. You might ask whether this uncertainty in energy could be avoided by not measuring the lifetime. The answer is no. Nature knows the lifetime, and so its brevity affects the energy of the particle. This is so well established experimentally that the uncertainty in decay energy is used to calculate the lifetime of short-lived states. Some nuclei and particles are so short-lived that it is difficult to measure their lifetime. But if their decay energy can be measured, its spread is ΔE, and this is used in the uncertainty principle ($\Delta E \Delta t \geq h/4\pi$) to calculate the lifetime Δt.

There is another consequence of the uncertainty principle for energy and time. If energy is uncertain by ΔE, then conservation of energy can be violated by ΔE for a time Δt. Neither the physicist nor nature can tell that conservation of energy has been violated, if the violation is temporary and smaller than the uncertainty in energy. While this sounds innocuous enough, we shall see in later chapters that it allows the temporary creation of matter from nothing and has implications for how nature transmits forces over very small distances.

Finally, note that in the discussion of particles and waves, we have stated that individual measurements produce precise or particle-like results. A definite position is determined each time we observe an electron, for example. But repeated measurements produce a spread in values consistent with wave characteristics. The great theoretical physicist Richard Feynman (1918–1988) commented, "What there are, are particles." When you observe enough of them, they distribute themselves as you would expect for a wave phenomenon. However, what there are as they travel we cannot tell because, when we do try to measure, we affect the traveling.

29.8 The Particle-Wave Duality Reviewed

Particle-wave duality—the fact that all particles have wave properties—is one of the cornerstones of quantum mechanics. We first came across it in the treatment of photons, those particles of EM radiation that exhibit both particle and wave properties, but not at the same time. Later it was noted that particles of matter have wave properties as well. The dual properties of particles and waves are found for all particles, whether massless like photons, or having a mass like electrons. (See **Figure 29.28**.)

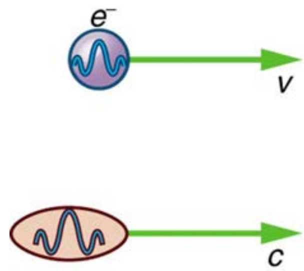

Figure 29.28 On a quantum-mechanical scale (i.e., very small), particles with and without mass have wave properties. For example, both electrons and photons have wavelengths but also behave as particles.

There are many submicroscopic particles in nature. Most have mass and are expected to act as particles, or the smallest units of matter. All these masses have wave properties, with wavelengths given by the de Broglie relationship $\lambda = h / p$. So, too, do combinations of these particles, such as nuclei, atoms, and molecules. As a combination of masses becomes large, particularly if it is large enough to be called macroscopic, its wave nature becomes difficult to observe. This is consistent with our common experience with matter.

Some particles in nature are massless. We have only treated the photon so far, but all massless entities travel at the speed of light, have a wavelength, and exhibit particle and wave behaviors. They have momentum given by a rearrangement of the de Broglie relationship, $p = h / \lambda$. In large combinations of these massless particles (such large combinations are common only for photons or EM waves), there is mostly wave behavior upon detection, and the particle nature becomes difficult to observe. This is also consistent with experience. (See **Figure 29.29**.)

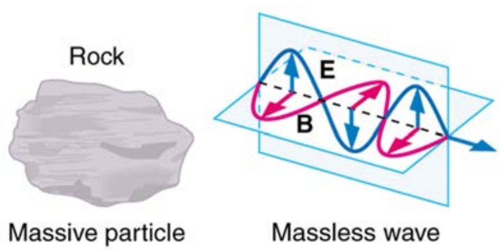

Figure 29.29 On a classical scale (macroscopic), particles with mass behave as particles and not as waves. Particles without mass act as waves and not as particles.

The particle-wave duality is a universal attribute. It is another connection between matter and energy. Not only has modern physics been able to describe nature for high speeds and small sizes, it has also discovered new connections and symmetries. There is greater unity and symmetry in nature than was known in the classical era—but they were dreamt of. A beautiful poem written by the English poet William Blake some two centuries ago contains the following four lines:

To see the World in a Grain of Sand

And a Heaven in a Wild Flower

Hold Infinity in the palm of your hand

And Eternity in an hour

Integrated Concepts

The problem set for this section involves concepts from this chapter and several others. Physics is most interesting when applied to general situations involving more than a narrow set of physical principles. For example, photons have momentum, hence the relevance of **Linear Momentum and Collisions**. The following topics are involved in some or all of the problems in this section:

- **Dynamics: Newton’s Laws of Motion**
- **Work, Energy, and Energy Resources**
- **Linear Momentum and Collisions**
- **Heat and Heat Transfer Methods**
- **Electric Potential and Electric Field**
- **Electric Current, Resistance, and Ohm’s Law**

- **Wave Optics**
- **Special Relativity**

Problem-Solving Strategy

1. Identify which physical principles are involved.
2. Solve the problem using strategies outlined in the text.

Example 29.10 illustrates how these strategies are applied to an integrated-concept problem.

