$$\begin{cases} \frac{dx}{dt} = ax - bxy \\ \frac{dy}{dt} = -cy + dxy \end{cases}$$

(a) $\begin{cases} 0 = ax - bxy \\ 0 = -cy + dxy \end{cases} \overset{\text{if } x \neq 0}{\Rightarrow} \begin{cases} y = \frac{a}{b} \\ x = \frac{c}{d} \end{cases}$, if $x = 0$, $(\bar{x}, \bar{y}) = (0,0)$

There are two fixed points on $(\frac{c}{d}, \frac{a}{b})$ and $(0,0)$

(b)

$$J(x,y) = \begin{bmatrix} \partial f_1/\partial x & \partial f_1/\partial y \\ \partial f_2/\partial x & \partial f_2/\partial y \end{bmatrix} = \begin{bmatrix} a-by & -bx \\ dy & -c+dx \end{bmatrix}$$

① $J(0,0) = \begin{bmatrix} a & 0 \\ 0 & -c \end{bmatrix} \Rightarrow \det\left(\begin{bmatrix} a-\lambda & 0 \\ 0 & -c-\lambda \end{bmatrix}\right) = -ac - a\lambda + \lambda c + \lambda^2$

$= \lambda^2 + (c-a)\lambda - ac$

② $J(\frac{c}{d}, \frac{a}{b}) = \begin{bmatrix} 0 & \frac{-bc}{d} \\ \frac{ad}{b} & 0 \end{bmatrix} \Rightarrow \det\left(\begin{bmatrix} -\lambda & -bc/d \\ \frac{ad}{b} & -\lambda \end{bmatrix}\right) = \lambda^2 + \frac{bc}{d}\frac{ad}{b}$

$= \lambda^2 + ac$

Characteristic equation

① $\lambda^2 + (c-a)\lambda - ac = 0$

② $\lambda^2 + ac = 0$

$$\lambda = \frac{-(c-a) \pm \sqrt{(c-a)^2 + 4ac}}{2}$$

$$\lambda = \sqrt{ac}\, i, -\sqrt{ac}\, i$$

$$= \frac{-(c-a) \pm (c+a)}{2} = a, -c$$

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(C)

① $(\bar{x}, \bar{y}) = (\frac{c}{d}, \frac{a}{b})$

$\because a > 0 \wedge c > 0$

$\therefore (\bar{x}, \bar{y}) = (\frac{c}{d}, \frac{a}{b})$ is a saddle point

② $(\bar{x}, \bar{y}) = (0, 0)$

$\because \mathrm{Re}\{\lambda\} = 0,\ \mathrm{Im}\{\lambda\} \neq 0$

$\therefore$ The point is unstable. This system is oscillating with a period $W = \sqrt{\lambda_1 \lambda_2} = \sqrt{ac}$