CSC215

Math and Computer Science

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Recursion

- Related to mathematical induction
  - Definition of the nth value in terms of the first n-1 values
  - A stopping condition

- In C++, it is a function that calls itself
  - Has what is known as a base case for stopping the function calls

Ask Yourself the Following

1. How to define the problem in terms of a smaller problem of the same type?
2. How does each recursive call diminish the size of the problem?
3. What instance of the problem can serve as the base case?
4. As the problem diminishes, will it reach the base case?

The example here are best done iteratively.

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Factorial

- N! = N * (n-1) * (n-2) * (n-3) *…….
- How to define in terms of the first n-1 terms
- N! = N * (n-1)!

- What will serve as our base case?
  - 0! = 1
  - Could use 1! But the smallest possible factorial is defined as 0! = 1

The Factorial Criteria

- N = 0 answer is 1
- N > 0 answer is N * (N-1)!

Drawing out N!

- Assume n = 6
  - 6! = 6 * 5!
  - 5! = 5 * 4!
  - 4! = 4 * 3!
  - 3! = 3 * 2!
  - 2! = 2 * 1!
  - 1! = 1 * 0!
  - 0! = 1 ← Good, I know the answer

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Substitute Going Back Up

- Answer to n = 6, 6! = 720
  - 6! = 6 * 5! = 6 * 120 = 720
  - 5! = 5 * 4! = 5 * 24 = 120
  - 4! = 4 * 3! = 4 * 6 = 24
  - 3! = 3 * 2! = 3 * 2 = 6
  - 2! = 2 * 1! = 2 * 1 = 2
  - 1! = 1 * 0! = 1 * 1 = 1
  - 0! = 1

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Writing the function

```
unsigned int factorial( unsigned int n )
{
    // write the base case first
    if( n == 0 )
        return 1;



}
```

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Writing the function

```
unsigned int factorial( unsigned int n )
{   int ans;
    // write the base case first
    if( n == 0 )
        return 1;

    // compute the answer
    ans = n * factorial( n - 1 );
    return ans;

}
```

Stepping Through the Function

```
int main()
{
    int result;
    int n = 4;
    result = factorial ( n );
    cout << "4! = " << result << endl;
    return 0;
}
```

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Memory (remember the scoping rules, f = factorial)

Memory	main
? = F(4)	result
4	n

Memory (remember the scoping rules, f = factorial)

Memory	main	F(4)
? = F(4)	Result	
4	n	
4		n
? = N *F(3)		ans

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Memory (remember the scoping rules, f = factorial)

Memory	main	F(4)	F(3)
? = F(4)	Result		
4	n		
4		n	
? = n * F(3)		ans	
3			n
? = n * F(2)			ans

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Memory (remember the scoping rules, f = factorial)

Memory	main	F(4)	F(3)	F(2)
? = F(4)	Result			
4	n			
4		n		
? = n * F(3)		ans		
3			n	
? = n * F(2)			ans	
2				n
? = n * F(1)				ans

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Memory (remember the scoping rules, f = factorial)

Memory	main	F(4)	F(3)	F(2)	F(1)
? = F(4)	Result				
4	n				
4		n			
? = n * F(3)		ans			
3			n		
? = n * F(2)			ans		
2				n	
? = n * F(1)				ans	
1					n
? = n * F(0)					ans

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Memory (remember the scoping rules, f = factorial)

Memory	main	F(4)	F(3)	F(2)	F(1)	F(0)
? = F(4)	Result					
4	n					
4		n				
? = n * F(3)		ans				
3			n			
? = n * F(2)			ans			
2				n		
? = n * F(1)				ans		
1					n	
? = n * F(0)					ans	
0						n
						ans

Base Case Reached

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Memory (remember the scoping rules, f = factorial)

Memory	main	F(4)	F(3)	F(2)	F(1)	F(0) = 1
? = F(4)	Result					
4	n					
4		n				
? = n * F(3)		ans				
3			n			
? = n * F(2)			ans			
2				n		
? = n * F(1)				ans		
1					n	
n * F(0) = 1					ans	

F(0) = 1

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Memory (remember the scoping rules, f = factorial)