Example 29.10 Recoil of a Dust Particle after Absorbing a Photon

The following topics are involved in this integrated concepts worked example:

Table 29.2 Topics

Photons (quantum mechanics)
Linear Momentum

A 550-nm photon (visible light) is absorbed by a $1.00\text{-}\mu g$ particle of dust in outer space. (a) Find the momentum of such a photon. (b) What is the recoil velocity of the particle of dust, assuming it is initially at rest?

Strategy Step 1

To solve an *integrated-concept problem*, such as those following this example, we must first identify the physical principles involved and identify the chapters in which they are found. Part (a) of this example asks for the *momentum of a photon*, a topic of the present chapter. Part (b) considers *recoil following a collision*, a topic of **Linear Momentum and Collisions**.

Strategy Step 2

The following solutions to each part of the example illustrate how specific problem-solving strategies are applied. These involve identifying knowns and unknowns, checking to see if the answer is reasonable, and so on.

Solution for (a)

The momentum of a photon is related to its wavelength by the equation:

$$p = \frac{h}{\lambda}. \tag{29.52}$$

Entering the known value for Planck's constant h and given the wavelength λ, we obtain

$$\begin{aligned} p &= \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{550\times10^{-9}\ \text{m}} \\ &= 1.21\times10^{-27}\ \text{kg}\cdot\text{m/s}. \end{aligned} \tag{29.53}$$

Discussion for (a)

This momentum is small, as expected from discussions in the text and the fact that photons of visible light carry small amounts of energy and momentum compared with those carried by macroscopic objects.

Solution for (b)

Conservation of momentum in the absorption of this photon by a grain of dust can be analyzed using the equation:

$$p_1 + p_2 = p'_1 + p'_2 (F_{\text{net}} = 0). \tag{29.54}$$

The net external force is zero, since the dust is in outer space. Let 1 represent the photon and 2 the dust particle. Before the collision, the dust is at rest (relative to some observer); after the collision, there is no photon (it is absorbed). So conservation of momentum can be written

$$p_1 = p'_2 = mv, \tag{29.55}$$

where p_1 is the photon momentum before the collision and p'_2 is the dust momentum after the collision. The mass and recoil velocity of the dust are m and v, respectively. Solving this for v, the requested quantity, yields

$$v = \frac{p}{m}, \tag{29.56}$$

where p is the photon momentum found in part (a). Entering known values (noting that a microgram is $10^{-9}\ \text{kg}$) gives

$$v = \frac{1.21\times10^{-27}\ \text{kg}\cdot\text{m/s}}{1.00\times10^{-9}\ \text{kg}} = 1.21\times10^{-18}\ \text{m/s}. \tag{29.57}$$

Discussion

The recoil velocity of the particle of dust is extremely small. As we have noted, however, there are immense numbers of photons in sunlight and other macroscopic sources. In time, collisions and absorption of many photons could cause a significant recoil of the dust, as observed in comet tails.

Glossary

atomic spectra: the electromagnetic emission from atoms and molecules

binding energy: also called the *work function*; the amount of energy necessary to eject an electron from a material

blackbody: an ideal radiator, which can radiate equally well at all wavelengths

blackbody radiation: the electromagnetic radiation from a blackbody

bremsstrahlung: German for *braking radiation*; produced when electrons are decelerated

characteristic x rays: x rays whose energy depends on the material they were produced in

Compton effect: the phenomenon whereby x rays scattered from materials have decreased energy

correspondence principle: in the classical limit (large, slow-moving objects), quantum mechanics becomes the same as classical physics

de Broglie wavelength: the wavelength possessed by a particle of matter, calculated by $\lambda = h/p$

gamma ray: also γ -ray; highest-energy photon in the EM spectrum

Heisenberg's uncertainty principle: a fundamental limit to the precision with which pairs of quantities (momentum and position, and energy and time) can be measured

infrared radiation: photons with energies slightly less than red light

ionizing radiation: radiation that ionizes materials that absorb it

microwaves: photons with wavelengths on the order of a micron (μm)

particle-wave duality: the property of behaving like either a particle or a wave; the term for the phenomenon that all particles have wave characteristics

photoelectric effect: the phenomenon whereby some materials eject electrons when light is shined on them

photon: a quantum, or particle, of electromagnetic radiation

photon energy: the amount of energy a photon has; $E = hf$

photon momentum: the amount of momentum a photon has, calculated by $p = \frac{h}{\lambda} = \frac{E}{c}$

