CSC215

Math and Computer Science

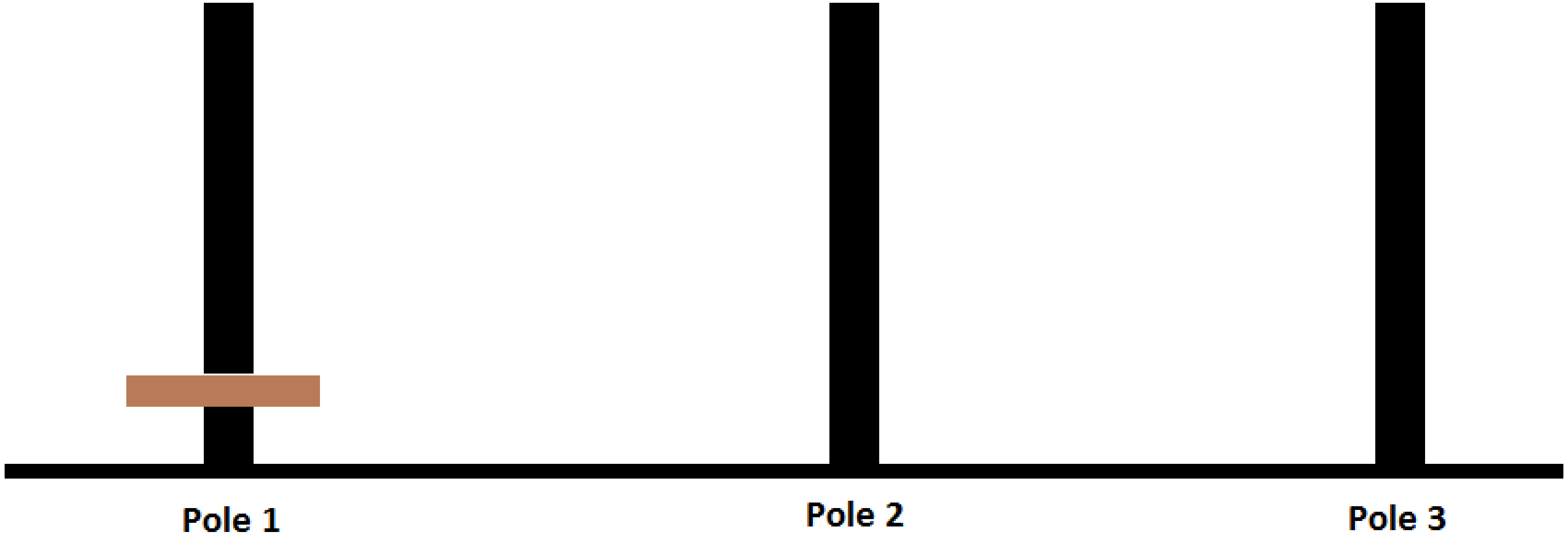
SOUTH DAKOTA SCHOOL OF MINES & TECHNOLOGY

A Legacy of Excellence

Towers of Hanoi

- There are n number of disks on a pole.
- You can move 1 disc at a time.
- You can place smaller discs on top of larger discs
- You must get all n discs to a particular pole.
- You have 1 spare pole to store discs when needed.

Example: 1 Disc



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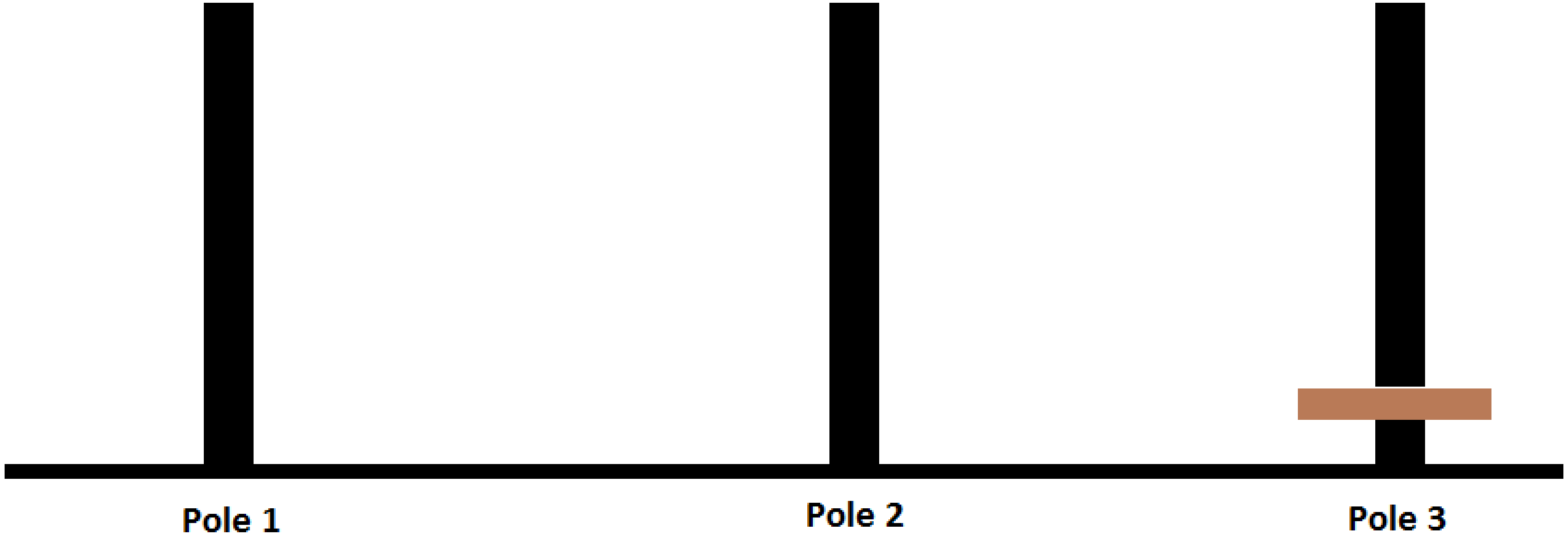
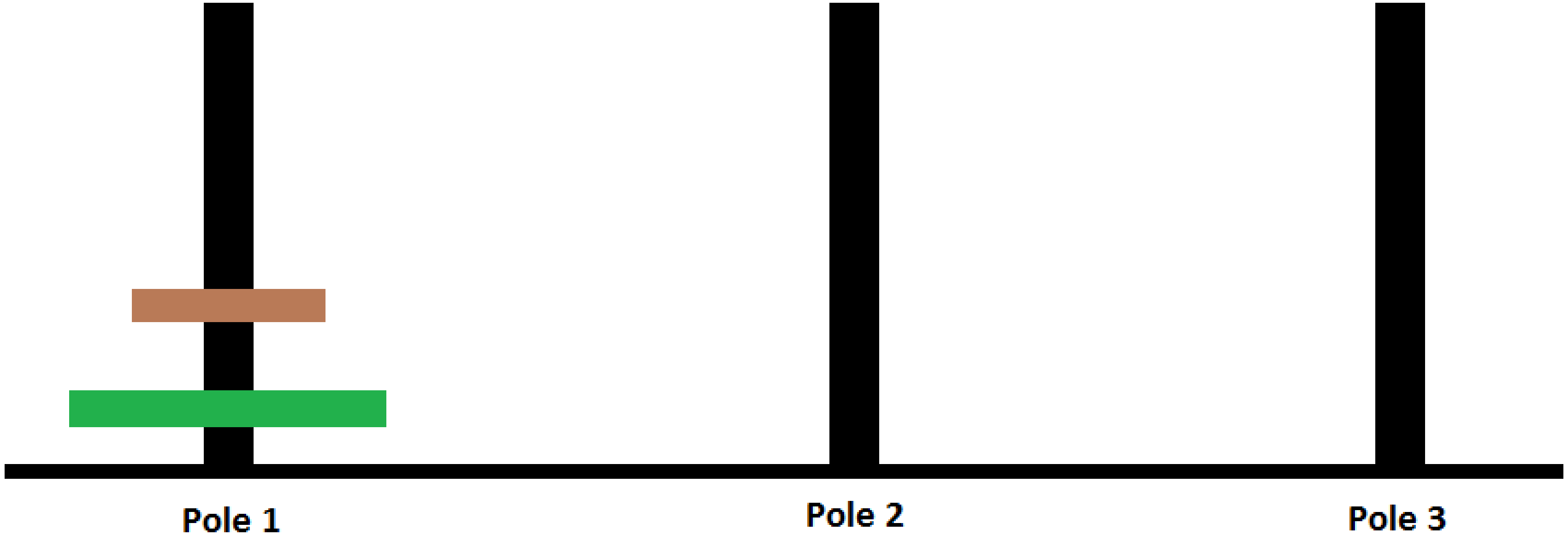
Example: 1 Disc

Problem is solved



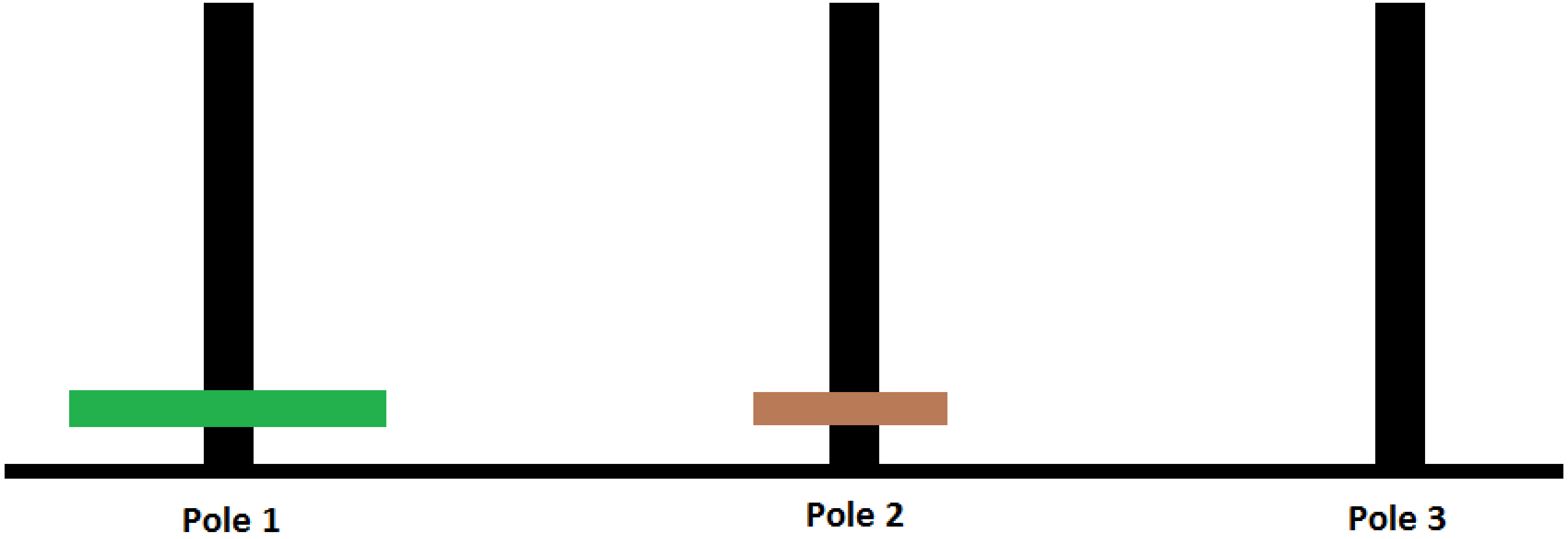
Example: 2 Discs

Can not move 2 discs from pole 1 to pole 3. Move the top disc to Pole 2 (move n-1 from src to spare)



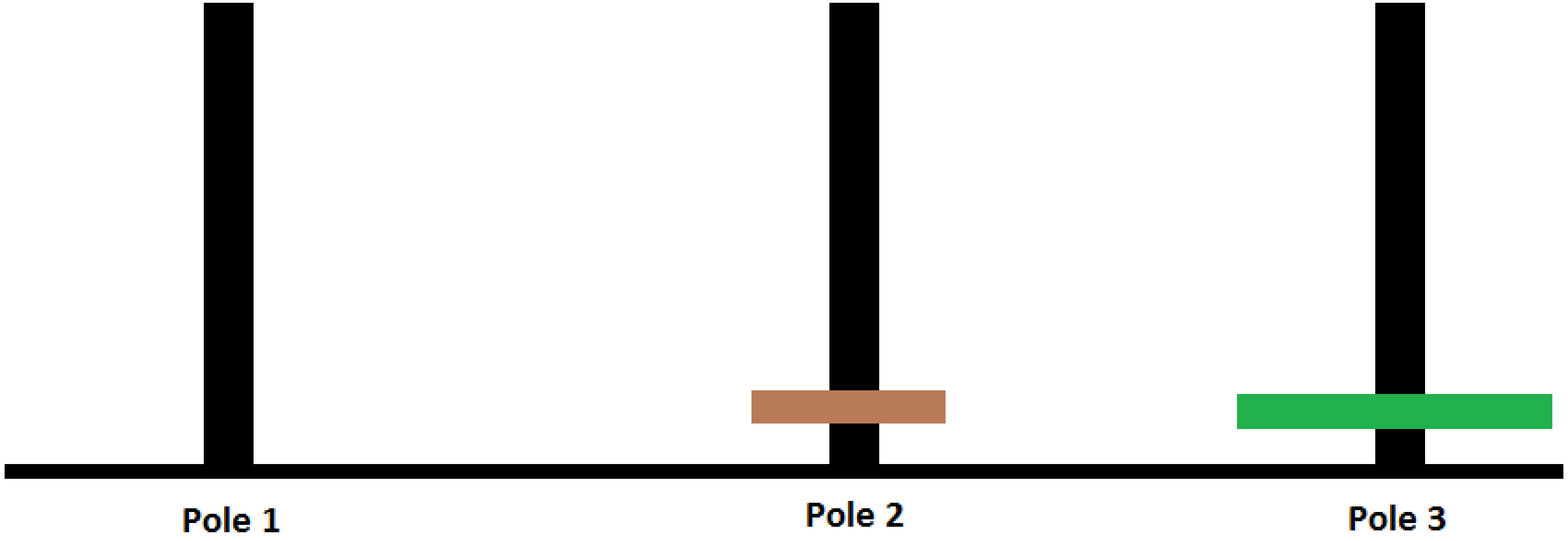
Example: 2 Discs

Now there is only 1 disc on pole 1, move it to pole 3



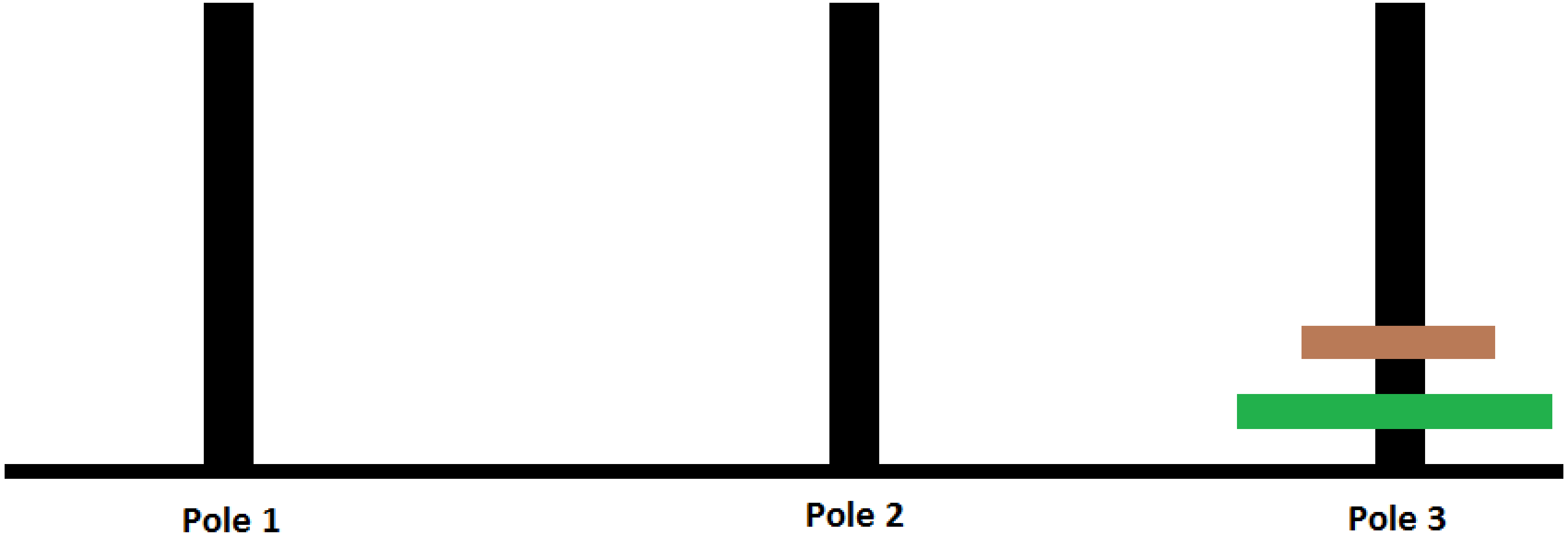
Example: 2 Disc

Now move the n-1 discs that were moved to the spare to the destination pole.



