

Steven
Glasford
3-5-20
Linear
programming
Assignment 2

①

1) Minimize z

$$z = -4x_1 - x_2$$

s.t.

$$3x_1 + 6x_2 \leq 15$$

$$8x_1 + 2x_2 \leq 12$$

$$x_1, x_2 \geq 0$$

canonical form

$$z = -4x_1 - x_2 - 0x_3 - 0x_4$$

$$z + 4x_1 + x_2 + 0x_3 + 0x_4 = 0$$

$$3x_1 + 6x_2 + x_3 = 15$$

$$8x_1 + 2x_2 + 0x_3 + 0x_4 = 12$$

$$\begin{array}{c|cccc|c} 1 & 4 & 1 & 0 & 0 & 0 \\ 0 & 3 & 6 & 1 & 0 & 15 \\ \frac{1}{8} & 8 & 2 & 0 & 1 & 12 \end{array}$$

$$\begin{array}{c|cccc|c} 1 & 4 & 1 & 0 & 0 & 0 \\ 0 & 3 & 6 & 1 & 0 & 15 \\ 0 & 1 & \frac{1}{4} & 0 & \frac{1}{8} & \frac{3}{2} \end{array}$$

$$\begin{array}{c|cccc|c} 1 & 0 & 0 & 0 & -\frac{1}{2} & -6 \\ 0 & 0 & \frac{2}{4} & 1 & -\frac{3}{8} & \frac{2}{2} \\ 0 & 1 & \frac{1}{4} & 0 & \frac{1}{8} & \frac{3}{2} \end{array}$$

$$\frac{15}{3} = 5$$

$$\frac{12}{8} = 1.5$$

$$\begin{array}{c|cccc|c} 1 & 4 & 1 & 0 & 0 & 0 \\ 0 & 3 & 6 & 1 & 0 & 15 \\ 0 & 8 & 2 & 0 & 1 & 12 \end{array}$$

$$\begin{array}{c|cccc|c} 1 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & \frac{1}{3} & 0 & 3 \\ 0 & 8 & 2 & 0 & 1 & 12 \end{array}$$

$$\begin{array}{c|cccc|c} 1 & 0 & -2 & -\frac{4}{3} & 0 & -12 \\ 0 & 1 & 2 & \frac{1}{3} & 0 & 3 \\ 0 & 0 & -14 & -\frac{8}{3} & 1 & -12 \end{array}$$

$$\begin{array}{c|cccc|c}
 2 & 1 & 0 & 0 & 0 & -\frac{1}{2} & -6 \\
 \frac{4}{21} \rightarrow & 0 & 0 & \frac{21}{4} & 1 & -\frac{3}{8} & \frac{1}{2} \\
 & 0 & 1 & \frac{1}{4} & 0 & \frac{1}{8} & \frac{3}{2}
 \end{array}$$

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Alternative answer pivot
want to obtain a number for x_2

$$\begin{array}{c|cccc|c}
 & 1 & 0 & 0 & 0 & -\frac{1}{2} & -6 \\
 \frac{1}{4} \rightarrow & 0 & 1 & 1 & \frac{4}{21} & -\frac{1}{14} & 2 \\
 & 0 & 1 & \frac{1}{4} & 0 & \frac{1}{8} & \frac{3}{2}
 \end{array}$$

$$\begin{array}{c|cccc|c}
 & 1 & 0 & 0 & 0 & -\frac{1}{2} & -6 \\
 & 0 & 0 & 1 & \frac{4}{21} & -\frac{1}{14} & 2 \\
 & 0 & 1 & 0 & \frac{1}{21} & \frac{1}{7} & 1
 \end{array}$$

$$x_1 = 1, x_2 = 2, z_{\min} = -6$$

③ ~~minimize~~ ~~z~~
~~maximize~~ ~~z~~
 $z = x_1 + 2x_2 - x_3$

st

$$4x_2 + x_3 \leq 40$$

$$x_1 - x_2 \leq 20$$

$$2x_1 + 4x_2 + 3x_3 \leq 60$$

$$x_1, x_2, x_3 \geq 0$$

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$$z = x_1 + 2x_2 - x_3 + 0x_4 + 0x_5 + 0x_6$$

$$z - x_1 - 2x_2 + x_3 - 0x_4 - 0x_5 - 0x_6 = 0$$

$$0x_1 + 4x_2 + x_3 + x_4 = 40$$

$$x_1 - x_2 + x_5 = 20$$

$$2x_1 + 4x_2 + 3x_3 + x_6 = 60$$

$$x_1, \dots, x_6 \geq 0$$

	z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
1	1	-1	-2	+1	0	0	0	0
2	0	0	4	1	1	0	0	40
3	0	1	-1	0	0	1	0	20
4	0	2	4	3	0	0	1	60