Planck's constant: $h = 6.626\times10^{-34}\ \text{J}\cdot\text{s}$

probability distribution: the overall spatial distribution of probabilities to find a particle at a given location

quantized: the fact that certain physical entities exist only with particular discrete values and not every conceivable value

quantum mechanics: the branch of physics that deals with small objects and with the quantization of various entities, especially energy

ultraviolet radiation: UV; ionizing photons slightly more energetic than violet light

uncertainty in energy: lack of precision or lack of knowledge of precise results in measurements of energy

uncertainty in momentum: lack of precision or lack of knowledge of precise results in measurements of momentum

uncertainty in position: lack of precision or lack of knowledge of precise results in measurements of position

uncertainty in time: lack of precision or lack of knowledge of precise results in measurements of time

visible light: the range of photon energies the human eye can detect

x ray: EM photon between γ -ray and UV in energy

Section Summary

29.1 Quantization of Energy

- The first indication that energy is sometimes quantized came from blackbody radiation, which is the emission of EM radiation by an object with an emissivity of 1.
- Planck recognized that the energy levels of the emitting atoms and molecules were quantized, with only the allowed values of $E = \left(n + \frac{1}{2}\right)hf,$ where n is any non-negative integer (0, 1, 2, 3, …).
- h is Planck's constant, whose value is $h = 6.626\times10^{-34}\ \text{J}\cdot\text{s}.$
- Thus, the oscillatory absorption and emission energies of atoms and molecules in a blackbody could increase or decrease only in steps of size $\Delta E = hf$ where f is the frequency of the oscillatory nature of the absorption and emission of EM radiation.
- Another indication of energy levels being quantized in atoms and molecules comes from the lines in atomic spectra, which are the EM emissions of individual atoms and molecules.

29.2 The Photoelectric Effect

- The photoelectric effect is the process in which EM radiation ejects electrons from a material.
- Einstein proposed photons to be quanta of EM radiation having energy $E = hf$, where f is the frequency of the radiation.
- All EM radiation is composed of photons. As Einstein explained, all characteristics of the photoelectric effect are due to the interaction of individual photons with individual electrons.
- The maximum kinetic energy KE_e of ejected electrons (photoelectrons) is given by $\text{KE}_e = hf - \text{BE}$, where hf is the photon energy and BE is the binding energy (or work function) of the electron to the particular material.

29.3 Photon Energies and the Electromagnetic Spectrum

- Photon energy is responsible for many characteristics of EM radiation, being particularly noticeable at high frequencies.
- Photons have both wave and particle characteristics.

29.4 Photon Momentum

- Photons have momentum, given by $p = \frac{h}{\lambda}$, where λ is the photon wavelength.
- Photon energy and momentum are related by $p = \frac{E}{c}$, where $E = hf = hc/\lambda$ for a photon.

29.5 The Particle-Wave Duality

- EM radiation can behave like either a particle or a wave.
- This is termed particle-wave duality.

29.6 The Wave Nature of Matter

- Particles of matter also have a wavelength, called the de Broglie wavelength, given by $\lambda = \frac{h}{p}$, where p is momentum.
- Matter is found to have the same *interference characteristics* as any other wave.

29.7 Probability: The Heisenberg Uncertainty Principle

- Matter is found to have the same interference characteristics as any other wave.
- There is now a probability distribution for the location of a particle rather than a definite position.
- Another consequence of the wave character of all particles is the Heisenberg uncertainty principle, which limits the precision with which certain physical quantities can be known simultaneously. For position and momentum, the uncertainty principle is $\Delta x \Delta p \geq \frac{h}{4\pi}$, where Δx is the uncertainty in position and Δp is the uncertainty in momentum.

- For energy and time, the uncertainty principle is $\Delta E \Delta t \geq \frac{h}{4\pi}$ where ΔE is the uncertainty in energy and Δt is the uncertainty in time.
- These small limits are fundamentally important on the quantum-mechanical scale.

29.8 The Particle-Wave Duality Reviewed

- The particle-wave duality refers to the fact that all particles—those with mass and those without mass—have wave characteristics.
- This is a further connection between mass and energy.

Conceptual Questions

29.1 Quantization of Energy

1. Give an example of a physical entity that is quantized. State specifically what the entity is and what the limits are on its values.

2. Give an example of a physical entity that is not quantized, in that it is continuous and may have a continuous range of values.

3. What aspect of the blackbody spectrum forced Planck to propose quantization of energy levels in its atoms and molecules?

4. If Planck's constant were large, say 10^{34} times greater than it is, we would observe macroscopic entities to be quantized. Describe the motions of a child's swing under such circumstances.

5. Why don't we notice quantization in everyday events?

29.2 The Photoelectric Effect

6. Is visible light the only type of EM radiation that can cause the photoelectric effect?

7. Which aspects of the photoelectric effect cannot be explained without photons? Which can be explained without photons? Are the latter inconsistent with the existence of photons?

8. Is the photoelectric effect a direct consequence of the wave character of EM radiation or of the particle character of EM radiation? Explain briefly.

9. Insulators (nonmetals) have a higher BE than metals, and it is more difficult for photons to eject electrons from insulators. Discuss how this relates to the free charges in metals that make them good conductors.

10. If you pick up and shake a piece of metal that has electrons in it free to move as a current, no electrons fall out. Yet if you heat the metal, electrons can be boiled off. Explain both of these facts as they relate to the amount and distribution of energy involved with shaking the object as compared with heating it.

29.3 Photon Energies and the Electromagnetic Spectrum

11. Why are UV, x rays, and γ rays called ionizing radiation?

12. How can treating food with ionizing radiation help keep it from spoiling? UV is not very penetrating. What else could be used?

13. Some television tubes are CRTs. They use an approximately 30-kV accelerating potential to send electrons to the screen, where the electrons stimulate phosphors to emit the light that forms the pictures we watch. Would you expect x rays also to be created?

14. Tanning salons use "safe" UV with a longer wavelength than some of the UV in sunlight. This "safe" UV has enough photon energy to trigger the tanning mechanism. Is it likely to be able to cause cell damage and induce cancer with prolonged exposure?

15. Your pupils dilate when visible light intensity is reduced. Does wearing sunglasses that lack UV blockers increase or decrease the UV hazard to your eyes? Explain.

16. One could feel heat transfer in the form of infrared radiation from a large nuclear bomb detonated in the atmosphere 75 km from you. However, none of the profusely emitted x rays or γ rays reaches you. Explain.

17. Can a single microwave photon cause cell damage? Explain.

18. In an x-ray tube, the maximum photon energy is given by $hf = qV$. Would it be technically more correct to say $hf = qV + \text{BE},$ where BE is the binding energy of electrons in the target anode? Why isn't the energy stated the latter way?

29.4 Photon Momentum

19. Which formula may be used for the momentum of all particles, with or without mass?

20. Is there any measurable difference between the momentum of a photon and the momentum of matter?

21. Why don't we feel the momentum of sunlight when we are on the beach?

29.6 The Wave Nature of Matter

22. How does the interference of water waves differ from the interference of electrons? How are they analogous?

23. Describe one type of evidence for the wave nature of matter.

24. Describe one type of evidence for the particle nature of EM radiation.

29.7 Probability: The Heisenberg Uncertainty Principle

25. What is the Heisenberg uncertainty principle? Does it place limits on what can be known?

29.8 The Particle-Wave Duality Reviewed

26. In what ways are matter and energy related that were not known before the development of relativity and quantum mechanics?

In what ways are matter and energy related that were not known before the development of relativity and quantum mechanics?

Problems & Exercises

29.1 Quantization of Energy

1. A LiBr molecule oscillates with a frequency of $1.7\times10^{13}\text{ Hz}$. (a) What is the difference in energy in eV between allowed oscillator states? (b) What is the approximate value of n for a state having an energy of 1.0 eV?

2. The difference in energy between allowed oscillator states in HBr molecules is 0.330 eV. What is the oscillation frequency of this molecule?

3. A physicist is watching a 15-kg orangutan at a zoo swing lazily in a tire at the end of a rope. He (the physicist) notices that each oscillation takes 3.00 s and hypothesizes that the energy is quantized. (a) What is the difference in energy in joules between allowed oscillator states? (b) What is the value of n for a state where the energy is 5.00 J? (c) Can the quantization be observed?

29.2 The Photoelectric Effect

4. What is the longest-wavelength EM radiation that can eject a photoelectron from silver, given that the binding energy is 4.73 eV? Is this in the visible range?