Example: 2 Disc

Problem is solved



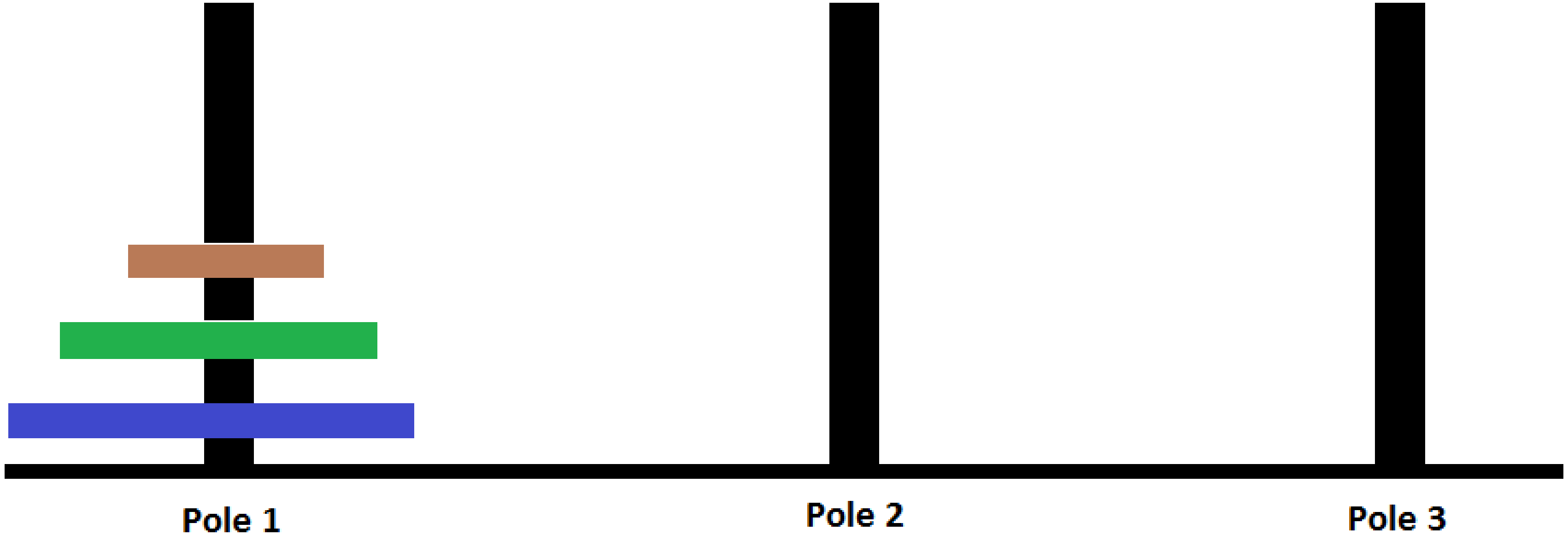
The Key = Slides 5, 6, 7, and 8

If pole 1 is your source and pole 3 is your destination

1. Move n-1 discs from your source to your spare.
2. Move 1 disc from your source to the destination.
3. Move n-1 discs from your spare to the destination.

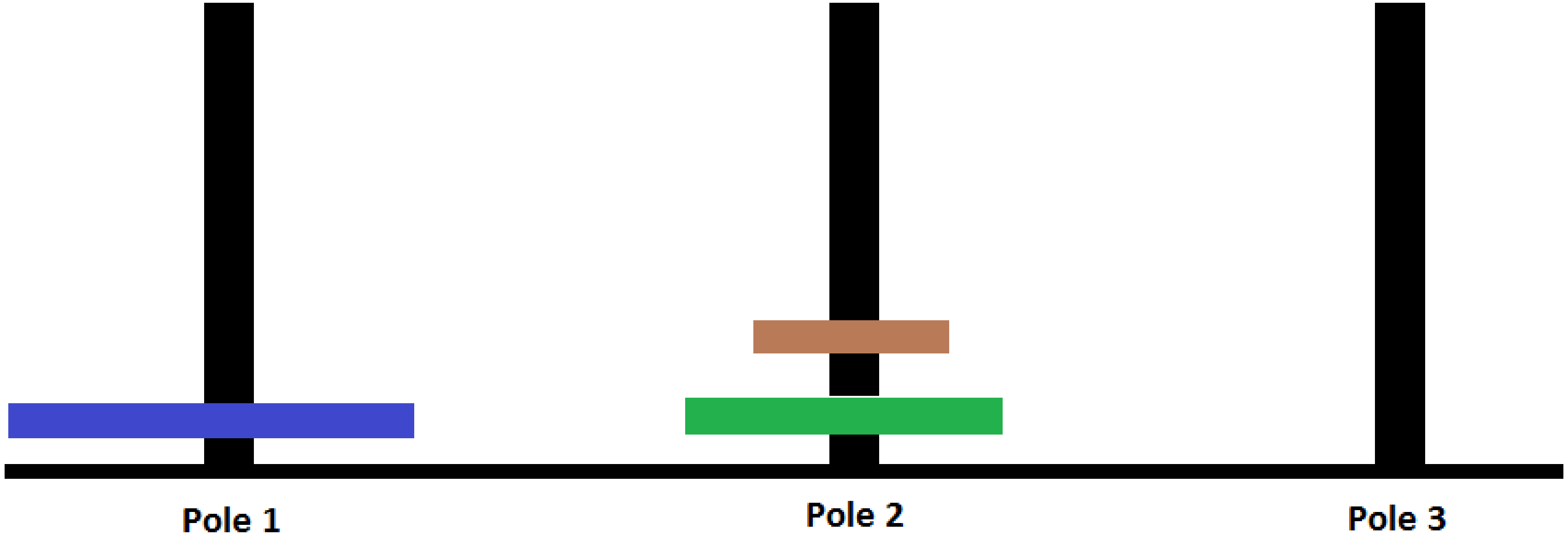
Example: 3 Discs

Move 3 discs from pole 1 to pole 3



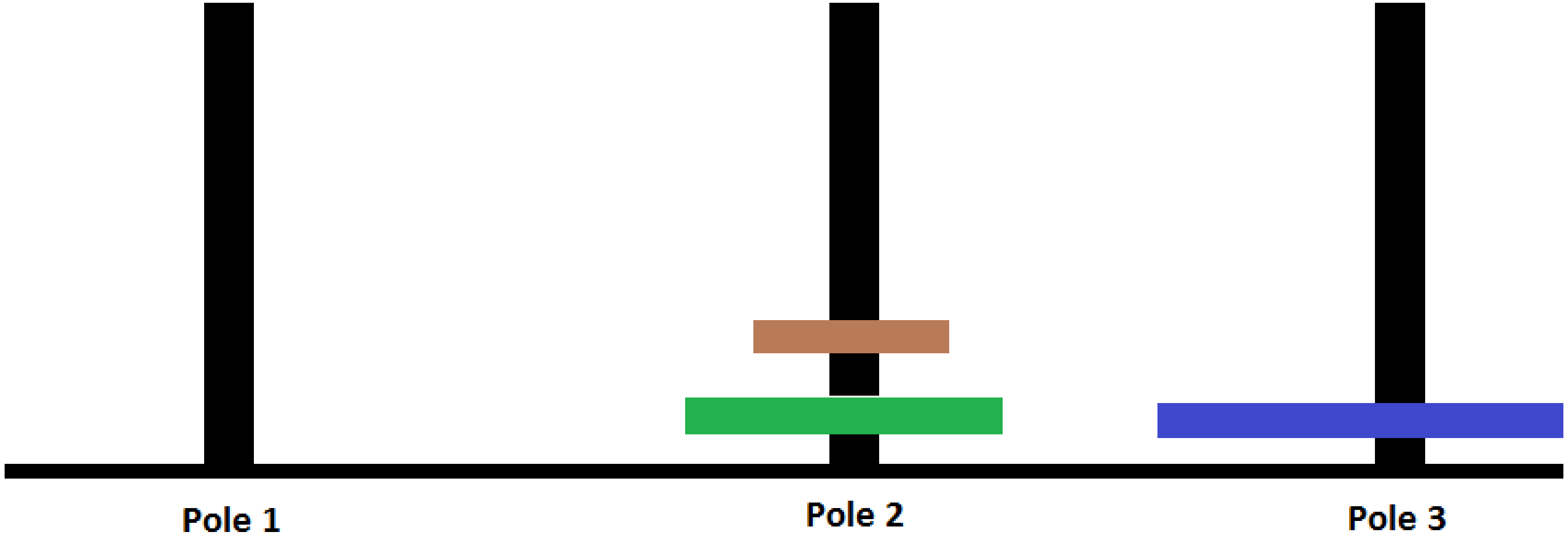
Example: 3 Discs

Move (n-1) discs from pole 1 to pole 2 (I know I am cheating, but this is the key steps.)