$$40/4 = 10$$

$$2/1 = 2$$

$$60/2 = 30$$

2	1	-1	-2	1	0	0	0	0
1	0	0	1	1/4	1/4	0	0	10
3	0	1	-1	0	0	1	0	20
4	0	2	4	3	0	0	1	60

$$\begin{array}{c}
 (4) \\
 \begin{array}{ccccccc|c}
 1 & -1 & 0 & \frac{3}{2} & \frac{1}{2} & 0 & 0 & 20 \\
 0 & 0 & 1 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 10 \\
 0 & 1 & 0 & \frac{1}{4} & \frac{1}{4} & 1 & 0 & 31 \\
 \frac{1}{2} & 0 & 0 & 2 & -1 & 0 & 1 & 20
 \end{array}
 \end{array}$$

$$\begin{array}{l}
 31/1 = 31 \\
 20/2 = 10
 \end{array}$$

$$\begin{array}{c}
 \begin{array}{ccccccc|c}
 1 & -1 & 0 & \frac{3}{2} & \frac{1}{2} & 0 & 0 & 20 \\
 0 & 0 & 1 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 10 \\
 0 & 1 & 0 & \frac{1}{4} & \frac{1}{4} & 1 & 0 & 31 \\
 0 & 1 & 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 10
 \end{array}
 \end{array}$$

$$\begin{array}{c}
 \begin{array}{ccccccc|c}
 1 & 0 & 0 & \frac{5}{2} & 0 & 0 & 0 & 30 \\
 0 & 0 & 1 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 10 \\
 0 & 0 & 0 & -\frac{3}{4} & \frac{3}{4} & 1 & -\frac{1}{2} & 21 \\
 0 & 1 & 0 & 1 & -\frac{1}{2} & 0 & \frac{1}{2} & 10
 \end{array}
 \end{array}$$

$$X_1 = 10, X_2 = 10, X_3 = 0, Z_{\min} = 30$$

```
In [7]: import pulp as p
```

```
In [8]: # problem 1
Lp_prob1 = p.LpProblem('Problem', p.LpMinimize)

x1 = p.LpVariable('x1', lowBound = 0)
y1 = p.LpVariable('y1', lowBound = 0)

# Objective function
Lp_prob1 += -4*x1 - y1

# Constraints
Lp_prob1 += 3*x1 + 6*y1 <= 15
Lp_prob1 += 8*x1 + 2*y1 <= 12
Lp_prob1 += x1 >= 0
Lp_prob1 += y1 >= 0

print(Lp_prob1)
status = Lp_prob1.solve()
print(p.LpStatus[status])
print(p.value(x1), p.value(y1), p.value(Lp_prob1.objective))
```

```
Problem:
MINIMIZE
-4*x1 + -1*y1 + 0
SUBJECT TO
_C1: 3 x1 + 6 y1 <= 15
_C2: 8 x1 + 2 y1 <= 12
_C3: x1 >= 0
_C4: y1 >= 0

VARIABLES
x1 Continuous
y1 Continuous

Optimal
1.0 2.0 -6.0
```



```

In [9]: # Problem 2
Lp_prob2 = p.LpProblem('Problem', p.LpMaximize)

x2 = p.LpVariable('x2', lowBound = 0)
y2 = p.LpVariable('y2', lowBound = 0)
z2 = p.LpVariable('z2', lowBound = 0)

# Objective function
Lp_prob2 += x2 + 2*y2 - z

# Constraints
Lp_prob2 += 4*y2 + z2 <= 40
Lp_prob2 += x2 - y2 <= 20
Lp_prob2 += 2*x2 + 4*y2 + 3*z2 <= 60
Lp_prob2 += x2 >= 0
Lp_prob2 += y2 >= 0
Lp_prob2 += z2 >= 0

# Produce the output
print(Lp_prob2)
status = Lp_prob2.solve()
print(p.LpStatus[status])
print(p.value(x2), p.value(y2), p.value(z2), p.value(Lp_prob2.objective))

```

```

Problem:
MAXIMIZE
1*x2 + 2*y2 + -1*z + 0
SUBJECT TO
_C1: 4 y2 + z2 <= 40

_C2: x2 - y2 <= 20

_C3: 2 x2 + 4 y2 + 3 z2 <= 60

_C4: x2 >= 0

_C5: y2 >= 0

_C6: z2 >= 0

VARIABLES
x2 Continuous
y2 Continuous
z Continuous
z2 Continuous

Optimal
23.333333 3.3333333 0.0 29.9999996

```

In []: