THE ISOPERIMETRIC PROBLEM

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the degree of Bachelor of Science of Mathematics

by

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Contents

Declaration

All sentences or passages quoted in this project dissertation from other people's work have been specifically acknowledged by clear cross referencing to author, work and page(s). I understand that failure to do this amounts to plagiarism and will be considered grounds for failure in this module and the degree examination as a whole.

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Abstract

In general, we want the maximum area whose boundary has a specific length.

For the 2-dimensional case.

For the 3-dimensional.

For the n-dimensional.

Manifolds?

Introduction

The isoperimetric problem,

Historical Notes

Something about historical notes. In the 2 dimensional case, a proof was given by Jakob Sternier, who was Riemann's teacher.

Important Preliminaries

We will take for granted the Jordan Curve Theorem

Theorem 1 (Jordan Curve Theorem). *A simple closed curve in the plane divides the plane into two regions, one compact and one non-compact, and in the common boundary of both regions*

Note. *When we talk of the region bounded by a simple closed curve in the plane, we always mean the compact region*

We have just used language such as closed, simple, bounded and compact. We give the definitions of these below.

Definition 1. *A closed curve, is a curve that changes direction but does not cross itself whilst changing direction*

The Isoperimetric Theorems for 2D, 3D and nD Cases

1.1 2 Dimensional Case (Plane)

Theorem 2. *Let C be a simple closed curve in the plane with length L and bounding a region of area A . Then $L^2 \leq 4\pi A$ with equality if and only if C is a circle.*

The circle therefore bounds the biggest area among all simple closed curves in the plane with a given length.

1.1.1 Unpacking Steiner's proof

The proof that I will be unpacking and taking a closer look at will be from the book (reference the book here), and is credited by them to Jakob Sternier.

Proof. We will begin the proof by assuming the solution exists □

1.2 3 Dimensional Case (Sphere)

1.3 n Dimensional Case ($\mathbb{R}^n$)

Manifolds