CSC 215

Math and Computer Science

SOUTH DAKOTA
SCHOOL OF MINES
& TECHNOLOGY

N - Queens

- Problem
  - Given an N x N checkerboard
  - Position N queens on the board so that no queen can attach another queen

- Movements
  - Any amount of squares Horizontally
  - Any amount of squares Vertically
  - Any amount of squares Diagonally

SOUTH DAKOTA SCHOOL OF MINES & TECHNOLOGY

By Hand N=4

?			

Try Putting a Queen ?, yes move to next column

SOUTH DAKOTA
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By Hand N=4

Q	?		

Try Putting a Queen ?, No move down a row

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By Hand N=4

Q			
	?		

Try Putting a Queen ?, No move down a row

SOUTH DAKOTA
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& TECHNOLOGY

By Hand N=4

Q			
	?		

Try Putting a Queen ?, Yes move to next column

SOUTH DAKOTA
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By Hand N=4

Q		?	
	Q		

Try Putting a Queen ?, No

SOUTH DAKOTA
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By Hand N=4

Q			
		?	
	Q		

Try Putting a Queen ?, No

SOUTH DAKOTA
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& TECHNOLOGY

By Hand N=4

Q			
	Q	?	

Try Putting a Queen ?, No

SOUTH DAKOTA
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By Hand N=4

Q			
	Q		
		?	

Try Putting a Queen ?, No, no more rows back up one column

SOUTH DAKOTA
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By Hand N=4

Q			
	?		

Try Putting a Queen ?, Yes, move to next column

SOUTH DAKOTA
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& TECHNOLOGY

By Hand N=4

Q		?	
	Q		

Try Putting a Queen ?, No

SOUTH DAKOTA
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& TECHNOLOGY

By Hand N=4

Q			
		?	
	Q		

Try Putting a Queen ?, Yes move to next column

SOUTH DAKOTA
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By Hand N=4

Q			?
		Q	
	Q		

Try Putting a Queen ?, No

SOUTH DAKOTA
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By Hand N=4

Q			
		Q	?
	Q		

Try Putting a Queen ?, No

SOUTH DAKOTA
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By Hand N=4

Q			
		Q	
			?
	Q		

Try Putting a Queen ?, No

SOUTH DAKOTA
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By Hand N=4

Q			
		Q	
	Q		?

Try Putting a Queen ?, No, back up one column

SOUTH DAKOTA
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& TECHNOLOGY

By Hand N=4

Q			
		?	
	Q		

Try Putting a Queen ?, No

SOUTH DAKOTA
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& TECHNOLOGY

By Hand N=4

Q			
	Q	?	

Try Putting a Queen ?, No back up one column

SOUTH DAKOTA
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& TECHNOLOGY

By Hand N=4

Q			

Try Putting a Queen ?, No more rows, back up one column

SOUTH DAKOTA
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& TECHNOLOGY

By Hand N=4

?			

Try Putting a Queen ?, Yes, move to next column

SOUTH DAKOTA
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By Hand N=4

	?		
Q			

Try Putting a Queen ?, No

SOUTH DAKOTA
SCHOOL OF MINES
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By Hand N=4

Q	?		

Try Putting a Queen ?, No

SOUTH DAKOTA
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& TECHNOLOGY

By Hand N=4

Q			
	?		

Try Putting a Queen ?, No

SOUTH DAKOTA
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& TECHNOLOGY

By Hand N=4

Q			
	?		

Try Putting a Queen ?, Yes, move to next column

SOUTH DAKOTA
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& TECHNOLOGY

By Hand N=4

		?	
Q			
	Q		

Try Putting a Queen ?, Yes, move to next column

SOUTH DAKOTA
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By Hand N=4

		Q	?
Q			
	Q		

Try Putting a Queen ?, No

SOUTH DAKOTA
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By Hand N=4

		Q	
Q			?
	Q		

Try Putting a Queen ?, No

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By Hand N=4

		Q	
Q			
			?
	Q		

Try Putting a Queen ?, Yes move to next column

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By Hand N=4

		Q	
Q			
			Q
	Q		

Checkerboard has N Queens, it is a solution, back up one column and try finding more

By Hand N=4

		Q	
Q			
	Q		?

Try Putting a Queen ?, No, back up one column

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By Hand N=4

Q		?	
	Q		

Try Putting a Queen ?, No

SOUTH DAKOTA
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By Hand N=4

Q			
		?	
	Q		

Try Putting a Queen ?, No

SOUTH DAKOTA
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By Hand N=4

Q			
	Q	?	

Try Putting a Queen ?, No back up one column

SOUTH DAKOTA
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By Hand N=4

Q			

Try Putting a Queen ?, No more rows back up one column

SOUTH DAKOTA
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& TECHNOLOGY

By Hand N=4

?			

Try Putting a Queen ?, Yes, move to next column

SOUTH DAKOTA
SCHOOL OF MINES
& TECHNOLOGY

By Hand N=4

	?		
Q			

Try Putting a Queen ?, Yes, move to next column

SOUTH DAKOTA
SCHOOL OF MINES
& TECHNOLOGY

By Hand N=4

	Q	?	
Q			

Try Putting a Queen ?, No

SOUTH DAKOTA
SCHOOL OF MINES
& TECHNOLOGY

By Hand N=4

	Q		
		?	
Q			

Try Putting a Queen ?, No

SOUTH DAKOTA
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By Hand N=4

	Q		
Q		?	

Try Putting a Queen ?, No

SOUTH DAKOTA
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By Hand N=4

	Q		
Q			
		?	

Try Putting a Queen ?, Yes, move to next column

SOUTH DAKOTA
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By Hand N=4

	Q		?
Q			
		Q	

Try Putting a Queen ?, No

SOUTH DAKOTA
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By Hand N=4

	Q		
			?
Q			
		Q	

Try Putting a Queen ?, Yes move to next column

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By Hand N=4

	Q		
			Q
Q			
		Q	

Have N queens on the board, it is a solution. Back up and find more.

By Hand N=4

	Q		
Q			?
		Q	

Try Putting a Queen ?, No

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By Hand N=4

	Q		
Q			
		Q	?

Try Putting a Queen ?, No out of rows, back up one column

SOUTH DAKOTA
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By Hand N=4

	Q		
Q			

Try Putting a Queen ?, out of rows, back up one column

SOUTH DAKOTA
SCHOOL OF MINES
& TECHNOLOGY

By Hand N=4

	?		
Q			

Try Putting a Queen ?, No

SOUTH DAKOTA
SCHOOL OF MINES
& TECHNOLOGY

By Hand N=4

Q	?		

Try Putting a Queen ?, No

SOUTH DAKOTA
SCHOOL OF MINES
& TECHNOLOGY

By Hand N=4

Q			
	?		

Try Putting a Queen ?, No, back up one column

SOUTH DAKOTA
SCHOOL OF MINES
& TECHNOLOGY

By Hand N=4

?			

Try Putting a Queen ?, Yes move to next column

SOUTH DAKOTA
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& TECHNOLOGY

By Hand N=4

	?		
Q			

Try Putting a Queen ?, Yes move to next column

SOUTH DAKOTA
SCHOOL OF MINES
& TECHNOLOGY

By Hand N=4

	Q	?	
Q			

Try Putting a Queen ?, No

SOUTH DAKOTA
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& TECHNOLOGY

By Hand N=4

	Q		
		?	
Q			

Try Putting a Queen ?, No

SOUTH DAKOTA
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& TECHNOLOGY

By Hand N=4

	Q		
		?	
Q			

Try Putting a Queen ?, Yes, move to next column

SOUTH DAKOTA
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& TECHNOLOGY

By Hand N=4

	Q		?
		Q	
Q			

Try Putting a Queen ?, No

SOUTH DAKOTA
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& TECHNOLOGY

By Hand N=4

	Q		
			?
		Q	
Q			

Try Putting a Queen ?, No

SOUTH DAKOTA
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By Hand N=4

	Q		
		Q	?
Q			

Try Putting a Queen ?, No

SOUTH DAKOTA
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& TECHNOLOGY

By Hand N=4

	Q		
		Q	
Q			?

Try Putting a Queen ?, No back up one column

SOUTH DAKOTA
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& TECHNOLOGY

By Hand N=4

	Q		
Q		?	

Try Putting a Queen ?, No back up one column

SOUTH DAKOTA
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& TECHNOLOGY

By Hand N=4

	?		
Q			

Try Putting a Queen ?, Yes, move to next column

SOUTH DAKOTA
SCHOOL OF MINES
& TECHNOLOGY

By Hand N=4

		?	
	Q		
Q			

Try Putting a Queen ?, No

SOUTH DAKOTA
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& TECHNOLOGY

By Hand N=4

	Q	?	
Q			

Try Putting a Queen ?, No

SOUTH DAKOTA
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& TECHNOLOGY

By Hand N=4

	Q		
		?	
Q			

Try Putting a Queen ?, No

SOUTH DAKOTA
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& TECHNOLOGY

By Hand N=4

	Q		
Q		?	

Try Putting a Queen ?, No back up one column

SOUTH DAKOTA
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& TECHNOLOGY

By Hand N=4

	?		
Q			

Try Putting a Queen ?, No

SOUTH DAKOTA
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By Hand N=4

Q	?		

Try Putting a Queen ?, No, back up one column

SOUTH DAKOTA
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Solving it Recursively

- Let the recursion handle not placing more than 1 queen in a column
- Store the solution in a 1d array
- Keep track of what rows are being used in a 1d array
- Keep track of what positive diagonals are being used in a 1d array
- Keep track of what negative diagonals are being used in a 1d array
- N the size of the board
- Pos for what column is being filled

Solving it Recursively – the Rows

- Visually – Board

	Q		
Q			
		Q	

usedRow

1
0
1
1

- P array (solution)

2	0	3	

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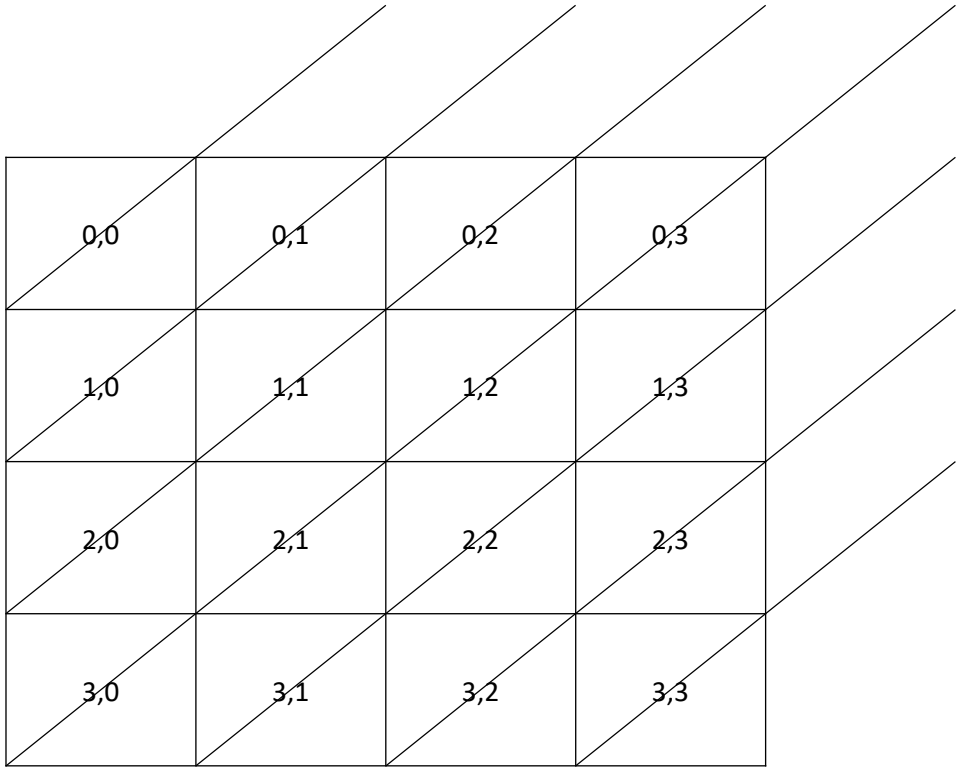
Solving it Recursively – Positive Diagonals



Looking for something to uniquely identify each diagonal.

Any thoughts?

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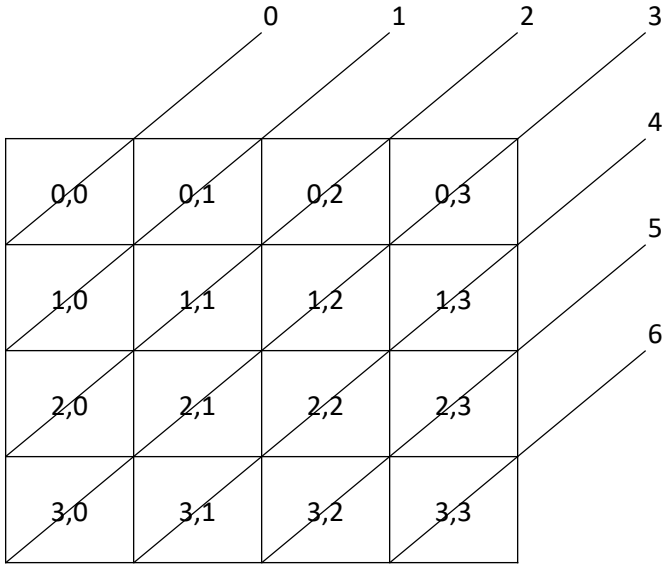
Solving it Recursively – Positive Diagonal



Add row and column together

0	0	0	0	0	0	0
0	1	2	3	4	5	6

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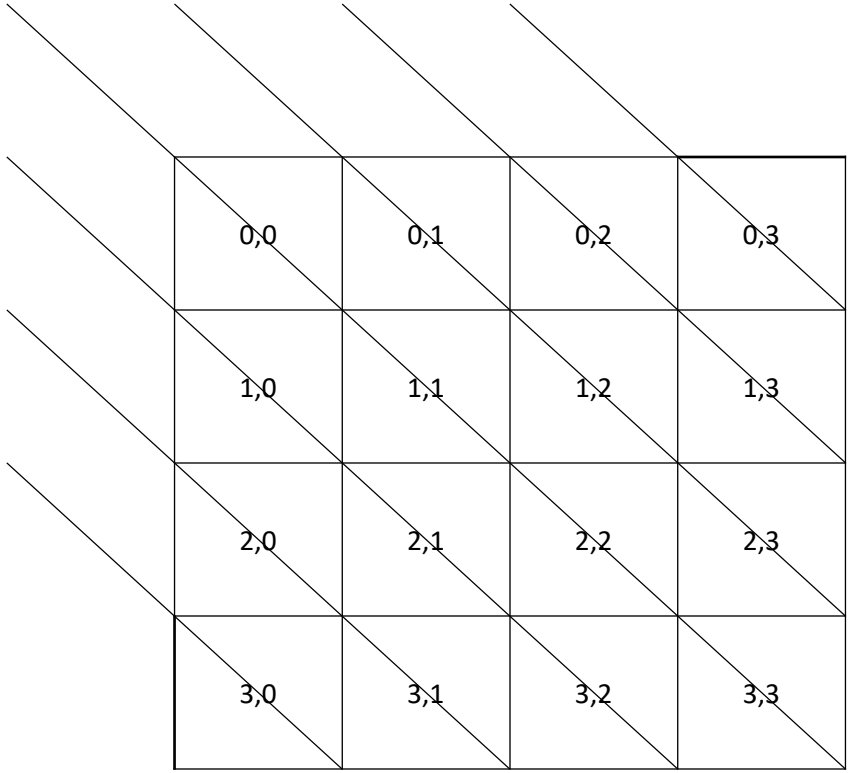
Solving it Recursively – Negative Diagonal



Any thoughts for the Negative Diagonal?

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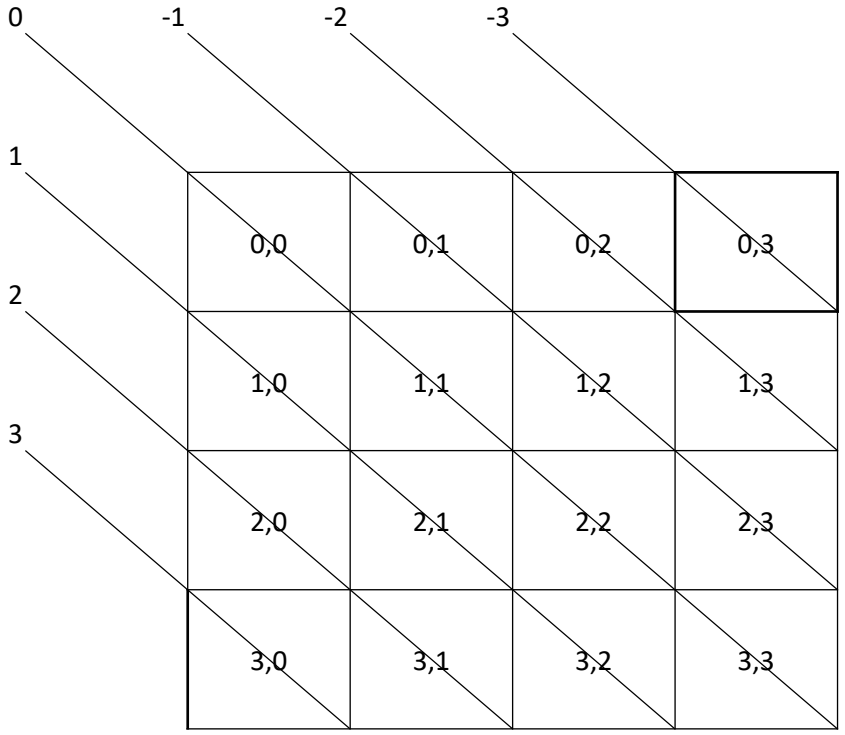
Solving it Recursively – Negative Diagonal



Row – Column, negatives are not valid indexes into an array

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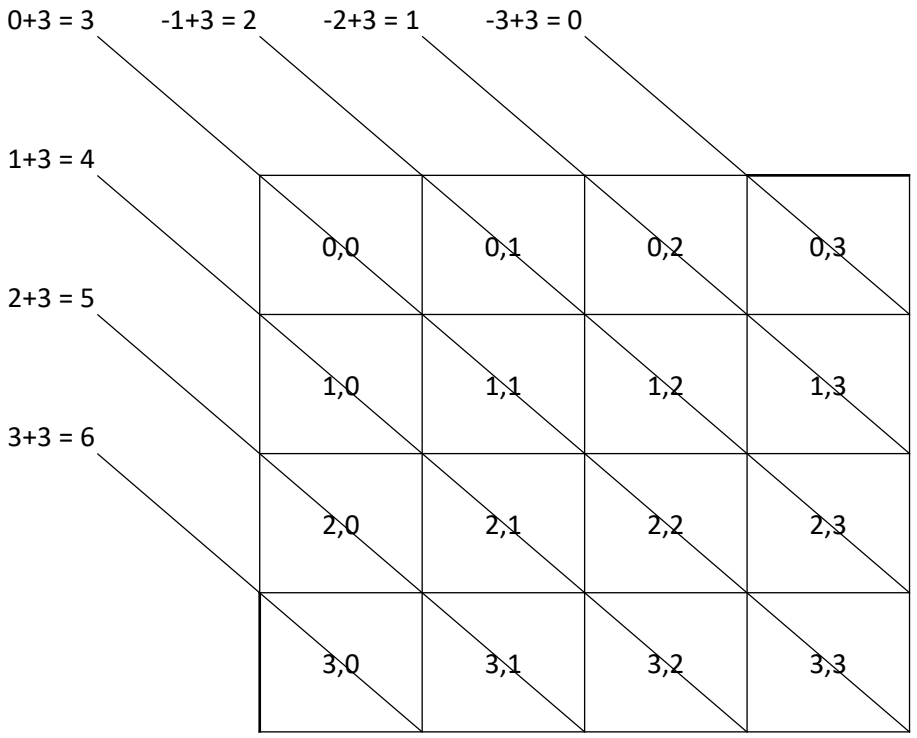
Solving it Recursively – Negative Diagonal



Row – Column + (n-1), unique numbers that are valid indexes into an array

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Solving it Recursively – Put it all together

0,0	Q	0,2	0,3
1,0	1,1	1,2	1,3
Q	2,1	2,2	2,3
3,0	3,1	Q	3,3

P	2	0	3	

Used Rows	1	0	1	1
	0	1	2	3

Used Pos	0	1	1	0	0	1	0	(row + col)
	0	1	2	3	4	5	6	

Used Neg	0	0	1	0	1	1	0	(row – col + (n-1))

SOUTH DAKOTA SCHOOL OF MINES & TECHNOLOGY

Solving it Recursively – Put it all together

0,0	Q	0,2	?
1,0	1,1	1,2	1,3
Q	2,1	2,2	2,3
3,0	3,1	Q	3,3

Position (pos) is the column
For loop (i) is the row

Can I put queen at 0,3
Used[i] is 1 – No

P	2	0	3	

Used Rows	1	0	1	1
	0	1	2	3

Used Pos	0	1	1	0	0	1	0	(row + col)
	0	1	2	3	4	5	6	

Used Neg	0	0	1	0	1	1	0	(row – col + (n-1))

SOUTH DAKOTA SCHOOL OF MINES & TECHNOLOGY

Solving it Recursively – Put it all together

0,0	Q	0,2	0,3
1,0	1,1	1,2	?
Q	2,1	2,2	2,3
3,0	3,1	Q	3,3

P	2	0	3	

Used Rows	1	0	1	1			
	0	1	2	3			
Used Pos	0	1	1	0	0	1	0
	0	1	2	3	4	5	6
Used Neg	0	0	1	0	1	1	0

(row + col)

(row – col + (n-1))

Position (pos) is the column
For loop (i) is the row

Can I put queen at 1,3
- Used[i] is 0 – good so far
- UsedPos[I + pos] is 0 – good so far
- UsedNeg[I – pos + 3] is 0 – place the queen

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Solving it Recursively – Put it all together

0,0	Q	0,2	0,3
1,0	1,1	1,2	Q
Q	2,1	2,2	2,3
3,0	3,1	Q	3,3

Position (pos) is the column
For loop (i) is the row

Put Queen in solution and mark all used arrays
make recursive call

P	2	0	3	1

Used Rows	1	1	1	1			
	0	1	2	3			
Used Pos	0	1	1	0	1	1	0
	0	1	2	3	4	5	6
Used Neg	0	1	1	0	1	1	0

(row + col)

(row – col + (n-1))

SOUTH DAKOTA SCHOOL OF MINES & TECHNOLOGY

Solving it Recursively – Put it all together

0,0	Q	0,2	0,3
1,0	1,1	1,2	1,3
Q	2,1	2,2	?
3,0	3,1	Q	3,3

Position (pos) is the column
For loop (i) is the row

Mark unused and try next position

P	2	0	3	

Used Rows	1	0	1	1
	0	1	2	3

Used Pos	0	1	1	0	0	1	0	(row + col)
	0	1	2	3	4	5	6	
Used Neg	0	0	1	0	1	1	0	(row – col + (n-1))

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