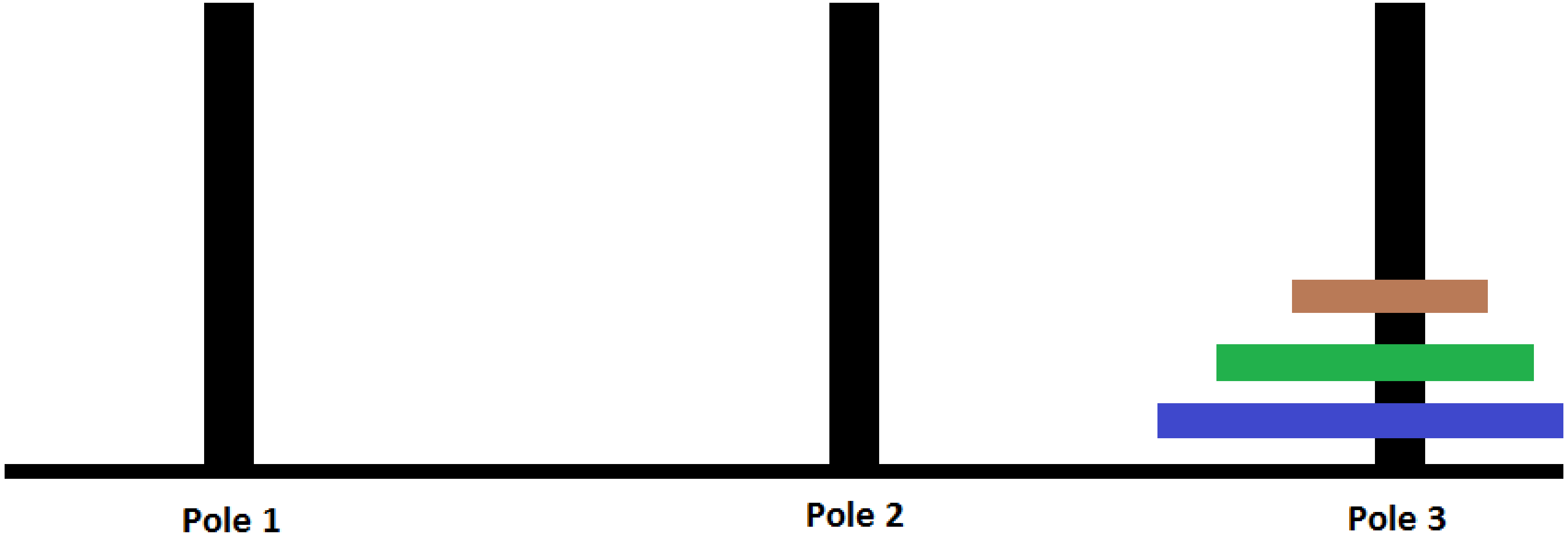
Example: 3 Discs

Move 1 disc from pole 1 to Pole 3



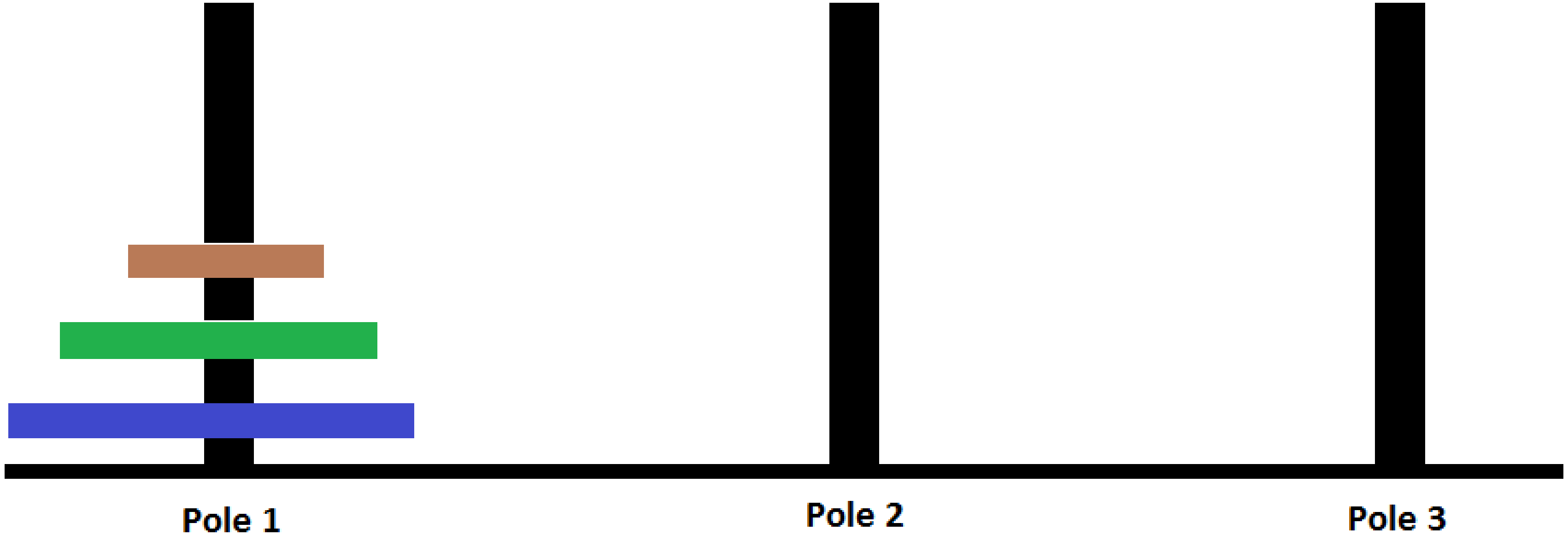
Example: 3 Discs

Move (n-1) discs from pole 2 to pole 3



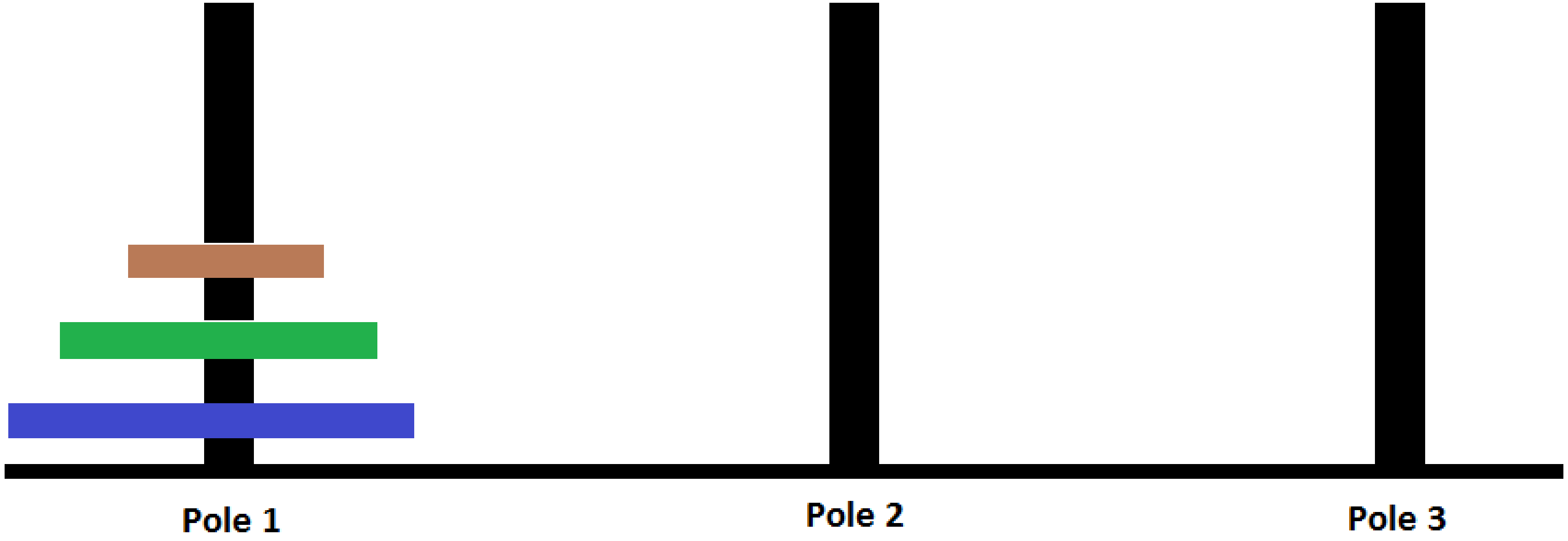
Example: 3 Discs (every step)

Move 3 discs from source (1) to destination (3).
New Problem: move 2 discs from source to spare(2).



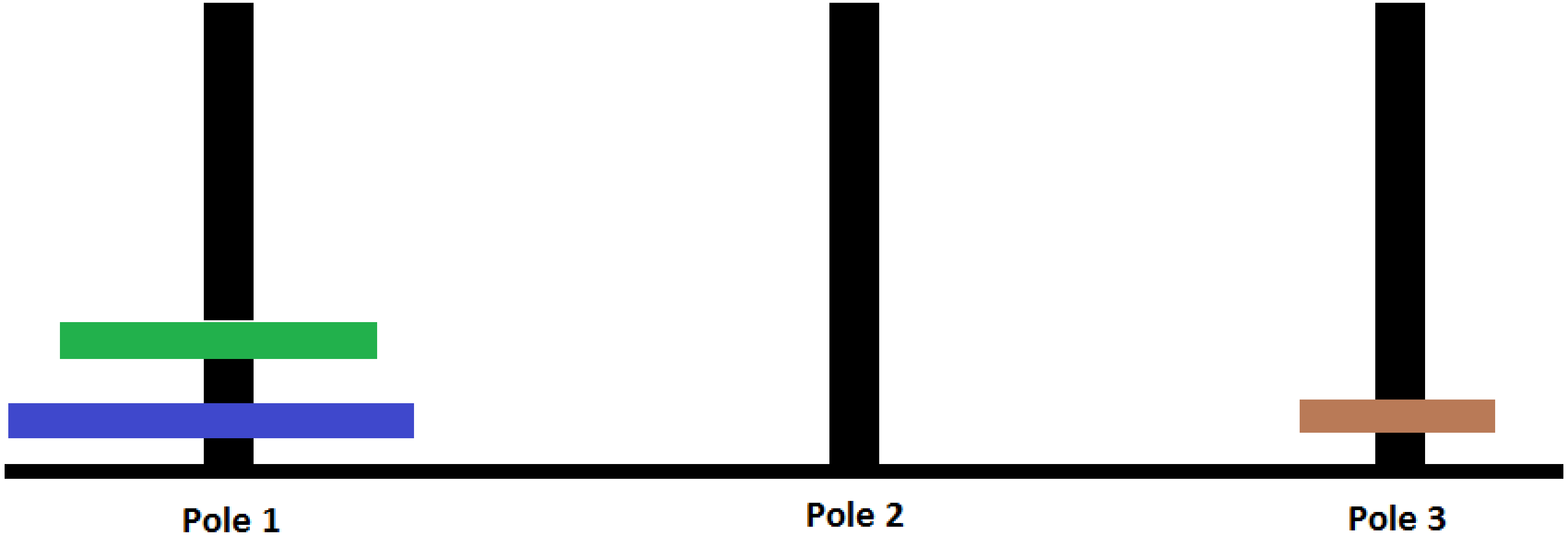
Example: 3 Discs

Move 2 discs from our source (1) to our destination (2).
New Problem: move 1 disc from Source (1) to spare (3).