5. Find the longest-wavelength photon that can eject an electron from potassium, given that the binding energy is 2.24 eV. Is this visible EM radiation?

6. What is the binding energy in eV of electrons in magnesium, if the longest-wavelength photon that can eject electrons is 337 nm?

7. Calculate the binding energy in eV of electrons in aluminum, if the longest-wavelength photon that can eject them is 304 nm.

8. What is the maximum kinetic energy in eV of electrons ejected from sodium metal by 450-nm EM radiation, given that the binding energy is 2.28 eV?

9. UV radiation having a wavelength of 120 nm falls on gold metal, to which electrons are bound by 4.82 eV. What is the maximum kinetic energy of the ejected photoelectrons?

10. Violet light of wavelength 400 nm ejects electrons with a maximum kinetic energy of 0.860 eV from sodium metal. What is the binding energy of electrons to sodium metal?

11. UV radiation having a 300-nm wavelength falls on uranium metal, ejecting 0.500-eV electrons. What is the binding energy of electrons to uranium metal?

12. What is the wavelength of EM radiation that ejects 2.00-eV electrons from calcium metal, given that the binding energy is 2.71 eV? What type of EM radiation is this?

13. Find the wavelength of photons that eject 0.100-eV electrons from potassium, given that the binding energy is 2.24 eV. Are these photons visible?

14. What is the maximum velocity of electrons ejected from a material by 80-nm photons, if they are bound to the material by 4.73 eV?

15. Photoelectrons from a material with a binding energy of 2.71 eV are ejected by 420-nm photons. Once ejected, how long does it take these electrons to travel 2.50 cm to a detection device?

16. A laser with a power output of 2.00 mW at a wavelength of 400 nm is projected onto calcium metal. (a) How many electrons per second are ejected? (b) What power is carried away by the electrons, given that the binding energy is 2.71 eV?

17. (a) Calculate the number of photoelectrons per second ejected from a 1.00-mm ^{2} area of sodium metal by 500-nm EM radiation having an intensity of 1.30 kW/m^2 (the intensity of sunlight above the Earth's atmosphere). (b) Given that the binding energy is 2.28 eV, what power is carried away by the electrons? (c) The electrons carry away less power than brought in by the photons. Where does the other power go? How can it be recovered?

18. Unreasonable Results

Red light having a wavelength of 700 nm is projected onto magnesium metal to which electrons are bound by 3.68 eV. (a) Use $\text{KE}_e = hf - \text{BE}$ to calculate the kinetic energy of the ejected electrons. (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

19. Unreasonable Results

(a) What is the binding energy of electrons to a material from which 4.00-eV electrons are ejected by 400-nm EM radiation? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

29.3 Photon Energies and the Electromagnetic Spectrum

20. What is the energy in joules and eV of a photon in a radio wave from an AM station that has a 1530-kHz broadcast frequency?

21. (a) Find the energy in joules and eV of photons in radio waves from an FM station that has a 90.0-MHz broadcast frequency. (b) What does this imply about the number of photons per second that the radio station must broadcast?

22. Calculate the frequency in hertz of a 1.00-MeV γ-ray photon.

23. (a) What is the wavelength of a 1.00-eV photon? (b) Find its frequency in hertz. (c) Identify the type of EM radiation.

24. Do the unit conversions necessary to show that $hc = 1240\text{ eV}\cdot\text{nm},$ as stated in the text.

25. Confirm the statement in the text that the range of photon energies for visible light is 1.63 to 3.26 eV, given that the range of visible wavelengths is 380 to 760 nm.

26. (a) Calculate the energy in eV of an IR photon of frequency $2.00\times10^{13}\text{ Hz}.$ (b) How many of these photons would need to be absorbed simultaneously by a tightly bound molecule to break it apart? (c) What is the energy in eV of a γ ray of frequency $3.00\times10^{20}\text{ Hz}?$ (d) How many tightly bound molecules could a single such γ ray break apart?

27. Prove that, to three-digit accuracy, $h = 4.14\times10^{-15}\text{ eV}\cdot\text{s},$ as stated in the text.

28. (a) What is the maximum energy in eV of photons produced in a CRT using a 25.0-kV accelerating potential, such as a color TV? (b) What is their frequency?

29. What is the accelerating voltage of an x-ray tube that produces x rays with a shortest wavelength of 0.0103 nm?

30. (a) What is the ratio of power outputs by two microwave ovens having frequencies of 950 and 2560 MHz, if they emit the same number of photons per second? (b) What is the ratio of photons per second if they have the same power output?