Memory	main	F(4)	F(3)	F(2)	F(1) = 1
? = F(4)	Result				
4	n				
4		n			
? = n * F(3)		ans			
3			n		
? = n * F(2)			ans		
2				n	
n * F(1) = 2				ans	

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Memory (remember the scoping rules, f = factorial)

Memory	main	F(4)	F(3)	F(2) = 2
? = F(4)	Result			
4	n			
4		n		
? = n * F(3)		ans		
3			n	
n * F(2) = 6			ans	

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Memory (remember the scoping rules, f = factorial)

Memory	main	F(4)	F(3) = 6
? = F(4)	Result		
4	n		
4		n	
n * F(3) = 24		ans	

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Memory (remember the scoping rules, f = factorial)

Memory	main	F(4) = 24
= F(4) = 24	Result	
4	n	

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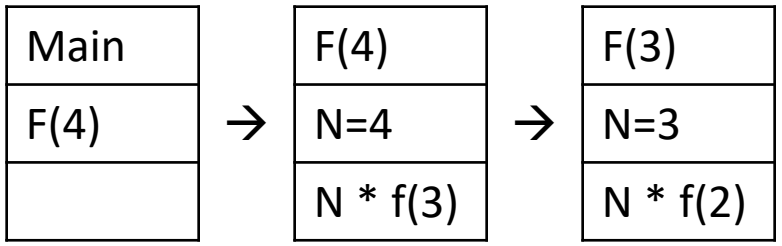
Box Method

Main
F(4)

→

F(4)
N=4
N * f(3)

Box Method



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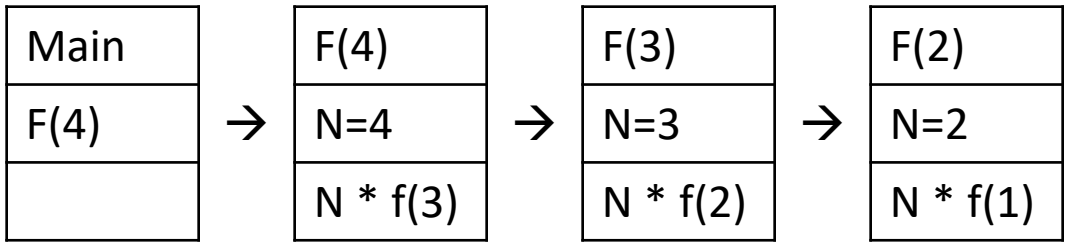
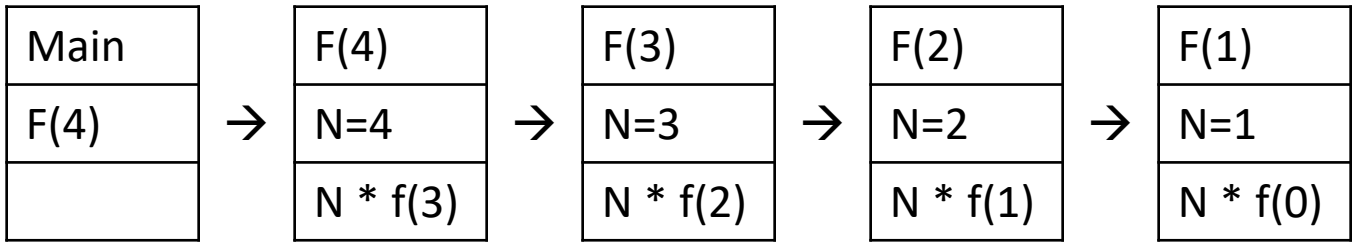
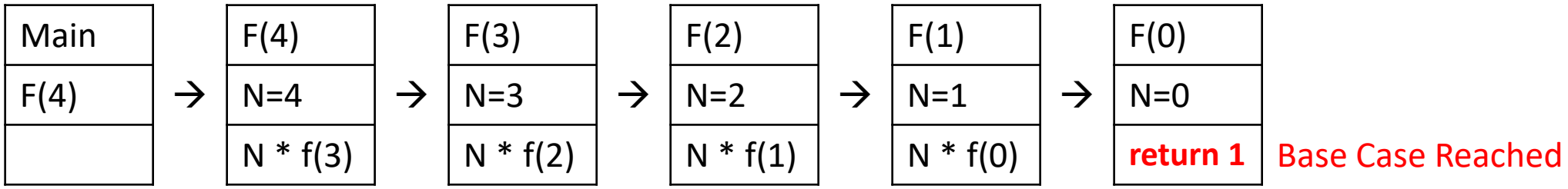
Box Method



