Homework 10

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Descrition

- Course: STAT638, 2022Fall
- Deadline: 2022/10/29, 12:01 pm > Read Chapter 10 in the Hoff book

Source code is shown here: https://stchiu.quarto.pub/stat638___hw10/

Computational Environment

Libraries

```
cd(@__DIR__)
using Pkg
Pkg.activate("hw10")

using Statistics
using Distributions
```

```
using LinearAlgebra
using KernelDensity
using Plots
```

```
  Activating project at `~/Documents/GitHub/STAT638_Applied-Bayes-Methods/hw/hw10`
```

Version

```
Pkg.status()
VERSION
```

```
Status `~/Documents/GitHub/STAT638_Applied-Bayes-Methods/hw/hw10/Project.toml`
  [31c24e10] Distributions v0.25.79
  [5ab0869b] KernelDensity v0.6.5
  [91a5bcdd] Plots v1.36.3
  [10745b16] Statistics
```

```
v"1.8.2"
```

Problem HW10-1

> Assume we have 4 observations, $(-1, 0, 1, 10)$, where the last observation can be thought of as an outlier. Assume that conditional on an unknown parameter θ, the data area i.i.d. from some population distribution. Assume a standard normal prior for θ.

(1)

> First, assume that the population distribution is a normal distribution with mean θ and variance 1. Draw samples from the posterior of θ using a Metropolis algorithm and also derive the exact posterior in closed form.

Exact

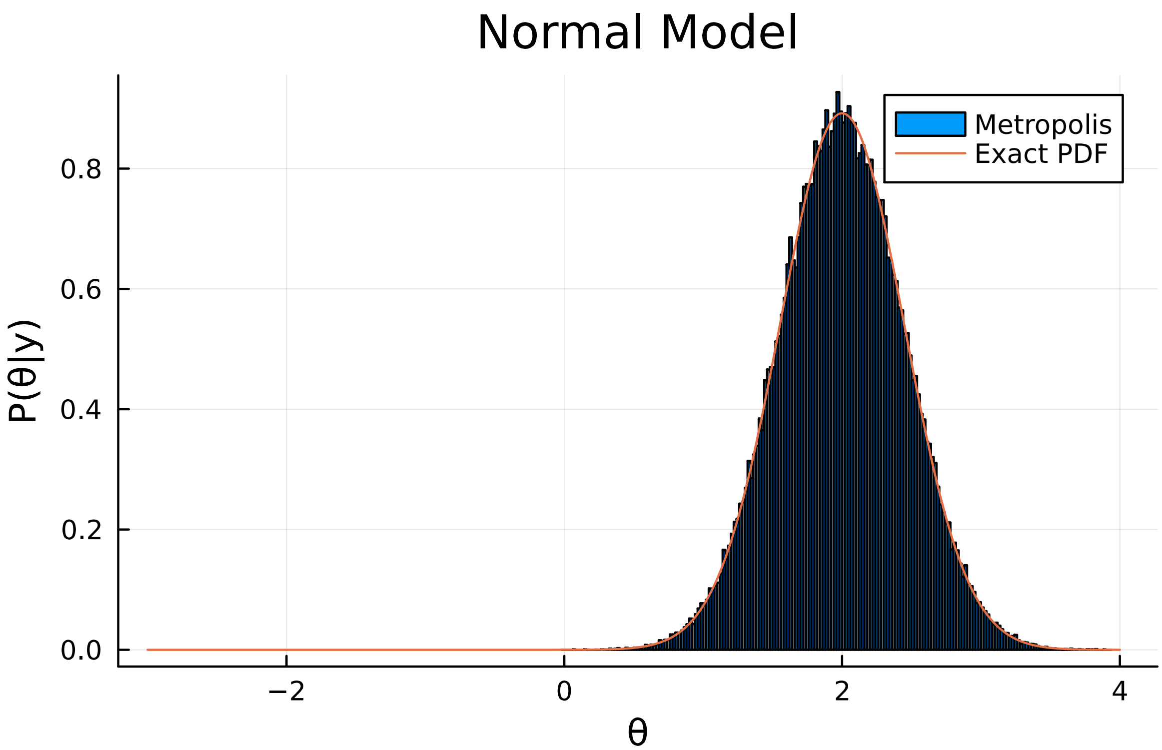
$$\theta \sim Normal(\mu = 0, \tau^2 = 1)$$
$$Y \sim Normal(\theta, \sigma^2 = 1)$$