31. How many photons per second are emitted by the antenna of a microwave oven, if its power output is 1.00 kW at a frequency of 2560 MHz?

32. Some satellites use nuclear power. (a) If such a satellite emits a 1.00-W flux of γ rays having an average energy of 0.500 MeV, how many are emitted per second? (b) These γ rays affect other satellites. How far away must another satellite be to only receive one γ ray per second per square meter?

33. (a) If the power output of a 650-kHz radio station is 50.0 kW, how many photons per second are produced? (b) If the radio waves are broadcast uniformly in all directions, find the number of photons per second per square meter at a distance of 100 km. Assume no reflection from the ground or absorption by the air.

34. How many x-ray photons per second are created by an x-ray tube that produces a flux of x rays having a power of 1.00 W? Assume the average energy per photon is 75.0 keV.

35. (a) How far away must you be from a 650-kHz radio station with power 50.0 kW for there to be only one photon per second per square meter? Assume no reflections or absorption, as if you were in deep outer space. (b) Discuss the implications for detecting intelligent life in other solar systems by detecting their radio broadcasts.

36. Assuming that 10.0% of a 100-W light bulb's energy output is in the visible range (typical for incandescent bulbs) with an average wavelength of 580 nm, and that the photons spread out uniformly and are not absorbed by the atmosphere, how far away would you be if 500 photons per second enter the 3.00-mm diameter pupil of your eye? (This number easily stimulates the retina.)

37. Construct Your Own Problem

Consider a laser pen. Construct a problem in which you calculate the number of photons per second emitted by the pen. Among the things to be considered are the laser pen's wavelength and power output. Your instructor may also wish for you to determine the minimum diffraction spreading in the beam and the number of photons per square centimeter the pen can project at some large distance. In this latter case, you will also need to consider the output size of the laser beam, the distance to the object being illuminated, and any absorption or scattering along the way.

29.4 Photon Momentum

38. (a) Find the momentum of a 4.00-cm-wavelength microwave photon. (b) Discuss why you expect the answer to (a) to be very small.

39. (a) What is the momentum of a 0.0100-nm-wavelength photon that could detect details of an atom? (b) What is its energy in MeV?

40. (a) What is the wavelength of a photon that has a momentum of $5.00\times10^{-29}\ \text{kg}\cdot\text{m/s}$? (b) Find its energy in eV.

41. (a) A γ-ray photon has a momentum of $8.00\times10^{-21}\ \text{kg}\cdot\text{m/s}$. What is its wavelength? (b) Calculate its energy in MeV.

42. (a) Calculate the momentum of a photon having a wavelength of $2.50\ \mu\text{m}$. (b) Find the velocity of an electron having the same momentum. (c) What is the kinetic energy of the electron, and how does it compare with that of the photon?

43. Repeat the previous problem for a 10.0-nm-wavelength photon.

44. (a) Calculate the wavelength of a photon that has the same momentum as a proton moving at 1.00% of the speed of light. (b) What is the energy of the photon in MeV? (c) What is the kinetic energy of the proton in MeV?

45. (a) Find the momentum of a 100-keV x-ray photon. (b) Find the equivalent velocity of a neutron with the same momentum. (c) What is the neutron's kinetic energy in keV?

46. Take the ratio of relativistic rest energy, $E = \gamma mc^2$, to relativistic momentum, $p = \gamma mu$, and show that in the limit that mass approaches zero, you find $E/p = c$.

47. Construct Your Own Problem

Consider a space sail such as mentioned in **Example 29.5**. Construct a problem in which you calculate the light pressure on the sail in N/m^2 produced by reflecting sunlight. Also calculate the force that could be produced and how much effect that would have on a spacecraft. Among the things to be considered are the intensity of sunlight, its average wavelength, the number of photons per square meter this implies, the area of the space sail, and the mass of the system being accelerated.

48. Unreasonable Results

A car feels a small force due to the light it sends out from its headlights, equal to the momentum of the light divided by the time in which it is emitted. (a) Calculate the power of each headlight, if they exert a total force of $2.00\times10^{-2}\ \text{N}$ backward on the car. (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

29.6 The Wave Nature of Matter

49. At what velocity will an electron have a wavelength of 1.00 m?

50. What is the wavelength of an electron moving at 3.00% of the speed of light?

51. At what velocity does a proton have a 6.00-fm wavelength (about the size of a nucleus)? Assume the proton is nonrelativistic. (1 femtometer = $10^{-15}\ \text{m.}$)

52. What is the velocity of a 0.400-kg billiard ball if its wavelength is 7.50 cm (large enough for it to interfere with other billiard balls)?