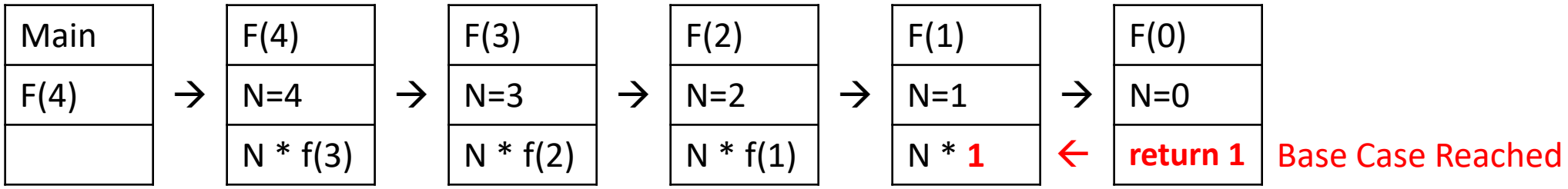
Box Method



Box Method


Main
F(4)
F(4)
N=4
N * f(3)
F(3)
N=3
N * f(2)
F(2)
N=2
N * f(1)
F(1)
N=1
N * f(0)
F(0)
N=0
return 1
Base Case Reached

Box Method



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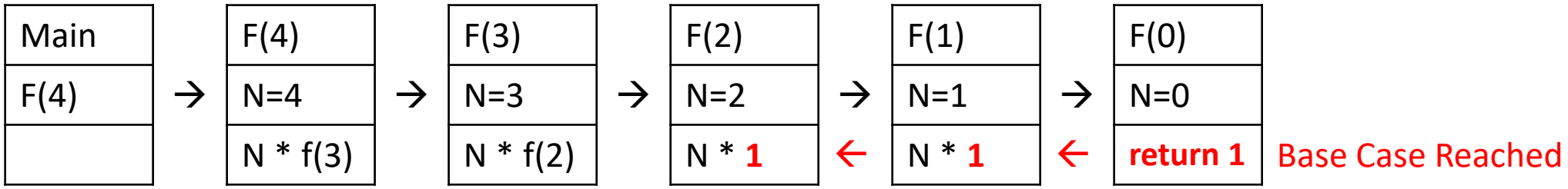
Box Method



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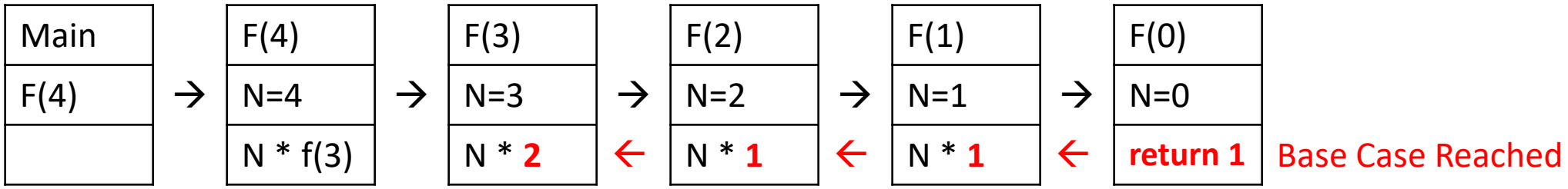
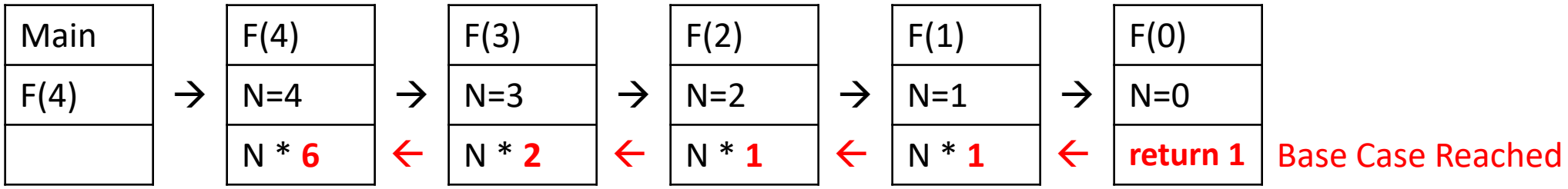
Box Method


