Computational Project 4

due: 11/02/17, in class

Last updated October 27, 2017.

Note: I strongly advise reading through the entire assignment before beginning. Part of the credit for this assignment will be based on efficiency. More credit will be given for solutions that require fewer function evaluations to calculate the integrals to the requested level of accuracy. Thus, plan ahead when choosing methods to implement in parts (1) and (2).

1. Write a MATLAB function that uses a Newton Cotes method to integrate any function given the following inputs: the function name, the integration range, and the number of intervals. It should output as an argument the numerical value of the integral. The function should stop with an error message if an inappropriate number of intervals are requested.

2. Demonstrate that your integration function actually works by making a plot of your routine's accuracy for integrating the following function using $N = N_{min}*[1\ 3\ 5\ 25\ 50\ 250\ 500]$ intervals over the range of $x = 3$ to 5,

   $$f(x) = \sin(x) + x^2 - 3.$$

   where N_{min} is the minimum number of intervals usable for your particular numerical integration method (i.e., 2 for Simpson's 1/3). For this part (and this part only!) you may use an analytical solution to estimate the accuracy of your numerical answer, just to help you with debugging this piece of your program.

3. Write a MATLAB function that performs adaptively integration using your code from part (1). The routine should take as input a user defined function, an integration range, and a desired absolute error tolerance. The output of the function should be the final numerical integral. At each iteration, the code should display* the numerical integral, its estimated absolute error, and the number of intervals being used.

   *Use the `fprintf` function to display floating point numbers to 10 significant digits.

4. The upward velocity of a rocket can be computed by the following formula:

   $$v = u \ln\left(\frac{m_0}{m_0 - qt}\right) - gt$$

   where v = upward velocity, u=velocity at which fuel is expelled relative to the rocket, m_0 = initial mass of the rocket at time t=0, q = fuel consumption rate, and g = downward acceleration of gravity (assumed constant = 9.8 m/s^2). If u = 1800 m/s, m_0 = 160,000 kg, and q = 2500 kg/s, determine how high the rocket will fly in 30

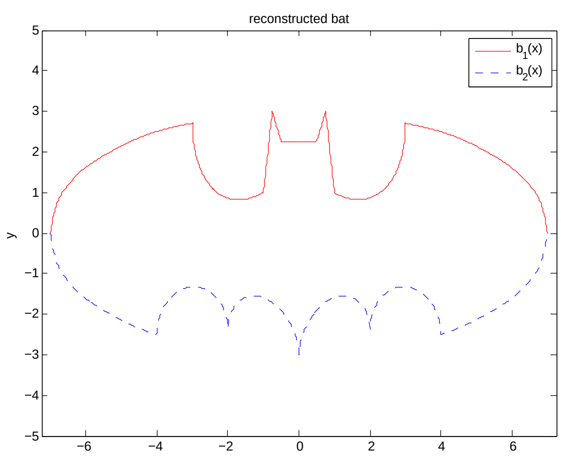
s. Find the answer accurate to the millimeter using your adaptive integration code from part (3). Note: you may integrate this by hand to check your answer, but your code should not use the true integral value as part of its solution method.

5. The overall goal of this problem is to find the area of the bat sign using numerical integration. The approach for finding the area of the bat sign will be to integrate two functions, $b_1(x)$ and $b_2(x)$, which evaluate the outline of the top and bottom halves of the sign. The total area is thus

$$I = \int_{-7}^{7} b_1(x)dx - \int_{-7}^{7} b_2(x)dx$$

Code for calculating b_1 and b_2 are provided on the course web site's homework page. Treat these codes like "black boxes", i.e., you can evaluate them for any x, but pretend you don't know anything about the underlying analytic relationships between $b_i(x)$ and x when solving this problem numerically (although feel free to look at the code if you are curious) *except* for the following: the symbol is symmetric about the y-axis, the x coordinate of the discontinuities in $b_1(x)$ are located at $x = \pm 0.5, \pm 0.75, \pm 1.0$ and ± 3.0, and the x coordinate of the discontinuities in $b_2(x)$ are located at $x = 0, \pm 2$ and ± 4. This information may or may not be useful to you, depending upon your approach.

Find the area of the bat sign accurate to within an absolute error of 0.01.



Remember to turn in both electronic and hard copies!