53. Find the wavelength of a proton moving at 1.00% of the speed of light.

54. Experiments are performed with ultracold neutrons having velocities as small as 1.00 m/s. (a) What is the wavelength of such a neutron? (b) What is its kinetic energy in eV?

55. (a) Find the velocity of a neutron that has a 6.00-fm wavelength (about the size of a nucleus). Assume the neutron is nonrelativistic. (b) What is the neutron's kinetic energy in MeV?

56. What is the wavelength of an electron accelerated through a 30.0-kV potential, as in a TV tube?

57. What is the kinetic energy of an electron in a TEM having a 0.0100-nm wavelength?

58. (a) Calculate the velocity of an electron that has a wavelength of $1.00\ \mu\text{m}$. (b) Through what voltage must the electron be accelerated to have this velocity?

59. The velocity of a proton emerging from a Van de Graaff accelerator is 25.0% of the speed of light. (a) What is the proton's wavelength? (b) What is its kinetic energy, assuming it is nonrelativistic? (c) What was the equivalent voltage through which it was accelerated?

60. The kinetic energy of an electron accelerated in an x-ray tube is 100 keV. Assuming it is nonrelativistic, what is its wavelength?

61. Unreasonable Results

(a) Assuming it is nonrelativistic, calculate the velocity of an electron with a 0.100-fm wavelength (small enough to detect details of a nucleus). (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

29.7 Probability: The Heisenberg Uncertainty Principle

62. (a) If the position of an electron in a membrane is measured to an accuracy of $1.00\ \mu\text{m}$, what is the electron's minimum uncertainty in velocity? (b) If the electron has this velocity, what is its kinetic energy in eV? (c) What are the implications of this energy, comparing it to typical molecular binding energies?

63. (a) If the position of a chlorine ion in a membrane is measured to an accuracy of $1.00\ \mu\text{m}$, what is its minimum uncertainty in velocity, given its mass is $5.86\times10^{-26}\ \text{kg}$? (b) If the ion has this velocity, what is its kinetic energy in eV, and how does this compare with typical molecular binding energies?

64. Suppose the velocity of an electron in an atom is known to an accuracy of $2.0\times10^{3}\ \text{m/s}$ (reasonably accurate compared with orbital velocities). What is the electron's minimum uncertainty in position, and how does this compare with the approximate 0.1-nm size of the atom?

65. The velocity of a proton in an accelerator is known to an accuracy of 0.250% of the speed of light. (This could be small compared with its velocity.) What is the smallest possible uncertainty in its position?

66. A relatively long-lived excited state of an atom has a lifetime of 3.00 ms. What is the minimum uncertainty in its energy?

67. (a) The lifetime of a highly unstable nucleus is $10^{-20}\ \text{s}$. What is the smallest uncertainty in its decay energy? (b) Compare this with the rest energy of an electron.

68. The decay energy of a short-lived particle has an uncertainty of 1.0 MeV due to its short lifetime. What is the smallest lifetime it can have?

69. The decay energy of a short-lived nuclear excited state has an uncertainty of 2.0 eV due to its short lifetime. What is the smallest lifetime it can have?

70. What is the approximate uncertainty in the mass of a muon, as determined from its decay lifetime?

71. Derive the approximate form of Heisenberg's uncertainty principle for energy and time, $\Delta E \Delta t \approx h$, using the following arguments: Since the position of a particle is uncertain by $\Delta x \approx \lambda$, where λ is the wavelength of the photon used to examine it, there is an uncertainty in the time the photon takes to traverse Δx. Furthermore, the photon has an energy related to its wavelength, and it can transfer some or all of this energy to the object being examined. Thus the uncertainty in the energy of the object is also related to λ. Find Δt and ΔE; then multiply them to give the approximate uncertainty principle.

29.8 The Particle-Wave Duality Reviewed

72. Integrated Concepts

The 54.0-eV electron in **Example 29.7** has a 0.167-nm wavelength. If such electrons are passed through a double slit and have their first maximum at an angle of 25.0°, what is the slit separation d?

73. Integrated Concepts

An electron microscope produces electrons with a 2.00-pm wavelength. If these are passed through a 1.00-nm single slit, at what angle will the first diffraction minimum be found?

74. Integrated Concepts

A certain heat lamp emits 200 W of mostly IR radiation averaging 1500 nm in wavelength. (a) What is the average photon energy in joules? (b) How many of these photons are required to increase the temperature of a person's shoulder by 2.0°C, assuming the affected mass is 4.0 kg with a specific heat of $0.83\ \text{kcal/kg}\cdot{}^\circ\text{C}$. Also assume no other significant heat transfer. (c) How long does this take?

75. Integrated Concepts

On its high power setting, a microwave oven produces 900 W of 2560 MHz microwaves. (a) How many photons per second is this? (b) How many photons are required to increase the temperature of a 0.500-kg mass of pasta by 45.0°C, assuming a specific heat of $0.900\ \text{kcal/kg}\cdot{}^\circ\text{C}$? Neglect all other heat transfer. (c) How long must the microwave operator wait for their pasta to be ready?

76. Integrated Concepts

(a) Calculate the amount of microwave energy in joules needed to raise the temperature of 1.00 kg of soup from 20.0°C to 100°C. (b) What is the total momentum of all the microwave photons it takes to do this? (c) Calculate the velocity of a 1.00-kg mass with the same momentum. (d) What is the kinetic energy of this mass?

77. Integrated Concepts

(a) What is γ for an electron emerging from the Stanford Linear Accelerator with a total energy of 50.0 GeV? (b) Find its momentum. (c) What is the electron's wavelength?

78. Integrated Concepts

(a) What is γ for a proton having an energy of 1.00 TeV, produced by the Fermilab accelerator? (b) Find its momentum. (c) What is the proton's wavelength?

79. Integrated Concepts

An electron microscope passes 1.00-pm-wavelength electrons through a circular aperture $2.00\ \mu m$ in diameter. What is the angle between two just-resolvable point sources for this microscope?

80. Integrated Concepts

(a) Calculate the velocity of electrons that form the same pattern as 450-nm light when passed through a double slit. (b) Calculate the kinetic energy of each and compare them. (c) Would either be easier to generate than the other? Explain.

81. Integrated Concepts

(a) What is the separation between double slits that produces a second-order minimum at 45.0° for 650-nm light? (b) What slit separation is needed to produce the same pattern for 1.00-keV protons.

82. Integrated Concepts

A laser with a power output of 2.00 mW at a wavelength of 400 nm is projected onto calcium metal. (a) How many electrons per second are ejected? (b) What power is carried away by the electrons, given that the binding energy is 2.71 eV? (c) Calculate the current of ejected electrons. (d) If the photoelectric material is electrically insulated and acts like a 2.00-pF capacitor, how long will current flow before the capacitor voltage stops it?

83. Integrated Concepts

One problem with x rays is that they are not sensed. Calculate the temperature increase of a researcher exposed in a few seconds to a nearly fatal accidental dose of x rays under the following conditions. The energy of the x-ray photons is 200 keV, and 4.00×10^{13} of them are absorbed per kilogram of tissue, the specific heat of which is $0.830\ \text{kcal/kg} \cdot ^\circ\text{C}$. (Note that medical diagnostic x-ray machines *cannot* produce an intensity this great.)

84. Integrated Concepts

A 1.00-fm photon has a wavelength short enough to detect some information about nuclei. (a) What is the photon momentum? (b) What is its energy in joules and MeV? (c) What is the (relativistic) velocity of an electron with the same momentum? (d) Calculate the electron's kinetic energy.

85. Integrated Concepts

The momentum of light is exactly reversed when reflected straight back from a mirror, assuming negligible recoil of the mirror. Thus the change in momentum is twice the photon momentum. Suppose light of intensity $1.00\ \text{kW/m}^2$ reflects from a mirror of area $2.00\ \text{m}^2$. (a) Calculate the energy reflected in 1.00 s. (b) What is the momentum imparted to the mirror? (c) Using the most general form of Newton's second law, what is the force on the mirror? (d) Does the assumption of no mirror recoil seem reasonable?

86. Integrated Concepts

Sunlight above the Earth's atmosphere has an intensity of $1.30\ \text{kW/m}^2$. If this is reflected straight back from a mirror that has only a small recoil, the light's momentum is exactly reversed, giving the mirror twice the incident momentum. (a) Calculate the force per square meter of mirror. (b) Very low mass mirrors can be constructed in the near weightlessness of space, and attached to a spaceship to sail it. Once done, the average mass per square meter of the spaceship is 0.100 kg. Find the acceleration of the spaceship if all other forces are balanced. (c) How fast is it moving 24 hours later?