Main
F(4)
F(4)
N=4
N * f(3)
F(3)
N=3
N * 2
F(2)
N=2
N * 1
F(1)
N=1
N * 1
F(0)
N=0
return 1
Base Case Reached

Box Method



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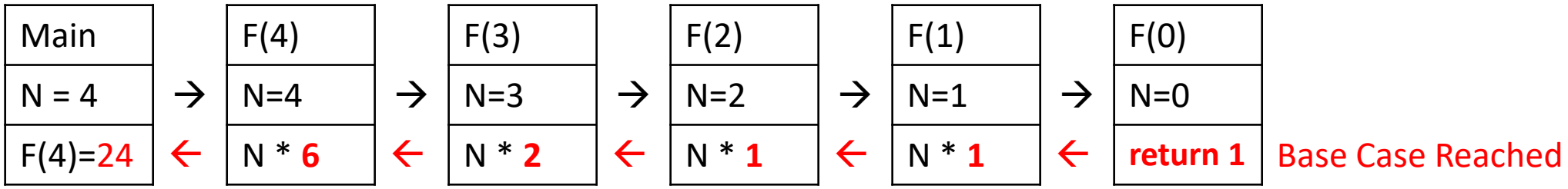
Box Method



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Factorial Iterative

```
unsigned int ifactorial( unsigned int n )
{
    int i;
    unsigned int ans = 1;
    for( i=1; i<=n; i++)
        ans *= i;
    return ans;
}
```

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Summation

$$\sum_{i=0}^{n} i = 0 + 1 + \ldots + (n-2) + (n-1) + n$$

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Summation

- How to define in terms of the first n-1 terms

$$\sum_{i=0}^{n} i = n + \sum_{i=0}^{n-1} i$$

- What will serve as our base case?
  - N = 0, answer is 0

Summation Criteria

N = 0 answer is 0

N > 0 answer is $n + \sum_{i=0}^{n-1} i$

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Drawing it out

- Assume n = 6
  - Sum(6) = 6 + Sum(5)
  - Sum(5) = 5 + Sum(4)
  - Sum(4) = 4 + Sum(3)
  - Sum(3) = 3 + Sum(2)
  - Sum(2) = 2 + Sum(1)
  - Sum(1) = 1 + Sum(0)
  - Sum(0) = 0 ← Good, I know the answer

Substitute Going Back Up

- Assume n = 6
  - Sum(6) = 6 + Sum(5) = 6 + 15 = 21
  - Sum(5) = 5 + Sum(4) = 5 + 10 = 15
  - Sum(4) = 4 + Sum(3) = 4 + 6 = 10
  - Sum(3) = 3 + Sum(2) = 3 + 3 = 6
  - Sum(2) = 2 + Sum(1) = 2 + 1 = 3
  - Sum(1) = 1 + Sum(0) = 1 + 0 = 1
  - Sum(0) = 0 ← Good, I know the answer

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Writing the function

```
unsigned int summation( unsigned int n )
{
    // write the base case first
    if( n == 0 )
        return 0;



}
```

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Writing the function

```
unsigned int summation( unsigned int n )
{   int ans;
    // write the base case first
    if( n == 0 )
        return 0;

    // compute the answer
    ans = n + summation( n - 1 );
    return ans;

}
```

Stepping Through the Function

```
int main()
{
    int result;
    int n = 4;
    result = summation ( n );
    cout << "sum of 0 to " << n << " = "
         << result << endl;
    return 0;
}
```