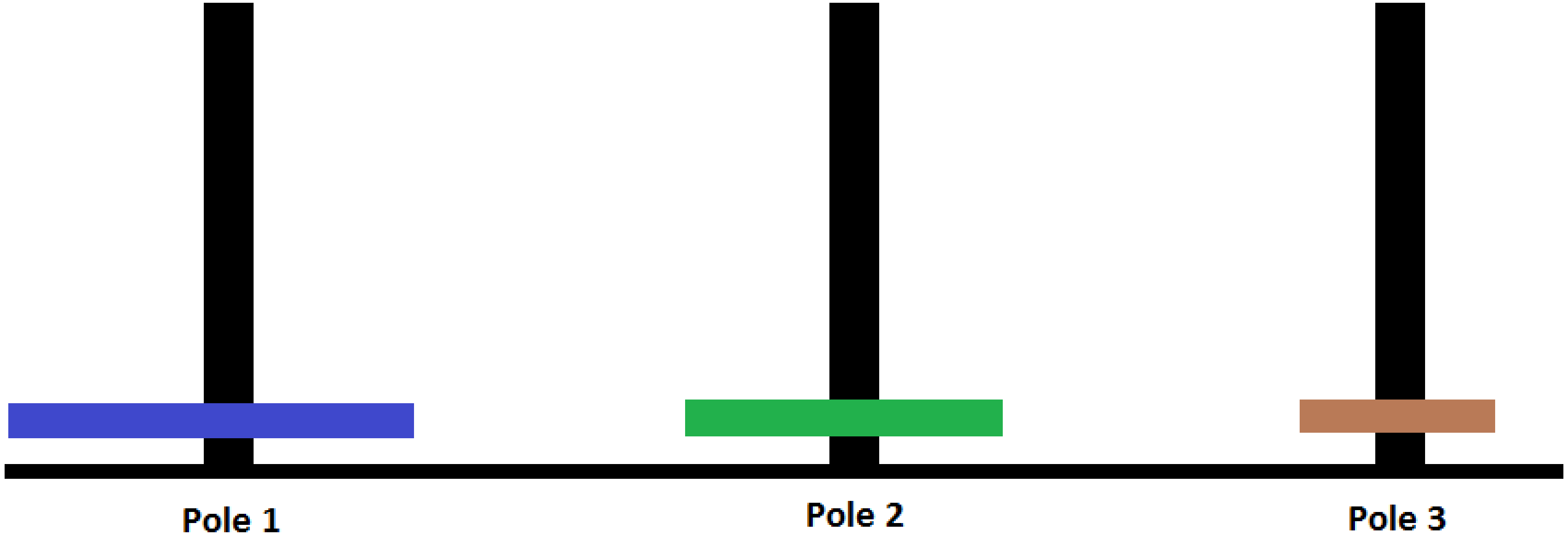
Example: 3 Discs

Only one disc move from source (1) to destination (3)



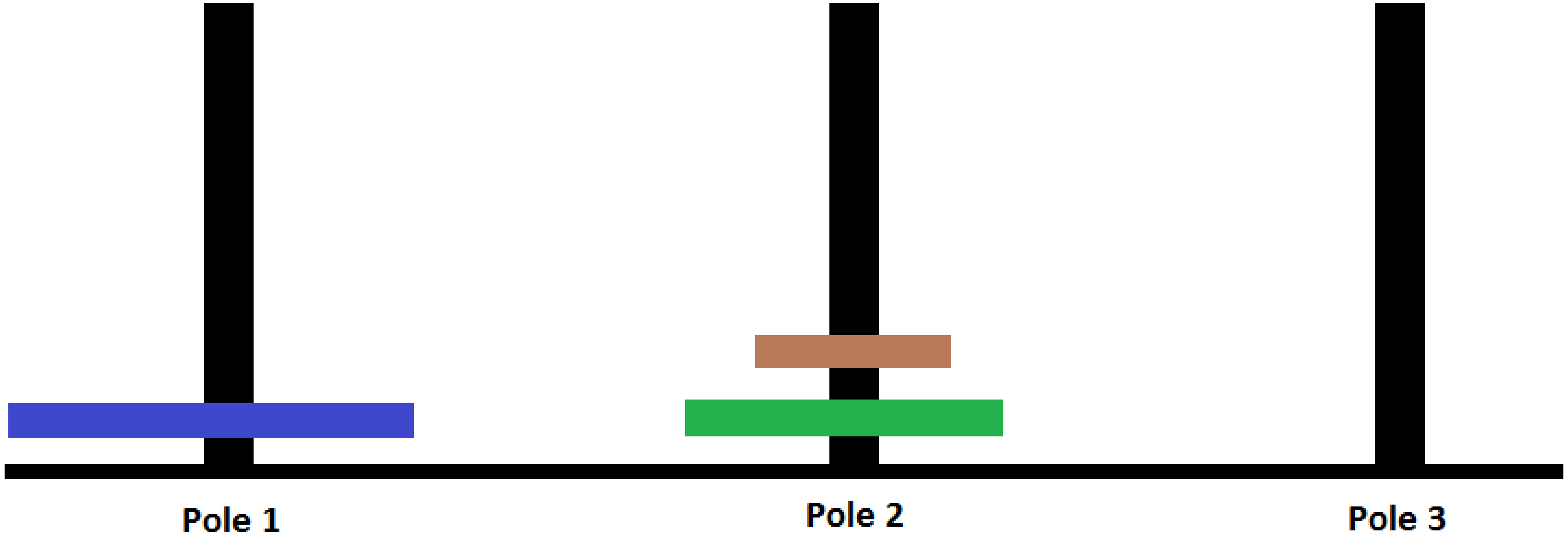
Example: 3 Discs

We moved 1 disc off the top, move 1 disc from source (1) to destination (2)
New Problem: move 1 disc from our spare(3) to the destination (2)



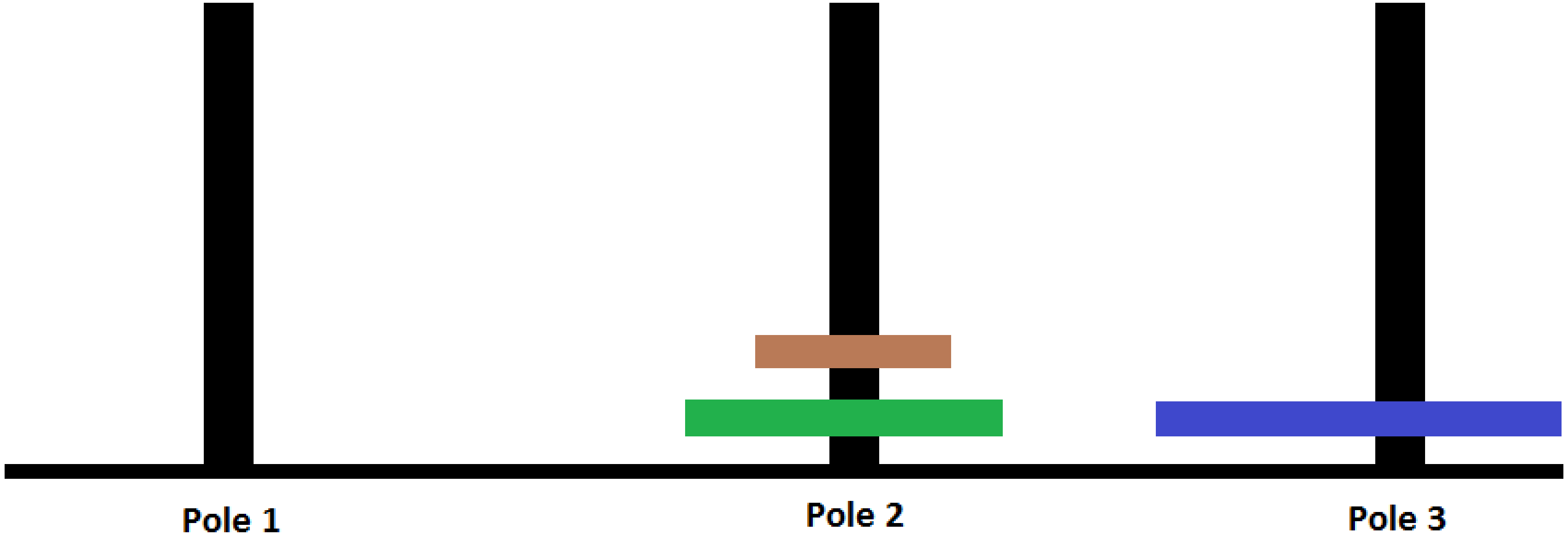
Example: 3 Discs

End of first stage (get n-1 discs from source to spare)
Move 1 disc form source (1) to destination (3)



Example: 3 Disc

Move 2 discs from our spare (2) to the destination (3)
End of second stage



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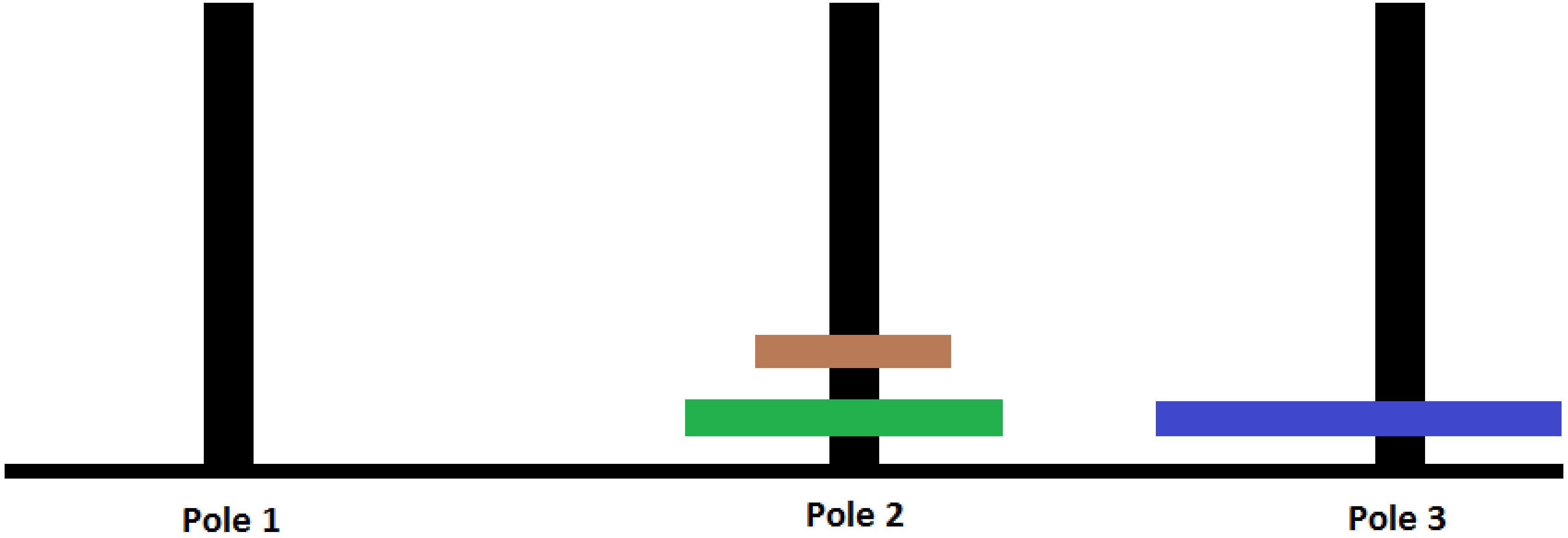
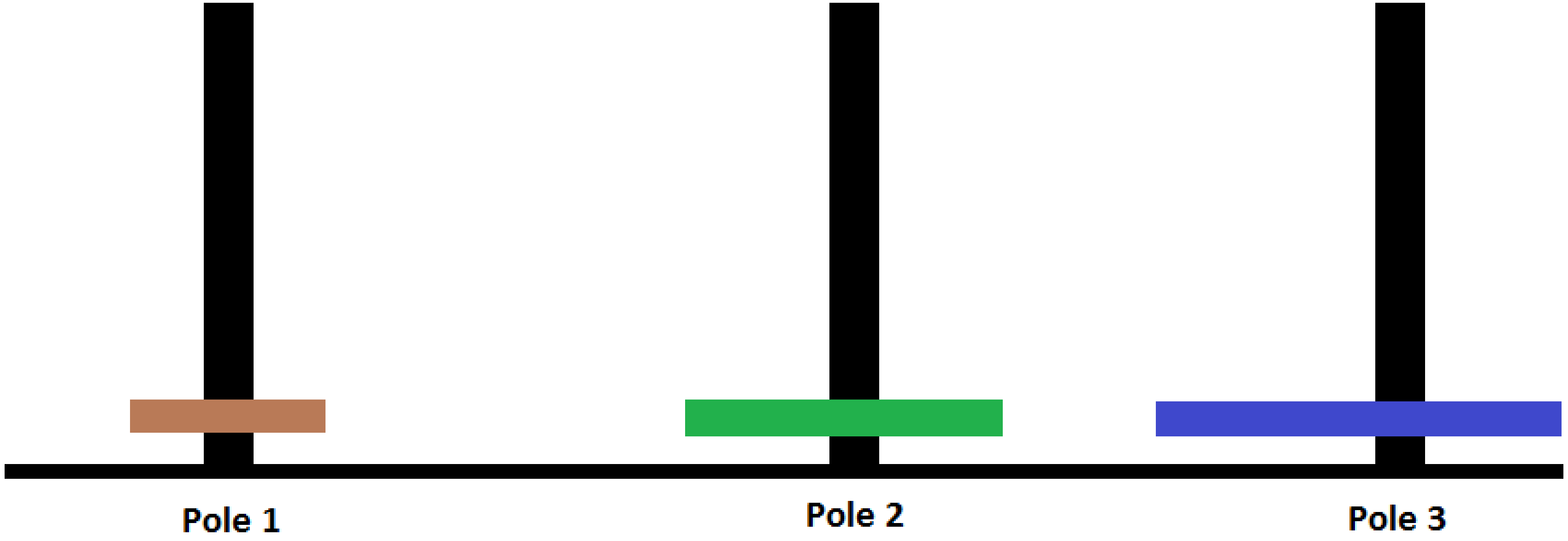
Example: 3 Discs

Move 2 discs from source(2) to destination(3)
New Problem: Move n-1 discs from source(2) to spare(1)



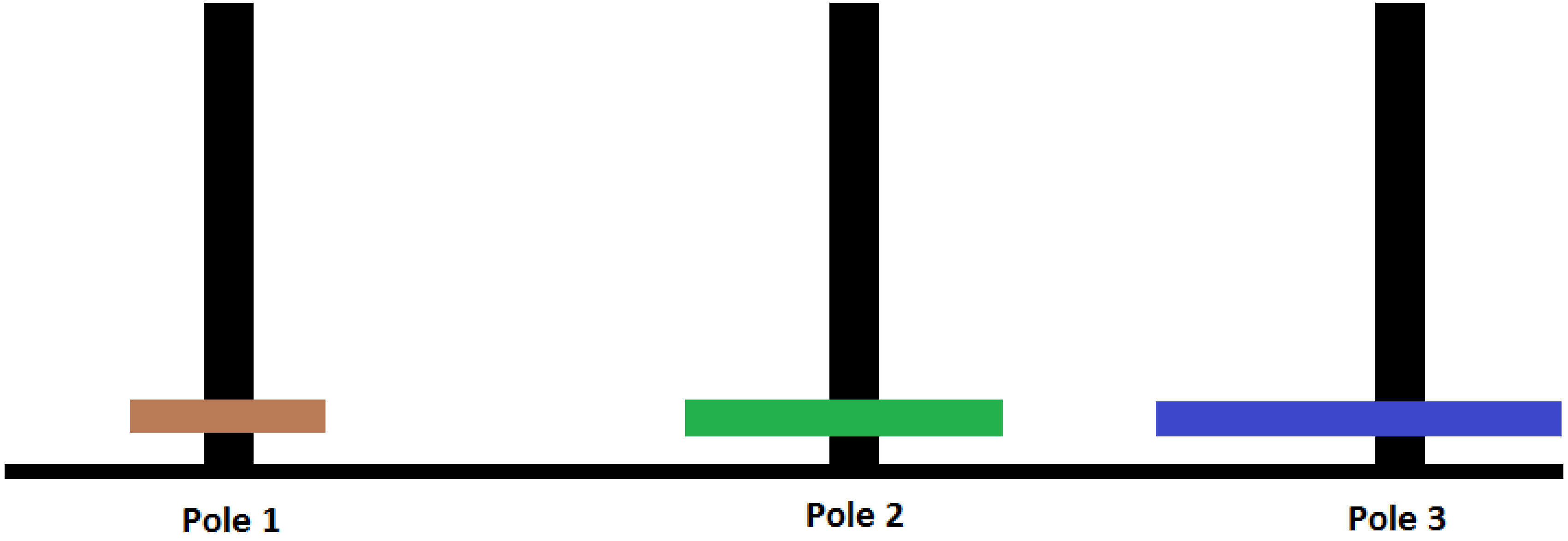
Example: 3 Disc

Move 1 disc from source (2) to destination (1)



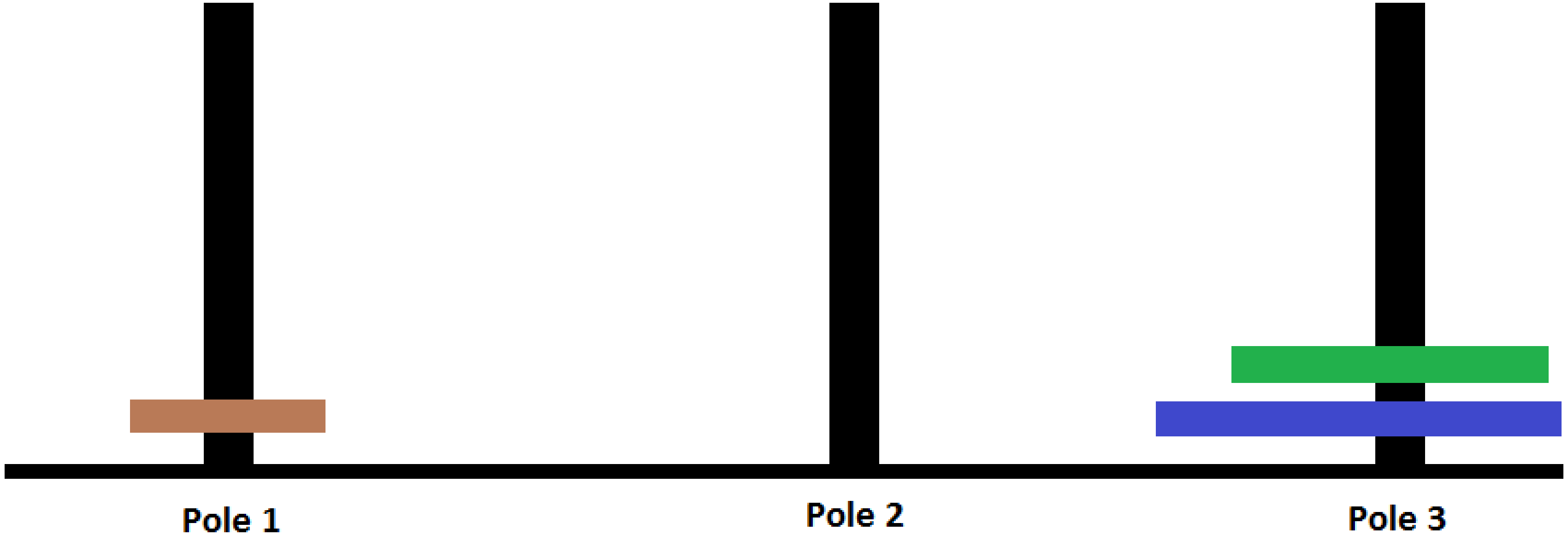
Example: 3 Discs

Move 1 disc from source (2) to destination (3)



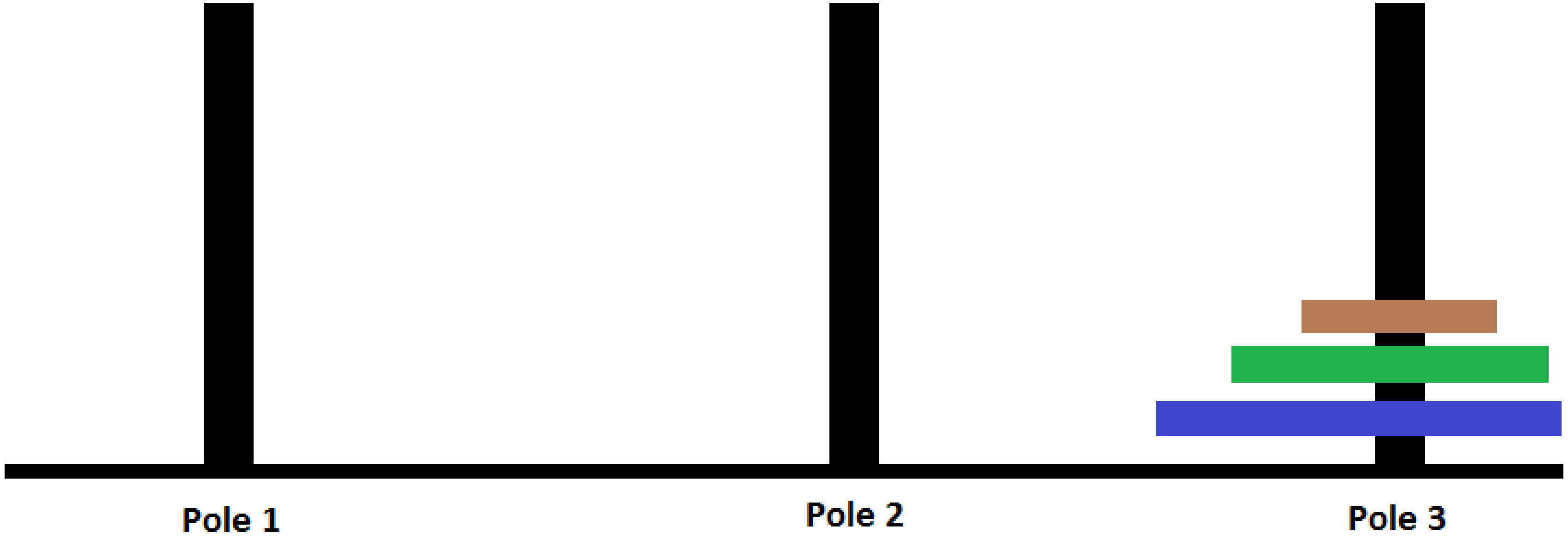
Example: 3 Disc

Move the discs on our spare(1) to our destination (3)



Example: 3 Disc

Problem Solved. End of third stage



Recap: N Discs

- 1st get n-1 number of discs off the source to the spare
- 2nd move the 1 remaining disc from the source to the destination
- 3rd move the n-1 discs from the spare to the destination

Towers of Hanoi - Base case

- N = 1 Move the disc from source to destination

Towers of Hanoi - Criteria

N = 1	Move the disc from source to destination
N > 1	Move n-1 discs from source to spare
	Move 1 disc from source to destination
	Move n-1 discs from spare to destination

Writing the Code – Base Case

```
void towers( int ndiscs, int srcpole, int destpole, int sparepole)
{
   // base case first, if only 1 disk output the move
   if( ndiscs == 1 )
   {
      cout << "Move 1 disc from pole " << srcpole << " to pole "
           << destpole << endl;
      return;
   }

}
```

Writing the Code – Recursive Solution

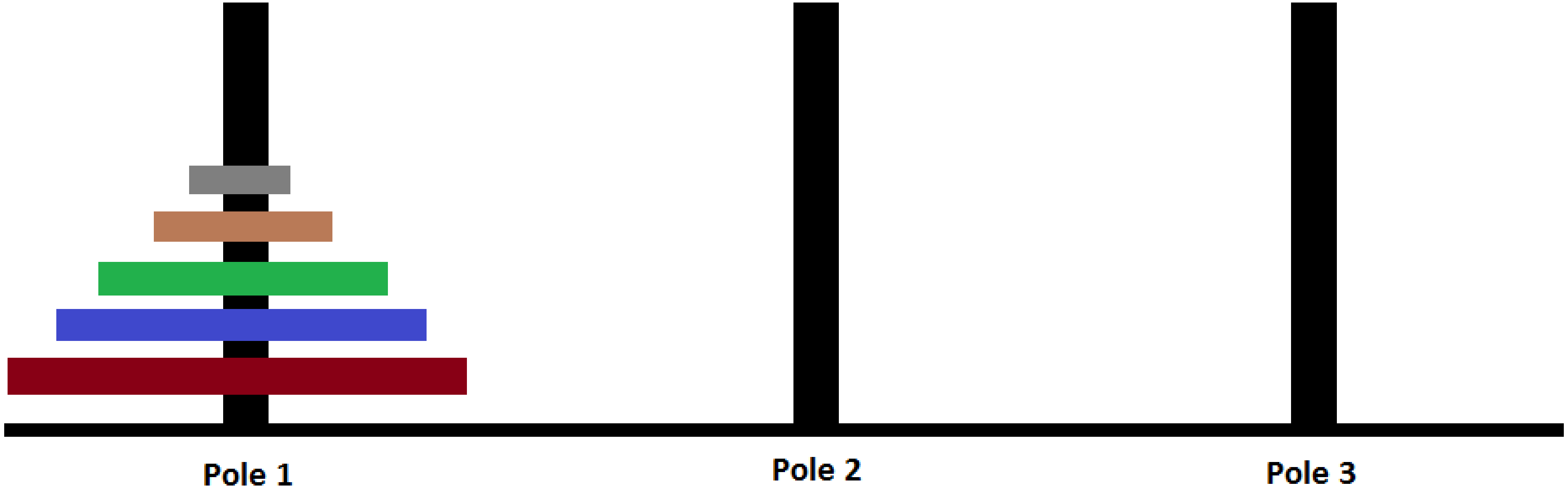
```
void towers( int ndiscs, int srcpole, int destpole, int sparepole)
{   // base case first, if only 1 disk output the move
    if( ndiscs == 1 )
    {  cout << "Move 1 disc from pole " << srcpole << " to pole " << destpole << endl;
       return;}

    // move n-1 discs from source to spare
    towers( ndiscs - 1, srcpole, sparepole, destpole );
    // force moving 1 disc
    towers( 1, srcpole, destpole, sparepole);
    // move n-1 discs from Spare to Destination
    towers( ndiscs - 1, sparepole, destpole, srcpole);
}
```

Calling the function

```cpp
int main()
{
    int ndiscs;

    cout << "Enter how many discs are on the pole: ";
    cin >> ndiscs;

    towers( ndiscs, 1, 3, 2 );

   return 0;
}
```

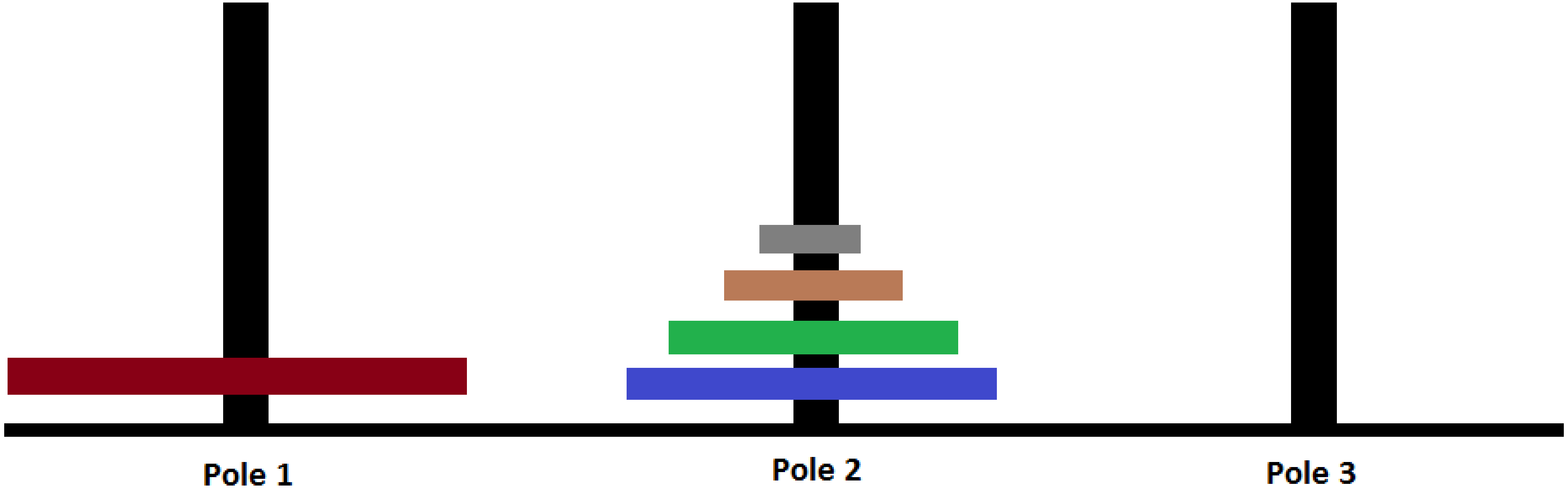
Example: 5 Disc

Move 4 (n-1) disc to pole 2



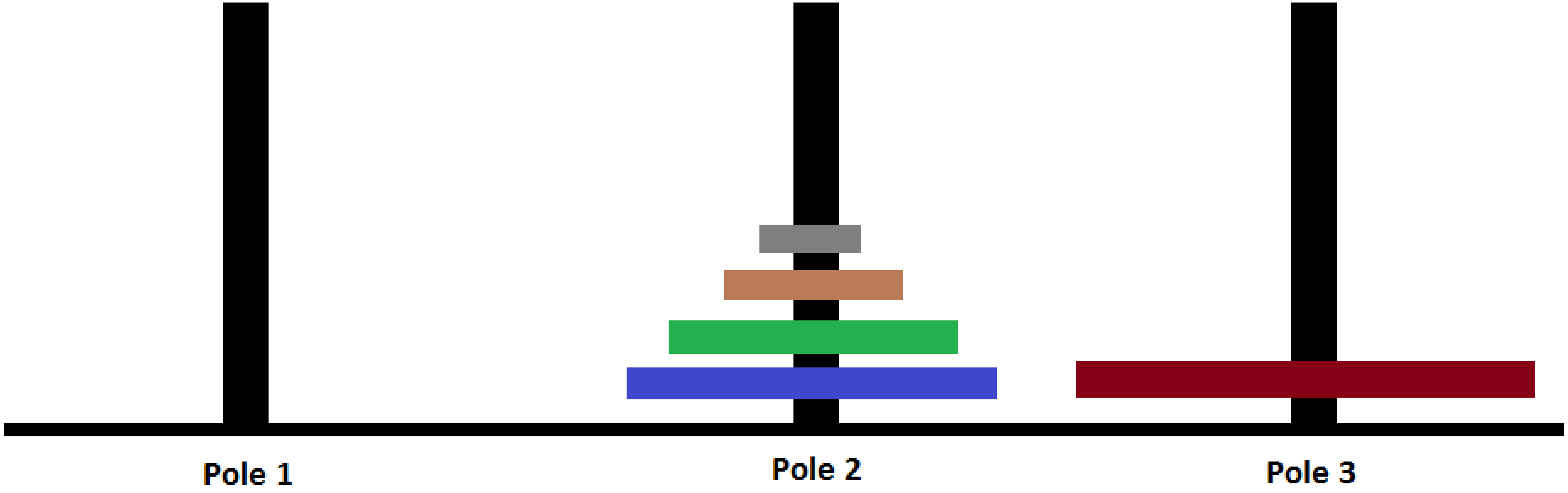
Example: 5 Disc

Move 1 disc from pole 1 to pole 3



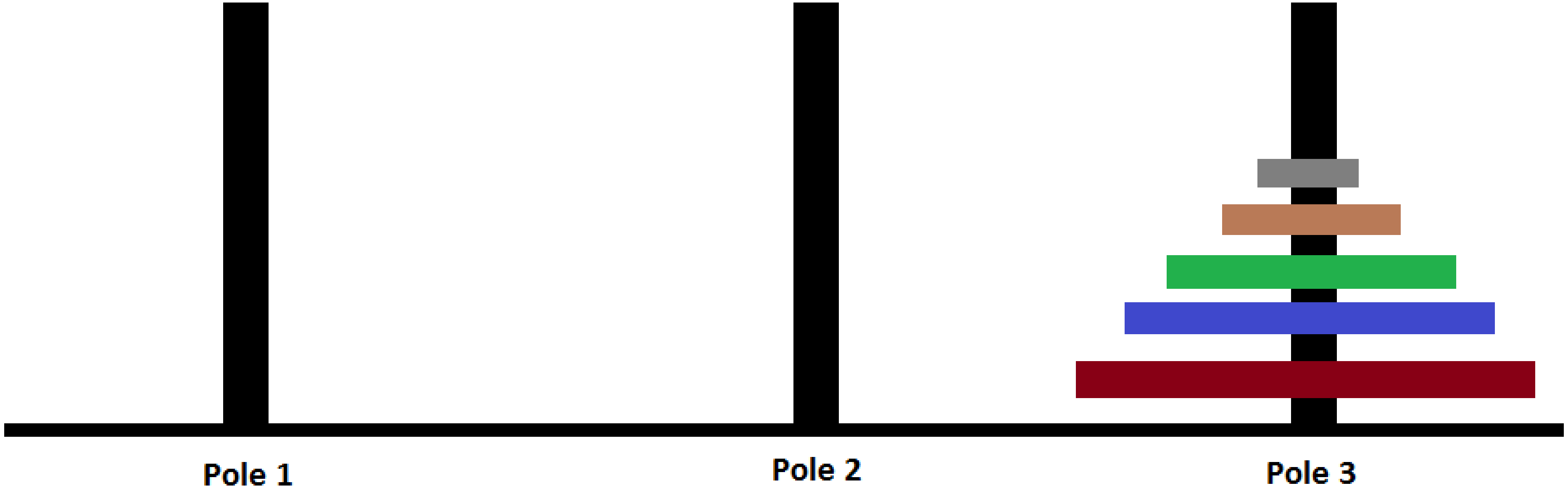
Example: 5 Disc

Move 4 (n-1) disc from pole 2 to pole 3



Example: 5 Disc

Solved



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Moving the N-1 (4) Discs, Src to Spare

```
T(4,1,2,3)
  T(3,1,3,2)
    T(2,1,2,3)
      T(1,1,3,2)
      T(1,1,2,3)
      T(1,3,2,3)
    T(1,1,3,2)
    T(2,2,3,2)
      T(1,2,1,3)
      T(1,2,3,1)
      T(1,1,3,1)
```

```
T(1,1,2,3)
```

```
T(3,3,2,3)
  T(2,3,1,2)
    T(1,3,2,1)
    T(1,3,1,2)
    T(1,2,1,2)
  T(1,3,2,1)
  T(2,1,2,1)
    T(1,1,3,2)
    T(1,1,2,3)
    T(1,3,2,3)
```

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Move the Bottom Disc, 1 Src to Dest

T(1,1,3,2)

Moving the N-1 (4) Discs, Spare to Dest

```
T(4,2,3,2)
  T(3,2,1,3)
    T(2,2,3,1)
      T(1,2,1,3)
      T(1,2,3,1)
      T(1,1,3,1)
    T(1,2,1,3)
    T(2,3,1,3)
      T(1,3,2,1)
      T(1,3,1,2)
      T(1,2,1,2)
```

```
T(1,2,3,1)
```

```
T(3,1,3,1)
  T(2,1,2,3)
    T(1,1,3,2)
    T(1,1,2,3)
    T(1,3,2,3)
  T(1,1,3,2)
  T(2,2,3,2)
    T(1,2,1,3)
    T(1,2,3,1)
    T(1,1,3,1)
```