$$P(\theta|y,\sigma^2) \sim Normal(\mu_n, \tau_n^2) \tag{1}$$

- $\mu_n = \bar{y}\frac{n/\sigma^2}{n/\sigma^2+1/\tau^2} + \mu\frac{1/\tau^2}{n/\sigma^2+1/\tau^2}$
- $\tau_n^2 = \frac{1}{n/\sigma^2+1/\tau^2}$

```
ys = [-1., 0., 1., 10.]



  = Normal()
k = 300000
 ² = 1.

function sampling_ma(like_dist,  , k,  ²)
     s = zeros(k)
    th = 0.
    for i in 1:k
          = th
        J = Normal( ,  ²)
          = rand(J)
        r_log = sum(logpdf.( like_dist( , 1), ys) .- logpdf.( like_dist( , 1), ys)) + logp

        # Accept
        u_log = log(rand(Uniform()))
        if r_log > u_log
             s[i] =
        else
             s[i] =
        end
        th =  s[i]
    end
    return  s
end

 s_n = sampling_ma(Normal,  , k,  ²)

# Exact PDF
n = length(ys)
  =( 1 / (n/ 1 + 1/ 1)^0.5)
  = mean(ys) * (n / 1) / (n/ 1 + 1/1) + 0
```

```
 _exact = Normal( ,   )
xs = collect(-3:0.01:4)
ys_exact = pdf.( _exact, xs)

# Display
p = histogram( s_n, normalize=:pdf, xlabel=" ", ylabel="P( |y)", label="Metropolis", title=
plot!(p, xs, ys_exact, label="Exact PDF")
```

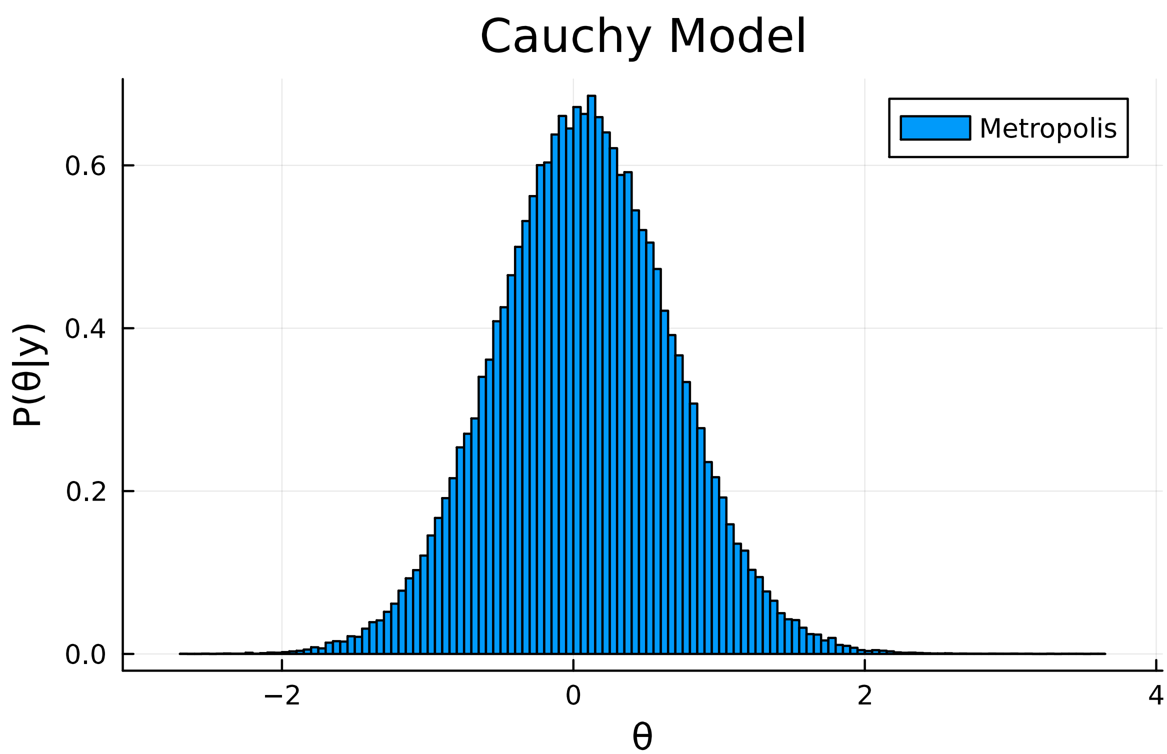


(2)

Now assume the population distribution is a Cauchy distribution with location parameter θ and scale 1. (This is equivalent to a nonstandardized t distribution with one degree of freedom and location parameter θ.) Draw samples from the posterior using the Metropolis algorithm.

```
 s_c = sampling_ma(Cauchy,  , k,  ²)
p2 = histogram( s_c, normalize=:pdf, xlabel=" ", ylabel="P( |y)", label="Metropolis", title=
```

```julia
display(p2)
```



(3)

> Plot the exact posterior density from part 1, together with kernel density estimates from the two Metropolis samplers. Describe how the outlier has affected the posteriors.

Cauchy distribution as likelihood distribution is less sensitive to the outliers than Normal distribution.

```julia
Un = kde( s_n)
Uc = kde( s_c)
pdf_n = [pdf(Un, x) for x in xs]
pdf_c = [pdf(Uc, x) for x in xs]
p3 = plot(xlabel=" ", ylabel="P( |y)", title="Pop dist. of Cuachy and Normal")
plot!(p3, xs, pdf_n, label="Normal")
plot!(p3, xs, pdf_c, label="Cauchy")
plot!(p3, xs, ys_exact, label="Exact PDF (Normal)")
```

```
scatter!(p3, ys, zeros(length(ys)), label="data")
display(p3)
```