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Memory (remember the scoping rules, s = summation)

Memory	main
? = S(4)	result
4	n

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Memory (remember the scoping rules, s = summation)

Memory	main	S(4)
? = S(4)	Result	
4	n	
4		n
? = N + S(3)		ans

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Memory (remember the scoping rules, s = summation)

Memory	main	S(4)	S(3)
? = S(4)	Result		
4	n		
4		n	
? = n + S(3)		ans	
3			n
? = n + S(2)			ans

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Memory (remember the scoping rules, s = summation)

Memory	main	S(4)	S(3)	S(2)
? = S(4)	Result			
4	n			
4		n		
? = n + S(3)		ans		
3			n	
? = n + S(2)			ans	
2				n
? = n + S(1)				ans

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Memory (remember the scoping rules, s = summation)

Memory	main	S(4)	S(3)	S(2)	S(1)
? = S(4)	Result				
4	n				
4		n			
? = n + S(3)		ans			
3			n		
? = n + S(2)			ans		
2				n	
? = n + S(1)				ans	
1					n
? = n + S(0)					ans

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Memory (remember the scoping rules, s = summation)

Memory	main	S(4)	S(3)	S(2)	S(1)	S(0)
? = S(4)	Result					
4	n					
4		n				
? = n + S(3)		ans				
3			n			
? = n + S(2)			ans			
2				n		
? = n + S(1)				ans		
1					n	
? = n + S(0)					ans	
0						n
						ans

Base Case Reached

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Memory (remember the scoping rules, s = summation)

Memory	main	S(4)	S(3)	S(2)	S(1)	S(0) = 0
? = S(4)	Result					
4	n					
4		n				
? = n + S(3)		ans				
3			n			
? = n + S(2)			ans			
2				n		
? = n + S(1)				ans		
1					n	
n + S(0) = 1					ans	

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Memory (remember the scoping rules, s = summation)

Memory	main	S(4)	S(3)	S(2)	S(1) = 1
? = S(4)	Result				
4	n				
4		n			
? = n + S(3)		ans			
3			n		
? = n + S(2)			ans		
2				n	
n + S(1) = 3				ans	

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Memory (remember the scoping rules, s = summation)

Memory	main	S(4)	S(3)	S(2) = 3
? = S(4)	Result			
4	n			
4		n		
? = n + S(3)		ans		
3			n	
n + S(2) = 6			ans	

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Memory (remember the scoping rules, s = summation)

Memory	main	S(4)	S(3) = 6
? = S(4)	Result		
4	n		
4		n	
n + S(3) = 10		ans	

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Memory (remember the scoping rules, s = summation)

Memory	main	S(4) = 10
= S(4) = 10	Result	
4	n	

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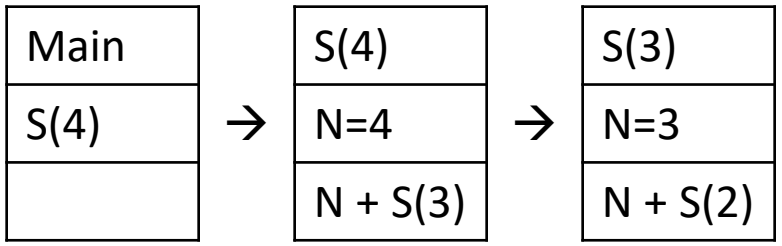
Box Method

Main
S(4)

→

S(4)
N=4
N + S(3)

Box Method



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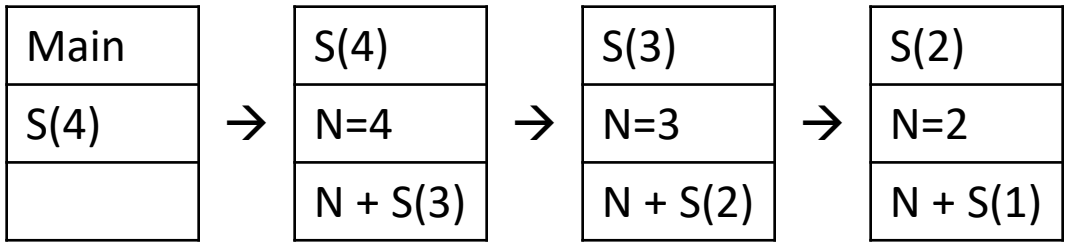
Box Method



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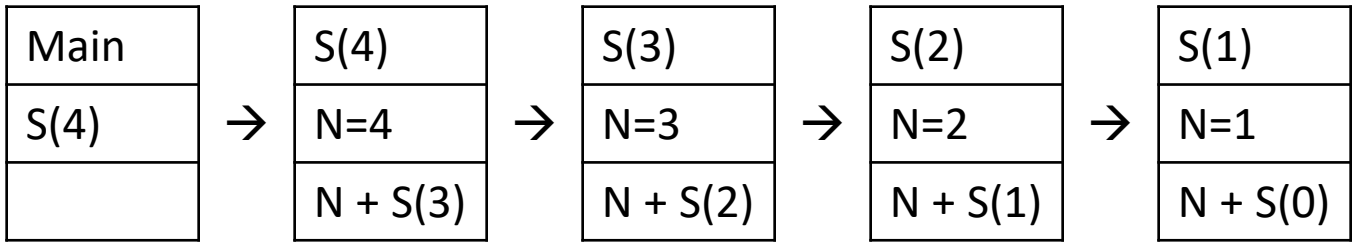
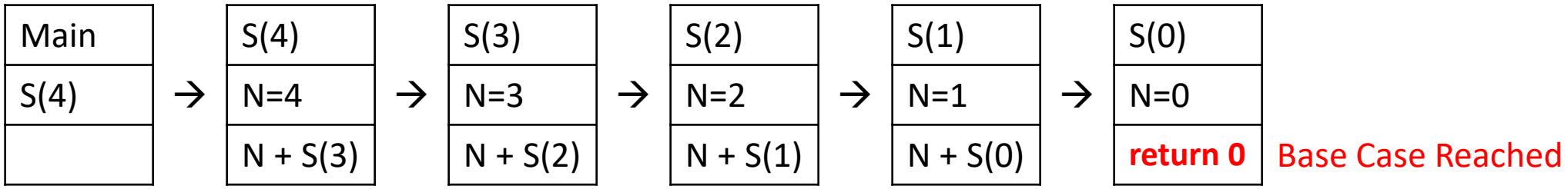
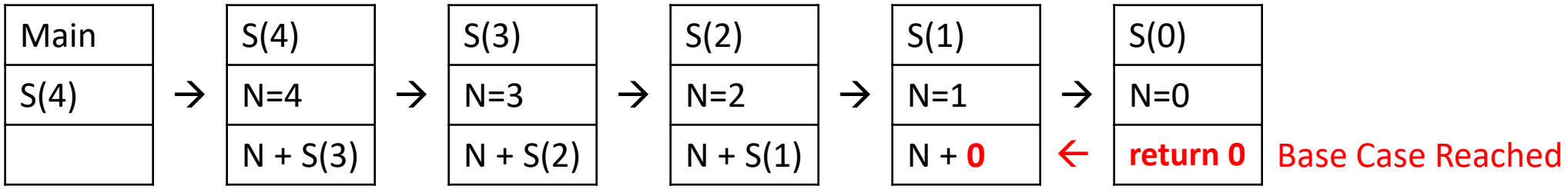
Box Method



Box Method


Main
S(4)
S(4)
N=4
N + S(3)
S(3)
N=3
N + S(2)
S(2)
N=2
N + S(1)
S(1)
N=1
N + S(0)
S(0)
N=0
return 0
Base Case Reached

Box Method



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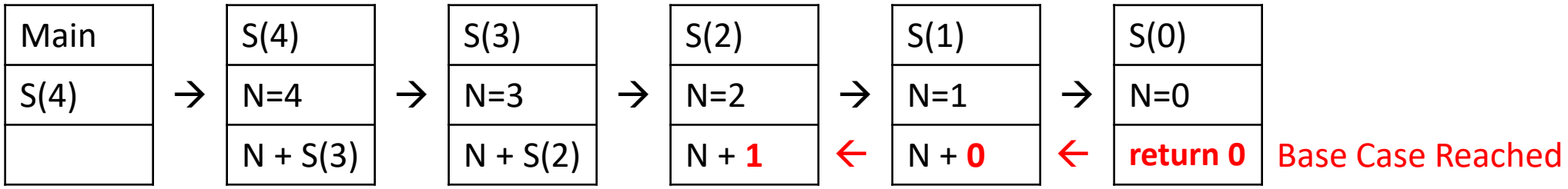
Box Method



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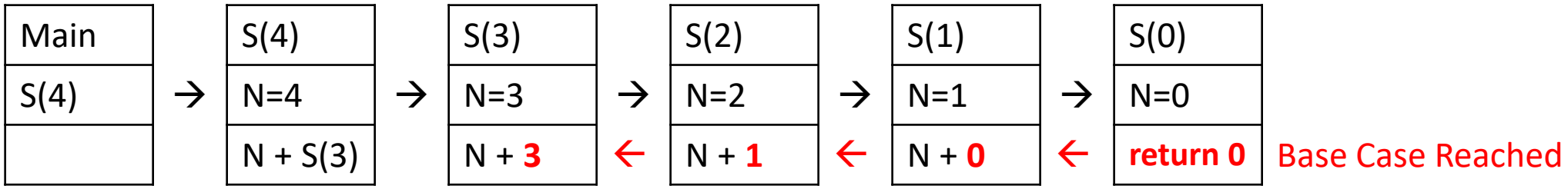
Box Method



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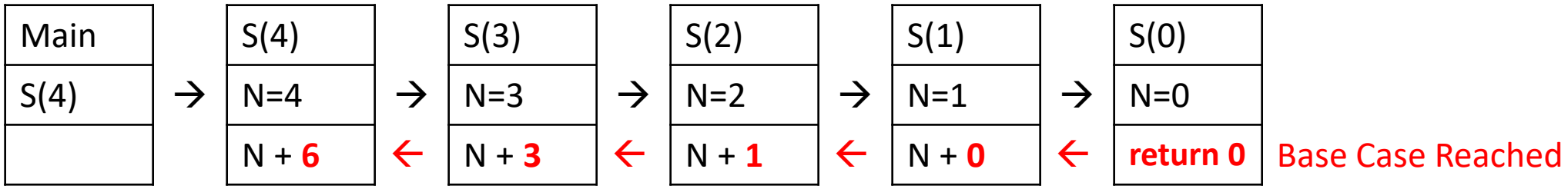
Box Method



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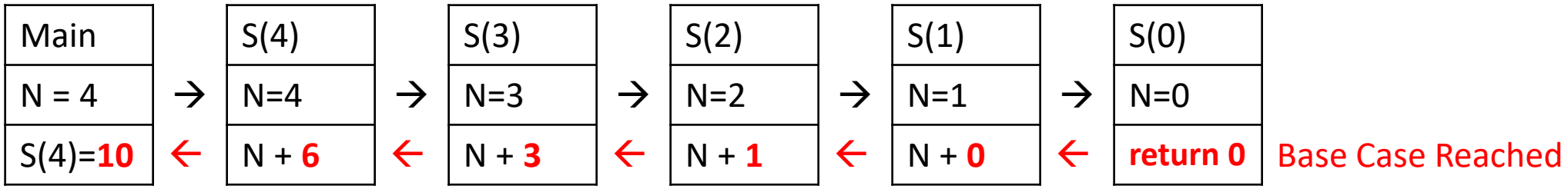
Box Method



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Summation Iterative

```
unsigned int isummation( unsigned int n )
{
    int i;
    unsigned int ans = 0;
    for( i=0; i<=n; i++)
        ans += i;
    return ans;
}
```

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You Try It at Home

- Given two integer number a and b where a is less than b and a is greater than or equal to 0. Sum the range from a to b.

Ie a = 3, b = 7

Sum(3,7) = 3 + 4 + 5 + 6 + 7 =25

Careful, Sum(3,3) = 3

To get started:

```
unsigned int sumRange(unsigned int a, unsigned int b );
```

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